

1. (10%) Please write down Maxwell equations in the form of differential equations, and
 - (a). (5%) Explain the physical meaning of each equation.
 - (b). (5%) from Helmholtz's theorem, explain why only four equations are enough for $\vec{E}$ and $\vec{B}$
2. (20%) For discrete charge, the electric field is given by $\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{q_k(\vec{R} - \vec{R}_k)}{|\vec{R} - \vec{R}_k|^3}$, for a dipole located at z axis and separated by a small distance d, as shown in Fig. 1.
 - a). (10%) Shown that for $R \gg d$, with $\vec{p} \equiv q\vec{d}$, the electric field at point S can be written as $\vec{E} \cong \frac{p}{4\pi\epsilon_0 R^3} (\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta)$ (hint: use the vector identity $\hat{a}_z = \hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta$, when you transform from Cartesian to spherical coordinate)
 - b). (5%) From the definition of electric potential $V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{q_k}{|\vec{R} - \vec{R}_k|}$, please calculate the electric potential at point S due to the dipole shown in Fig. 1.
 - c). (5%) Used $\vec{E} = -\nabla V$, and $\nabla V = \hat{a}_R \frac{\partial V}{\partial R} + \hat{a}_\theta \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{a}_\phi \frac{1}{R \sin\theta} \frac{\partial V}{\partial \phi}$, shown that one will get the same result as in (a).

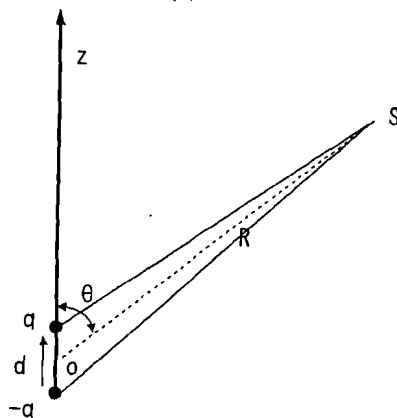


Figure 1: A Electric dipole

3. (10%) Please answer the following questions:
 - (a). What are the units for the electric and magnetic fields under the SI units system? (2%)
 - (b). What is the definition of the magnetic flux? (2%)
 - (c). What is the definition of the inductance? (2%)
 - (d). What is the definition of the mutual inductance? (2%)
 - (e). How to calculate the static magnetic energy stored in an inductor with the inductance=L and the static electric current=I ? (2%)
4. (15%) Let us consider a dielectric medium with a one-dimensional spatially varying

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permittivity constant $\epsilon(z)$ and a one-dimensional spatially varying permeability constant $\mu(z)$.

(a). (5%) Please write down the four time-varying Maxwell's equations for such a dielectric medium.

(b). (10%) Now consider the special case when the amplitude of the time-varying electric field at the harmonic frequency ω is constant in the x- and y-directions, its phase is with a propagation constant k_y in the y-direction, and its polarization is in the

x-direction: $\vec{E}(x, y, z, t) = \text{Re}[E(z)e^{-jk_y y + j\omega t} \vec{a}_x]$. Here $\vec{a}_x$ is the unit vector along

the x-direction. Please derive a single propagation equation for $E(z)$ from the general Maxwell's equations.

5. (10%) Let us consider an artificial medium with $\epsilon = \epsilon_1(1 - js)$ and $\mu = \mu_1(1 - js)$. Here $s \ll 1$ is a positive real number and ϵ_1, μ_1 are also both positive real. If a plane wave with unit power density at the frequency ω is normal incident on this artificial medium (with a planar interface) from left to right and if the medium in the left (incident) side is with $\epsilon = \epsilon_1$ and $\mu = \mu_1$,

(a). (5%) Please determine the power density of the reflected plane wave.

(b). (5%) Please determine the power density of the transmitted plane wave after its propagating inside the artificial medium with a distance L.

6. (25%) Circular waveguide

For a metallic circular waveguide:

(a) (5%) A 20(GHz) signal is to be transmitted inside a hollow circular conducting pipe as operated for TM_{11} mode. Please determine the inside diameter of the pipe.

(b) (5%) What waveguide modes can be propagating in the pipe?

TM Mode:

$$E_z^0 = C_n J_n(hr) \cos n\phi$$

$$E_r^0 = -\frac{j\beta}{h} C_n J'_n(hr) \cos n\phi$$

$$E_\phi^0 = \frac{j\beta n}{h^2 r} C_n J_n(hr) \sin n\phi$$

$$H_r^0 = -\frac{j\omega\epsilon n}{h^2 r} C_n J_n(hr) \sin n\phi$$

$$H_\phi^0 = -\frac{j\omega\epsilon}{h} C_n J'_n(hr) \cos n\phi$$

$$H_z^0 = 0$$

TE Mode:

$$H_z^0 = C'_n J_n(hr) \cos n\phi$$

$$H_r^0 = -\frac{j\beta}{h} C'_n J'_n(hr) \cos n\phi$$

$$H_\phi^0 = \frac{j\beta n}{h^2 r} C'_n J_n(hr) \sin n\phi$$

$$E_r^0 = \frac{j\omega\mu n}{h^2 r} C'_n J_n(hr) \sin n\phi$$

$$E_\phi^0 = -\frac{j\omega\mu}{h} C'_n J'_n(hr) \cos n\phi$$

$$E_z^0 = 0$$

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Zeros of $J_n(x)$, x_{np}

$\begin{matrix} n \\ p \end{matrix}$	$n=0$	$n=1$	$n=2$
1	2.405	3.832	5.136
2	5.520	7.016	8.417

Zeros of $J'_n(x)$, x'_{np}

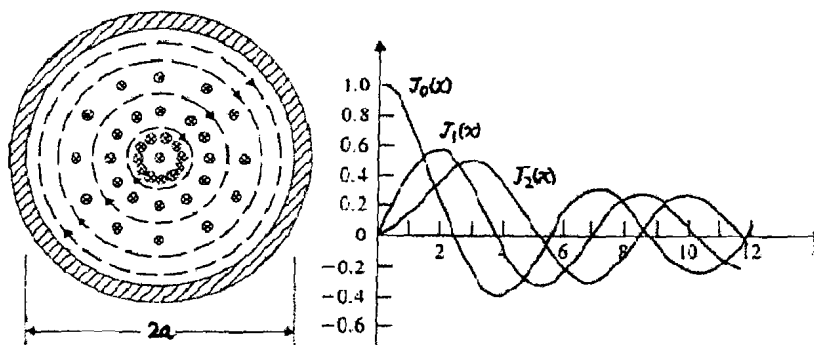
$\begin{matrix} n \\ p \end{matrix}$	$n=0$	$n=1$	$n=2$
1	3.832	1.841	3.054
2	7.016	5.331	6.706

- (c) (15%) If we make it as a resonator, and operate at TM_{010} mode (equation are listed), what kind of excitation will be more suitable, probe or loop? Why? and Where is the appropriate location for the excitation.

TM_{010}

$$E_z = C_0 J_0(hr) = C_0 J_0\left(\frac{2.405}{a} r\right)$$

$$H_\phi = -\frac{jC_0}{\eta_0} J'_0(hr) = \frac{jC_0}{\eta_0} J_1\left(\frac{2.405}{a} r\right)$$



Transverse section field pattern of TM_{010} mode in a circular cylindrical cavity resonator

Bessel function of the first kind

7. (10%) Impedance matching of Transmission line

(a) (5%) Please explain why does transmission line need impedance matching?

(b) (5%) A $100\ \Omega$ transmission line is connected to three loads as shown below. Each load consists of a quarter-wavelength transmission line and resistive termination. Please find the characteristic impedance Z_1 , Z_2 , and Z_3 such that the loads receive equal power with in-phase voltage.

