`Chapter 7 Forces in Beams and Cables

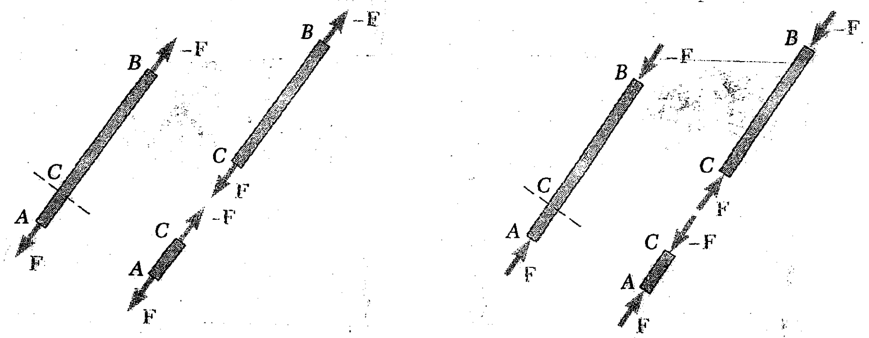
7.1 Introduction

beams : long, straight prismatic members

cables : flexible members, tension only

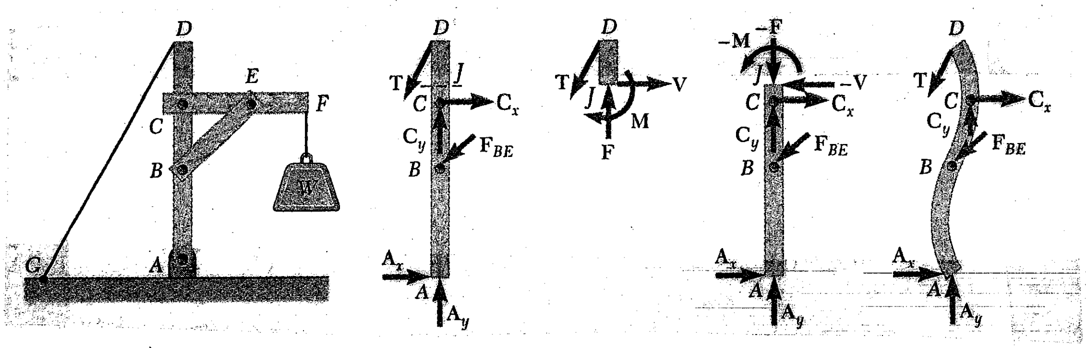
7.2 Internal Force in Members

consider a straight two-force member AB



in the case of straight two-force member, the internal force on each part of the member are equivalent to an axial force

next, consider a multiforce member



consider the free body DJ

F = vertical component of T

V = horizontal component of T

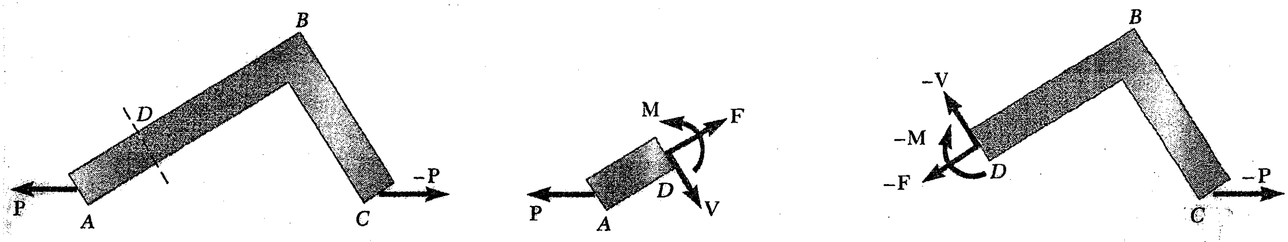
M = moment of T about J

it clearly that the action of internal force in member AD is not limited to producing tension or compression, it also produce shear and bending the force F is called an axial force

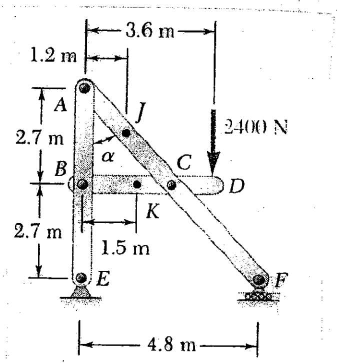
the force V is called a shearing force

the moment M is known as the bending moment at J

a two-force member which is not straight, the internal forces are also equivalent to a force-couple system

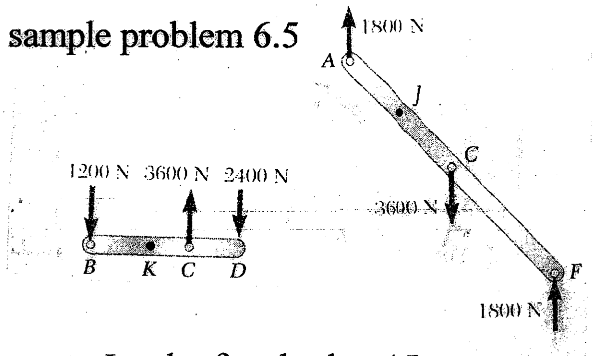


Sample Problem 7.1

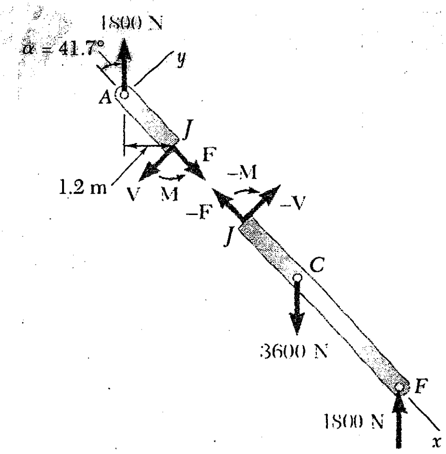


determine the internal forces at points J and K

the reactions and forces acting on each members are determined in sample problem 6.5



(a) internal forces at J, take free body AJ

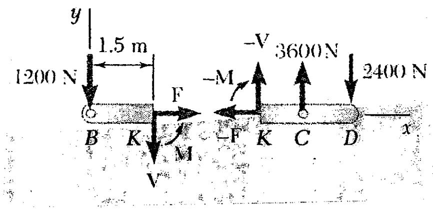


$\Sigma F_x = 0 \qquad F = 1800 \cos \alpha = 1344 \text{ N} \searrow$

$\Sigma F_y = 0 \qquad V = 1800 \sin \alpha = 1197 \text{ N} \swarrow$

$\Sigma M_J = 0 \qquad M = 1800 \times 1.2 = 2160 \text{ N-m}$

(b) internal forces at K, take free body BK



$\Sigma F_x = 0 \qquad F = 0$

$\Sigma F_y = 0 \qquad V = 1200 \text{ N}\uparrow$

$\Sigma M_K = 0 \qquad M = 1200 \times 1.5 = 1800 \text{ N-m}$

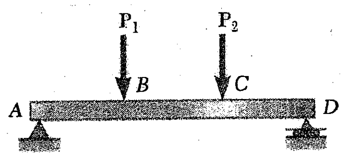
Beams

7.3 Various Types of Loading and Support

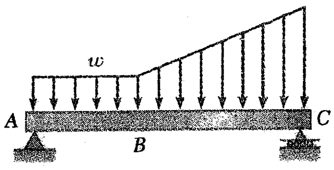
in most cases, the loads are perpendicular to the axis of the beam and will cause only shear and bending in the beam

a beam may be subjected to concentrated loads P_1, P_2 ··· (expressed in Newton), and distributed load ω (expressed in N/m), or combination of both

when the load z per unit length has a constant value, the load is said to be uniform distributed



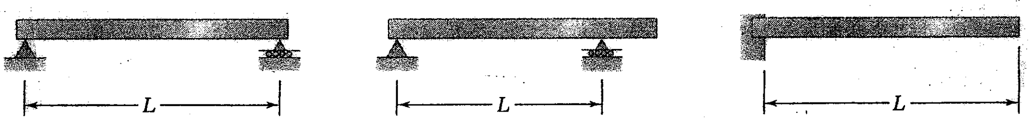
(*a*) Concentrated loads



(*b*) Distributed load

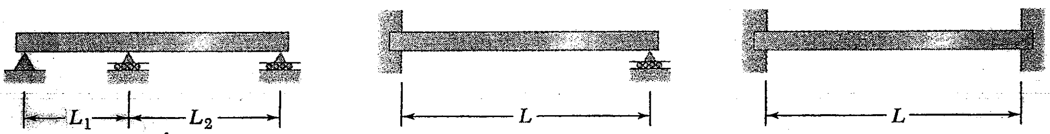
beams are classified according to the way in which they are supported

statically determinate beam



(*a*) Simply supported beam (*b*) Overhanging beam (*c*) Cantilever beam

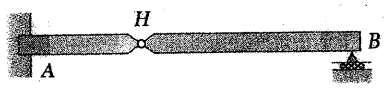
statically indeterminate beam



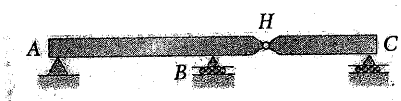
(*d*) Continuous beam (*e*) Beam fixed at one end and simply supported at the other end (*f*) Fixed beam

the distance L between supports is called the span

two or more beams are connected by hinges to form a single continuous structure is called combined beam

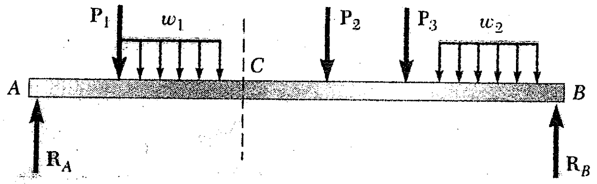


(*a*)



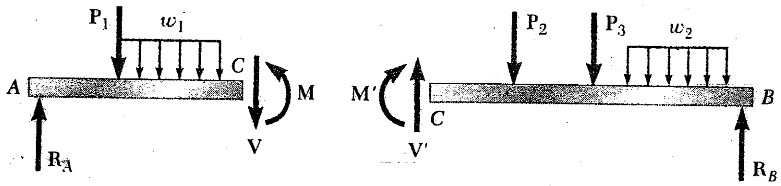
7.4 Shear and Bending Moment in a Beam

consider a beam subjected to various concentrated and distributed loads, determine the reactions R_A and R_B first



to determine the internal force at C,

take free body AC

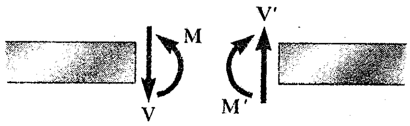


$\Sigma F_y = 0$ to determine V

$\Sigma M_C = 0$ to determine M

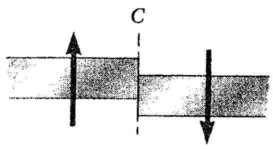
V and V', M and M' have same magnitude, but opposite direction

the shear force V and the bending moment M at a given point are said to be positive when the internal forces and couples acting on each portion of the beam are directed as shown



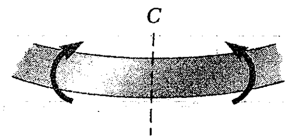
(a) Internal forces at section (positive shear and positive bending moment)

1. the shear at C is positive when the external forces acting on the beam tend to shear off the beam at C as indicated



(b) Effect of external forces (positive shear)

2. the moment at C is positive when the external forces acting on the beam tend to bend the beam at C as indicated

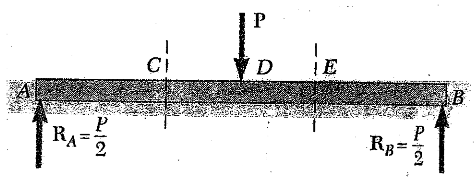


(c) Effect of external forces (positive bending moment)

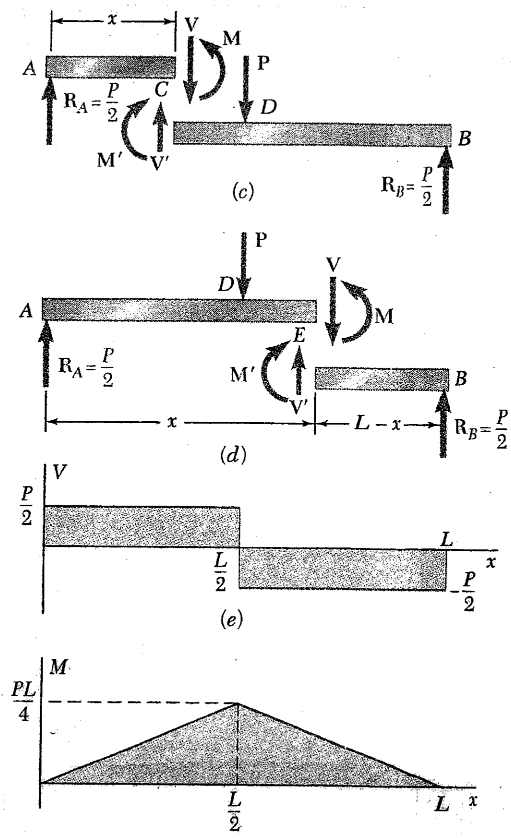
7.5 Shear and Bending-Moment Diagrams

record the values of shear and bending moment at any point of a beam by plotting these values against the distance x measured from the beam are called the shear force diagram and bending moment diagram, respectively

consider the beam AB, cut the beam at point C between A and D



$V = P/2 \qquad M = Px/2$



V and M are positive at point C

cut the beam at point E between D and B

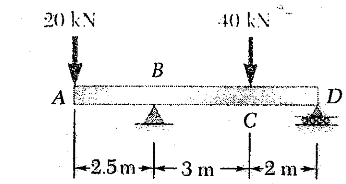
$$V = -P/2 \qquad M = P(L-x)/2$$

V is negative and M is positive

the shear-force diagram and bending-moment diagram can be plotted

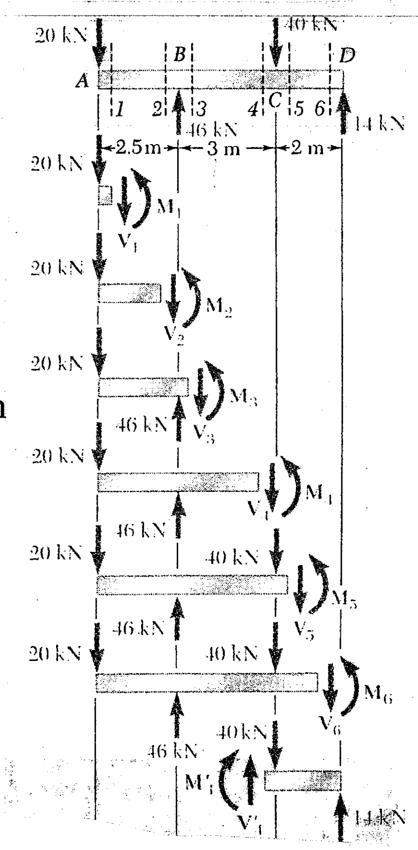
when the beam subjected only to concentrated load, the shear force is of constant value between loads and the bending moment varies linearly between loads

Sample Problem 7.2



draw the V- and M- diagrams

$R_B = 46$ kN $\qquad R_D = 14$ kN



at section 1

$$V_1 + 20 = 0 \qquad V_1 = -20 \text{ kN}$$

$$M_1 = 20 \times 0 = 0$$

at section 2

$$V_2 + 20 = 0 \qquad V_2 = -20 \text{ kN}$$

$$M_2 + 20 \times 2.5 = 0 \qquad M_2 = -50 \text{ kN-m}$$

at section 3 (in a similarly way)

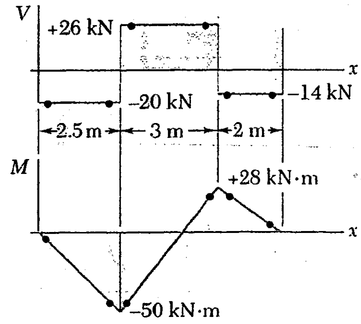
$$V_3 = 26 \text{ kN} \qquad M_3 = -50 \text{ kN-m}$$

at section 4

$$V_4 = 26 \text{ kN} \qquad M_4 = 28 \text{ kN-m}$$

at section 5

$$V_5 = -14 \text{ kN} \qquad M_5 = 28 \text{ kN-m}$$

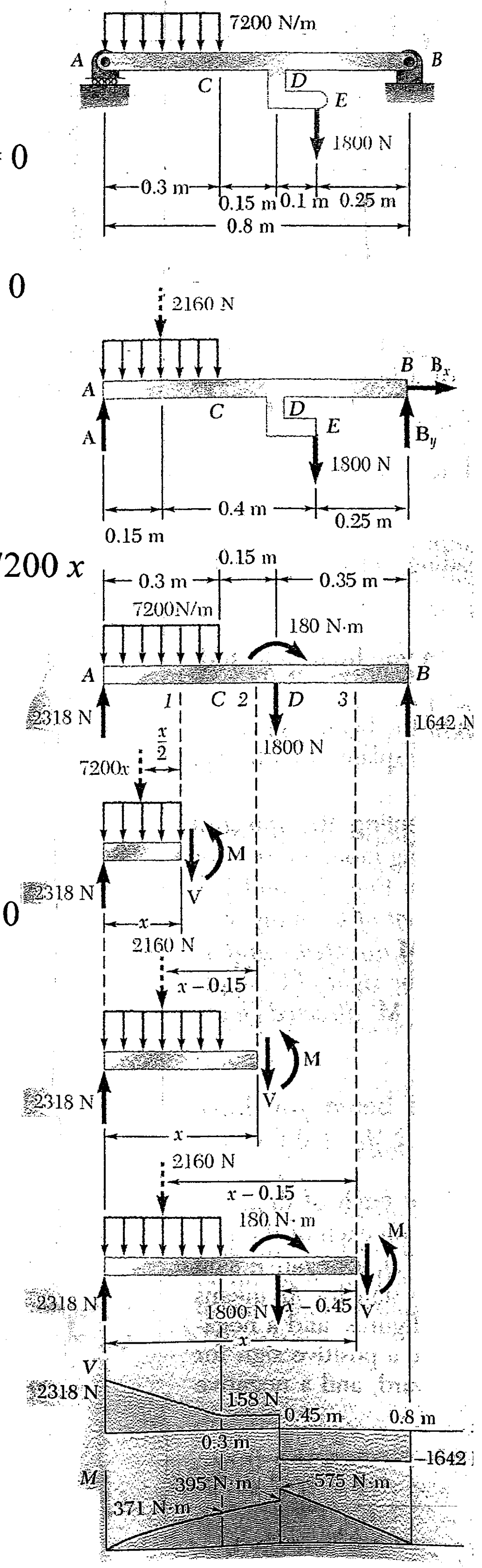


at section 6

$V_6 = -14$ kN $\qquad M_6 = 0$

the V-diagram and M-diagram can be plotted

Sample Problem 7.3



draw the V- and M- diagrams

$\Sigma M_A = 0 \quad B_y \times 0.8 - 2160 \times 0.15 - 1800 \times 0.55 = 0$

$B_y = 1642$ N $\uparrow$

$\Sigma M_B = 0 \quad 2160 \times 0.65 + 1800 \times 0.25 - A \times 0.8 = 0$

$A = 2318$ N $\uparrow$

$\Sigma F_x = 0 \quad B_x = 0$

from $A \sim C \quad 0 < x < 0.3$ m

$\Sigma F_y = 0 \quad 2318 - 7200\,x - V = 0 \qquad V = 2318 - 7200\,x$

$\Sigma M_1 = 0 \quad -2318\,x - 7200\,x\,(\tfrac{1}{2}x) + M = 0$

$M = 2318\,x - 3600\,x^2$

from $C \sim D \quad 0.3$ m $< x < 0.45$ m

$\Sigma F_y = 0 \quad 2318 - 2160 - V = 0 \qquad V = 158$ N

$\Sigma M_2 = 0 \quad -2318\,x - 2160 \times (x - 0.15) + M = 0$

$M = 324 + 158\,x$

from $D \sim B \quad 0.45$ m $< x < 0.8$ m

$\Sigma F_y = 0 \quad 2318 - 2160 - 1800 - V = 0$

$V = -1642$ N

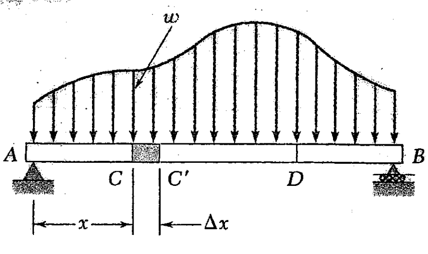
$\Sigma M_3 = 0 \quad -2318\,x - 2160\,(x - 0.15) - 180 +$

$1800\,(x - 0.45) + M = 0$

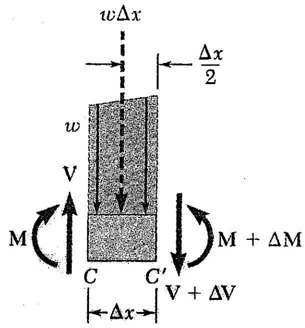
$M = 1314 - 1642\,x$

the V-diagram and M-diagram can be plotted

7.6 Relations among Load, Shear, and Bending Moment



(a)



consider a simply supported beam carrying a distributed load ω per unit length

relationship between load and shear

$$V - \omega \Delta x - (V + \Delta V) = 0$$

$$\Delta V = -\omega \Delta x$$

as $\Delta x \rightarrow 0$, it is obtained

$$dV / dx = -\omega$$

or $$dV = -\omega \, dx$$

integrating the above equation between points C and D

$$\int_{x_C}^{x_D} dV = -\int_{x_C}^{x_D} \omega \, dx$$

$$V_D - V_C = -\int_{x_C}^{x_D} \omega \, dx$$

i.e. $V_D - V_C$ = - (area under load curve between C and D)

the formula $dV / dx = -\omega$ is not valid at a point where the concentrated load is applies [$dV / dx = \infty$ for concentrated load, and the shear force has a sudden jump or drop at this point]

relationship between shear force and bending moment

$$\Sigma M_{C'} = 0$$

$$(M + \Delta M) - M - V \Delta x + \omega \Delta x (\Delta x/2) = 0$$

$$\Delta M / \Delta x = V - \omega \Delta x / 2$$

for $\Delta x \rightarrow 0$ $$dM / dx = V$$

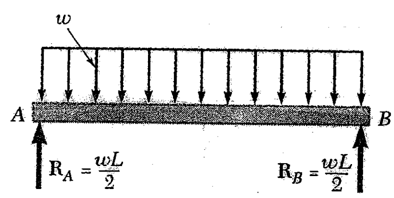
i.e. the slope dM/dx of the bending-moment curve is equal to the value of shear force at that point integrating the above equation between points C and D

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

or $\quad M_D - M_C =$ area under shear curve between C and D

[the area under shear force should be considered positive and negative for the shear force are positive and negative, respectively]

for a simply beam supported to uniform loading



$$R_A = R_B = \omega L / 2$$

$$V_A = R_A = \omega L / 2$$

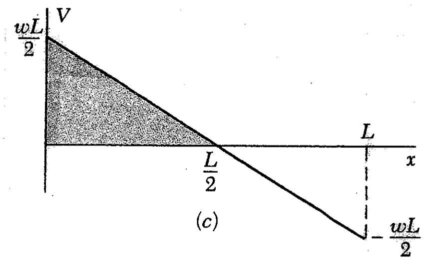
$$V - V_A = - \int_0^x \omega\,dx$$

$\therefore \quad V = V_A + \omega x = \omega L / 2 - \omega x = \omega\,(L/2 - x)$

$$M - M_A = \int_0^x V\,dx = \omega \int_0^x (L/2 - x)\,dx$$

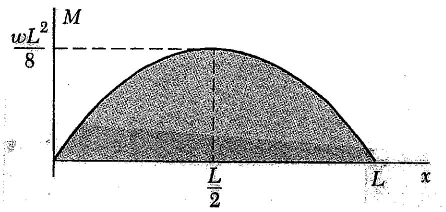
$\because \quad M_A = 0$

thus $\quad M = \frac{1}{2}\,\omega\,(Lx - x^2)$



the bending moment curve is a parabola

M_{max} occurs at $x = L/2$ since dM/dx (shear force) is equal zero at this point



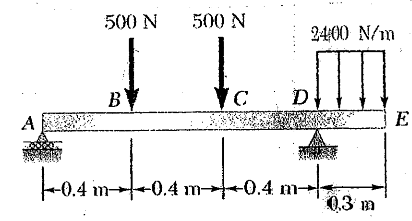
$$M_{max} = \frac{1}{2}\,\omega\,(L^2/2 - L^2/4) = \omega L^2 / 8$$

in most engineering application, the moment needs to be known only at a few specific points

loading $\rightarrow$ shear force diagram $\rightarrow$ some specific points ($V = 0$) $\rightarrow$ M_{max} or M_{min}

load curve : horizontal straight line (0 degree) $\rightarrow$ shear curve : oblique straight line (1st degree) $\rightarrow$ moment curve : parabola curve (2nd degree) load curve : oblique straight line (1st degree) $\rightarrow$ shear curve : parabola curve (2nd degree) $\rightarrow$ moment curve : cubic curve (3rd degree)

thus the shear and bending moment will always be, respectively, one and two degrees higher than the load curve

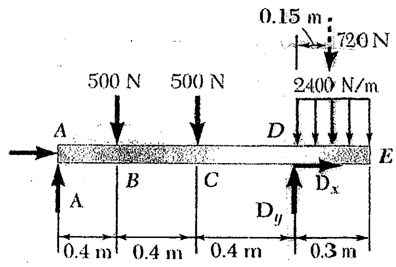


Sample Problem 7.4

draw the V-diagram and M-diagram for the beam

$\Sigma M_D = 0 \quad 500 \times 0.8 + 500 \times 0.4 - 720 \times 0.15 - A \times 0.12 = 0$

$A = 410 \text{ N} \uparrow$

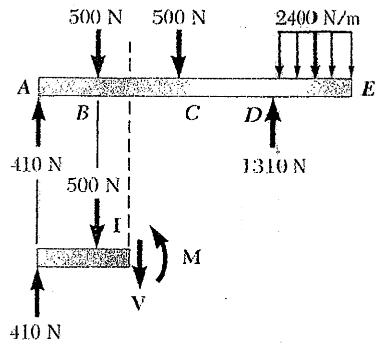


$\Sigma F_y = 0 \quad 410 - 500 - 500 - 720 + D_y = 0$

$D_y = 1310 \text{ N} \uparrow$

$\Sigma F_x = 0 \quad D_x = 0$

the shear-force diagram and bending-moment diagram can be plotted



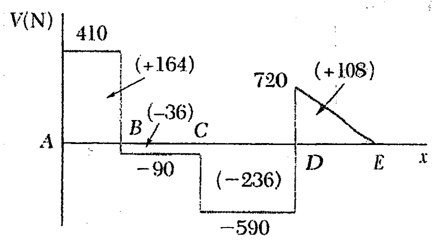
for the shear force-force diagram

$V_{BC} = 410 - 500 = -90 \text{ N}$

$V_{CD} = -90 - 500 = -590 \text{ N}$

for the bending-moment diagram $(M_A = 0)$

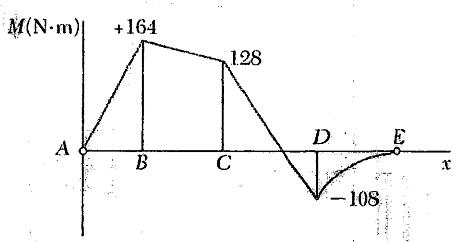
$M_B - M_A = 164$	$M_B = 164$ N-m
$M_C - M_B = -36$	$M_C = 128$ N-m
$M_D - M_C = -236$	$M_D = -108$ N-m
$M_E - M_D = 108$	$M_E = 0$



note that the slopes of moment diagram at points B, C and D are discontinuous because the shear forces are discontinuous at that points

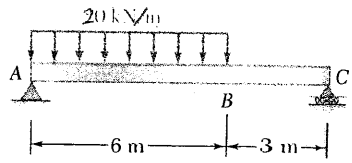
M_{max} at point B $(V = 0) = 164$ N-m

M_{min} at point D $(V = 0) = -108$ N-m

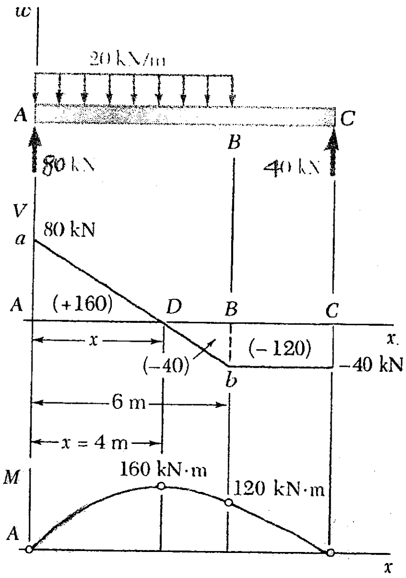


Sample Problem 7.5

draw the V- and M-diagrams and determine the location and magnitude of M_{max}



$R_A = 80$ kN $\qquad R_B = 40$ kN



$V_A = R_A = 80$ kN

$V_B = V_A - \omega \times 6 = 80 - 120 = -40$ kN

$x / 80 = (6 - x) / 40 \qquad x = 4$ m

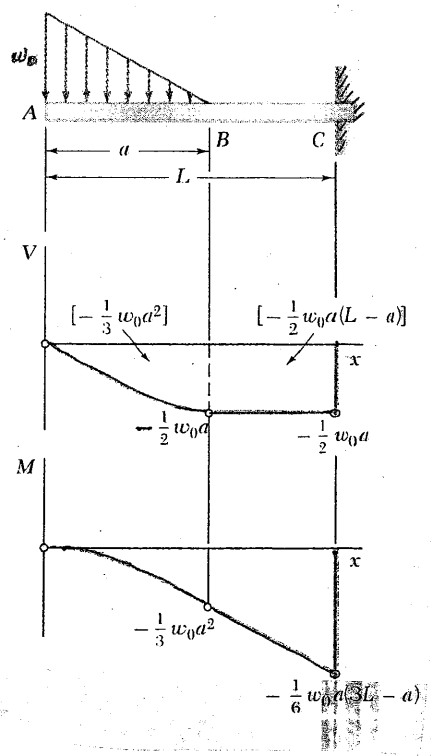
$M_D - M_A = 160 \qquad M_D = M_{max} = 160$ kN-m

$M_B - M_D = -40 \qquad M_B = 120$ kN-m

$M_C - M_B = -120 \qquad M_C = 0$

Sample Problem 7.6

sketch the shear and moment diagrams



$V_A = 0$

$V_B - V_A = \frac{1}{2} \omega_0 a \qquad V_B = -\frac{1}{2} \omega_0 a$

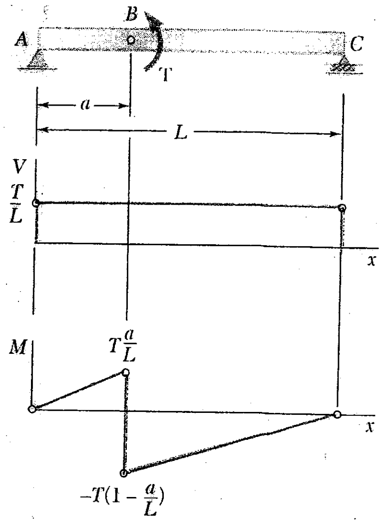
$V_C = V_B$

$M_A = 0$

$M_B - M_A = -\frac{1}{3} \omega_0 a^2 \qquad M_B = -\frac{1}{3} \omega_0 a^2$

$M_C - M_B = -\frac{1}{2} \omega_0 a (L - a)$

$M_C = -\omega_0 a (3L - a) / 6$



Sample Problem 7.7

draw the shear and moment diagrams for the beam

$R_A = T/L \uparrow \qquad R_B = T/L \downarrow$

for the bending moment diagram, a couple T is applied at B, the diagram is discontinuous at B, the bending moment decreases suddenly by an amount equal to T

Cables

7.7 Cables with Concentrated Loads

7.8 Cables with Distributed Loads

7.9 Parabolic Cable

7.10 Catenary