Chapter 5 Distributed Forces : Centroids and Centers of Gravity

5.1 Introduction

Actually, the earth exerts a force on each of particles forming the body, all the small forces may be replaced by a single equivalent force W which is applied at the center of gravity of the rigid body

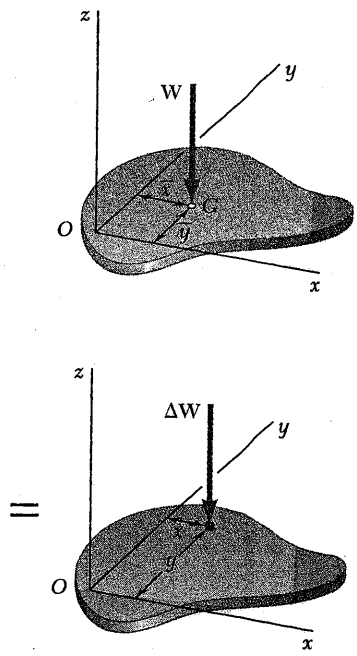
AREAS AND LINES

5.2 Center of Gravity of a Two-Dimensional Body

consider a flat horizontal plate, we divide the plate into n small elements

ΔW_1 at (x_1, y_1)

ΔW_2 at (x_2, y_2) $\quad\cdots\quad$ ΔW_n at (x_n, y_n)



then $\qquad W = \sum_{i=1}^{n} \Delta W_i$

ΣM_y : $\qquad \underline{x}\, W = \sum_{i=1}^{n} x_i \,\Delta W_i$

ΣM_x : $\qquad \underline{y}\, W = \sum_{i=1}^{n} y_i \,\Delta W_i$

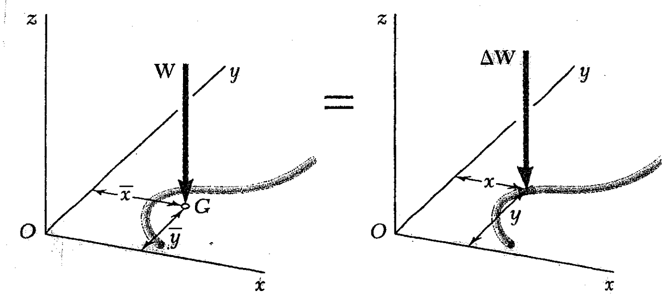
the point $G(\underline{x}, \underline{y})$ is the position of the center of gravity of the plate

increase the number of elements and decrease the size of the elements, then

$$W = \int_V dW$$

$$\underline{x}\, W = \int_V x\, dW \qquad\qquad \underline{y}\, W = \int_V y\, dW$$

for a wire lying in x-y plane, the center of gravity can be determine by the same equation



in general, $G(\bar{x}, \bar{y})$ is not located on the wire

5.3 Centroids of Areas and Lines

in the case of a homogeneous plate of uniform thickness

$$\Delta W = \rho\, g\, t\, \Delta A$$

ρ : density of material $\qquad$ g : acceleration of gravity

t : thickness $\qquad$ ΔA : area of element

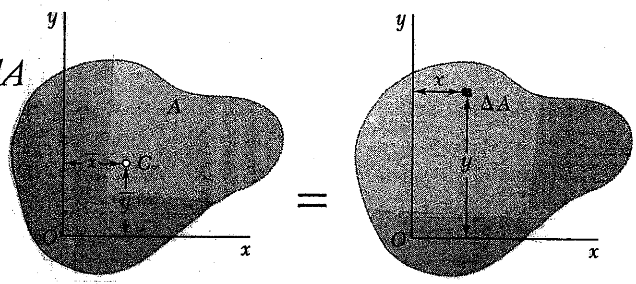
then $\qquad W = \gamma\, t\, A$

ΣM_y : $\qquad \bar{x}\, W = \int x\, dW = \int x\, \gamma\, t\, dA$

$$\bar{x}\, \gamma\, t\, A = \gamma\, t \int x\, dA$$

$$\bar{x}\, A = \int x\, dA$$

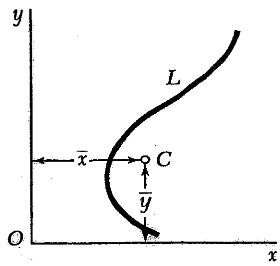
similarly $\quad \Sigma M_x$: $\qquad \bar{y}\, A = \int y\, dA$



then the centroid $(\bar{x}, \bar{y})$ is the same as the center of gravity, the integral $\int x\, dA$ and $\int y\, dA$ are called the first moment of area A with respect to y and x axes, respectively

if the plate is not homogeneous, the above equations cannot used to determine the center of gravity

in the case of homogeneous wire of uniform cross section

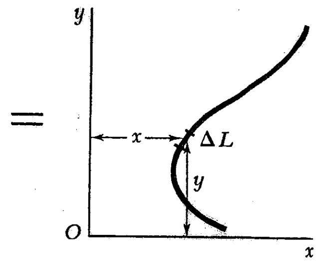


$$\Delta W = \rho\, g\, a\, \Delta L$$

a : cross-sectional area of the wire $\qquad$ ΔL : length of element

similarly as above

$$\bar{x}\, L = \int x\, dL \qquad\qquad \bar{y}\, L = \int y\, dL$$



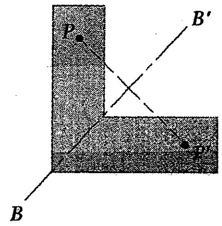
5.4 First Moments of Areas and Lines

define Q_x, Q_y the first moment of area A with respect to y and x axes, respectively, then

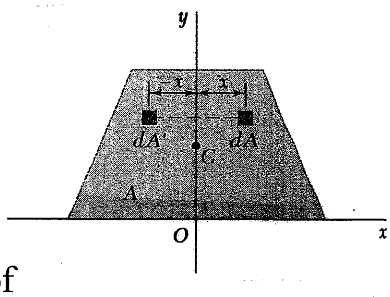
$$Q_x = \int y\,dA = \underline{y}\,A \qquad\qquad Q_y = \int x\,dA = \underline{x}\,A$$

similarly for a lines

$$Q_x = \int y\,dL = \underline{y}\,L \qquad\qquad Q_y = \int x\,dL = \underline{x}\,L$$

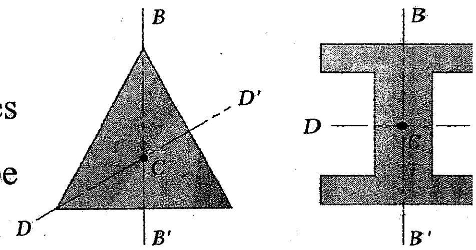


an area A is said to be symmetrical about an axis BB' if to every point P of the area corresponds a point P' of the same area such that the line PP' is perpendicular to BB' and is divided into two equal parts by that axis



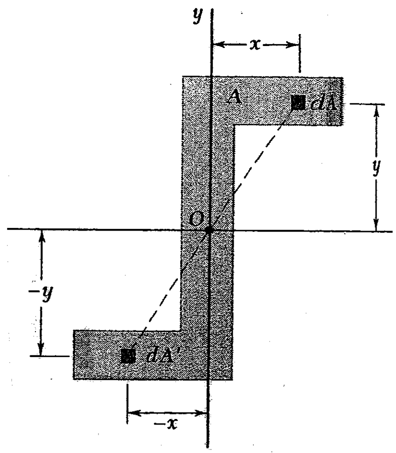
a line L is said to be symmetrical about BB' if it satisfies similar condition

when the area A or line L possesses an axis of symmetry BB', the centroid of area or line must be located on the axis



if an area A or line L possesses two or more axes of symmetry, the centroid of area or line must be located at the intersection of the axes

an area is said to be symmetrical about a center O if to every point P of the area corresponds a point P' of the area such that the line PP' is divided into two equal parts by O, a line L is said to be symmetrical about O if it satisfies similar condition



when the area of line possesses a center of symmetry O, the point O must be the centroid of the area of line centroid of common shapes of areas and lines are listed in figures 5.8A and 5.8B in the textbook

5.5 Composite Plates and Wires

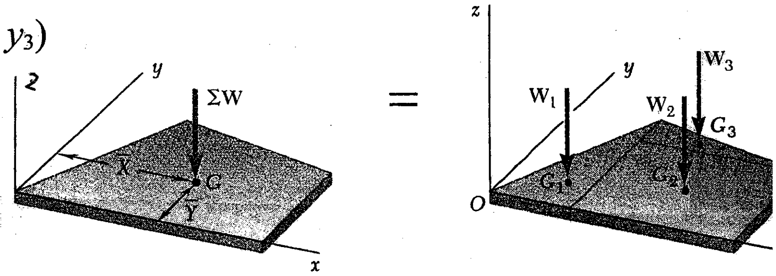
in many instances, a flat plate can be divided into several regular shapes, the center of gravity G can be determined from the centers of gravity of the various parts

$G_1(x_1, y_1) \qquad G_2(x_2, y_2) \qquad G_3(x_3, y_3)$

$$W = W_1 + W_2 + W_3$$

ΣM_y: $\quad \underline{X}\,\Sigma W = \Sigma x_i W_i$

ΣM_x: $\quad \underline{Y}\,\Sigma W = \Sigma y_i W_i$

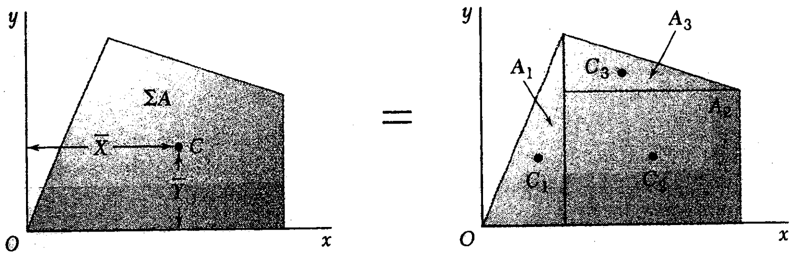


if the plate is homogeneous and of uniform thickness, the center of gravity coincides with centroid C of its area

$$A = \Sigma A_i$$

ΣM_y: $\quad \underline{X}\,\Sigma A = \Sigma x_i A_i$

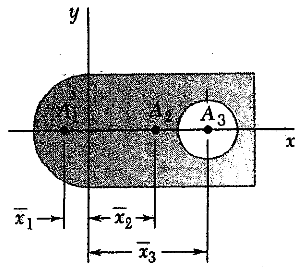
ΣM_x: $\quad \underline{Y}\,\Sigma A = \Sigma y_i A_i$



the first moment of area, may be positive or negative

negative : an area whose centroid located to the left of the y axis or the area of a hole

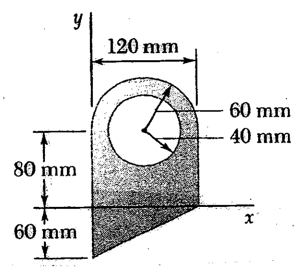
	$\underline{x}$	A	$\underline{x}\,A$
A_1 semicircle	-	+	-
A_2 full rectangle	+	+	+
A_3 circular hole	+	-	-

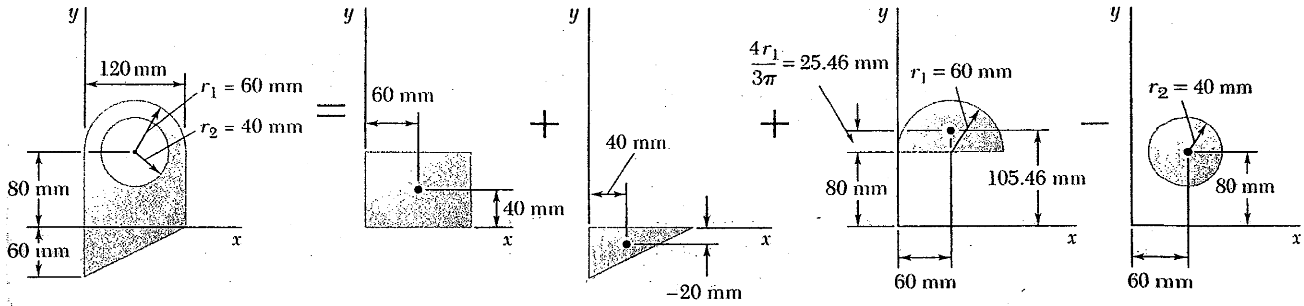


the center of gravity and centroids of a composite wire can be determined by the similar way

Sample Problem 5.1

determine Q_x, Q_y and the centroid of the composite area





	A (mm^2)	$\underline{x}$ (mm)	$\underline{y}$ (mm)	$\underline{x}A$ (mm^3)	$\underline{y}A$ (mm^3)
I	9.6×10^3	60	40	576×10^3	384×10^3
II	3.6×10^3	40	-20	144×10^3	-72×10^3
III	5.655×10^3	60	105.46	393.3×10^3	596.4×10^3
IV	-5.027×10^3	60	80	-301.6×10^3	-402.2×10^3
Σ	13.828×10^3			757.7×10^3	506.2×10^3

$Q_x = \Sigma \underline{y} A = 506.2 \times 10^3 \text{ mm}^3$

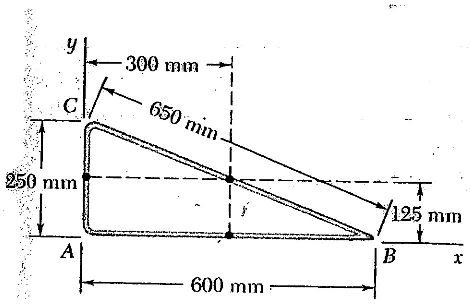
$Q_y = \Sigma \underline{x} A = 757.7 \times 10^3 \text{ mm}^3$

$\underline{X} = \Sigma \underline{x} A / A = 757.7 \times 10^3 / 13.828 \times 10^3 = 54.8 \text{ mm}$

$\underline{Y} = \Sigma \underline{y} A / A = 506.2 \times 10^3 / 13.828 \times 10^3 = 36.6 \text{ mm}$

Sample Problem 5.2

determine the center of gravity of a thin homogeneous wire

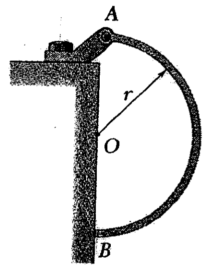


segment	L (mm)	$\underline{x}$ (mm)	$\underline{y}$ (mm)	$\underline{x}L$ (mm^2)	$\underline{y}L$ (mm^2)
AB	600	300	0	18×10^4	0
BC	650	300	125	19.5×10^4	8.125×10^4
CA	250	0	125	0	3.125×10^4
Σ	1500			37.5×10^4	11.25×10^4

$\underline{X} = \Sigma \underline{x} L / L = 37.5 \times 10^4 / 1500 = 250 \text{ mm}$

$\underline{Y} = \Sigma \underline{y} L / L = 11.25 \times 10^4 / 1500 = 75 \text{ mm}$

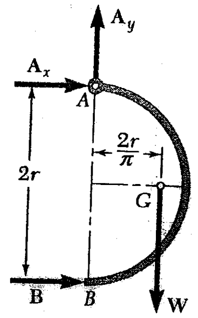
Sample Problem 5.3

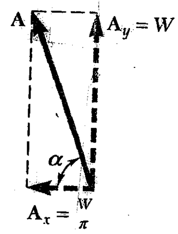


determine the reactions at A and B

$\Sigma M_A = 0 \quad B_x\,(2r) = W\,(2r/\pi)$

$B_x = W/\pi \rightarrow$





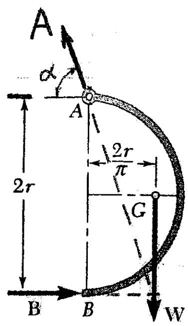
$\Sigma F_x = 0 \quad A_x + B_x = 0$

$A_x = -W/\pi \leftarrow$

$\Sigma F_y = 0 \quad A_y - W = 0 \qquad A_y = W \uparrow$

$A = A_x + A_y = 1.049\,W \;\nwarrow\; 72.3^{\circ}$

the concept of three-force body can be used



$a = 72.3^{\circ}$

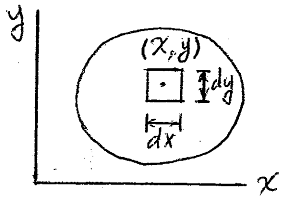
$B_x = W/\tan a = 0.318\ \mathrm{W} \rightarrow$

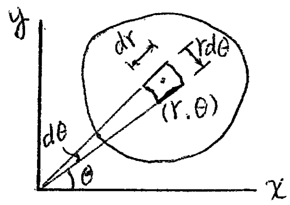
$A = W/\sin a = 1.049\ \mathrm{W} \nwarrow$

5.6 Determination of Centroids by Integration

$\underline{x}\,A = \int x\,dA \qquad \underline{y}\,A = \int y\,dA$

this is a double integration





$dA = dx\,dy$

$\underline{x}_i = x \quad \underline{y}_i = y$

$dA = r\,dr\,d\theta$

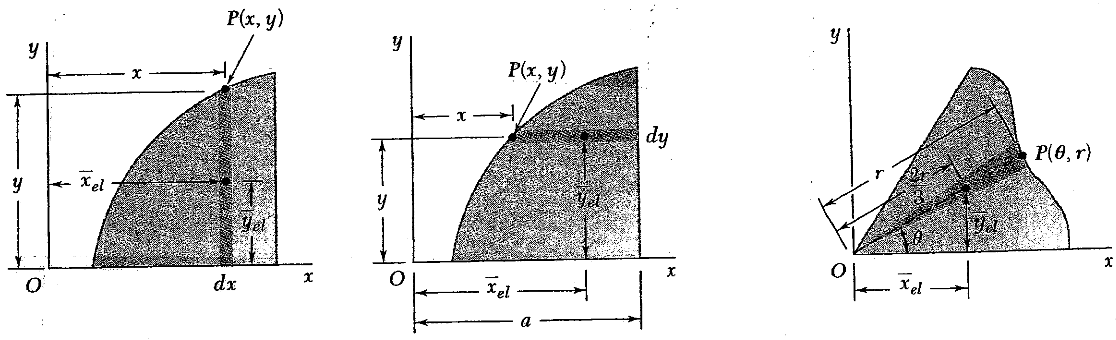
$\underline{x}_i = r\cos\theta \quad \underline{y}_i = r\sin\theta$

denoted by $\underline{x}_{el}$ and $\underline{y}_{el}$ the coordinates of the centroid of the element dA, then

$Q_y = \underline{x}\,A = \int \underline{x}_{el}\,dA \qquad Q_x = \underline{y}\,A = \int \underline{y}_{el}\,dA$

$\bar{x}_{el}$ and $\bar{y}_{el}$ should be expressed in terms of the coordinates of a point located on the curve bounding the area under consideration

three common types of elements are general used to evaluate the integration



$\bar{x}_{el} = x \quad \bar{y}_{el} = y/2$ $\quad$ $\bar{x}_{el} = (a+x)/2 \quad \bar{y}_{el} = y$ $\quad$ $\bar{x}_{el} = (2r/3)\cos\theta \quad \bar{y}_{el} = (2r/3)\sin\theta$

$dA = y\,dx$ $\quad$ $dA = (a - x)dy$ $\quad$ $dA = r^2\, d\theta / 2$

the double integral can be reduced to a single integral

similarly, the centroid of a line is defined by

$$\bar{x} L = \int x\, dL \qquad \bar{y} L = \int y\, dL$$

dL should be replaced by one of the following

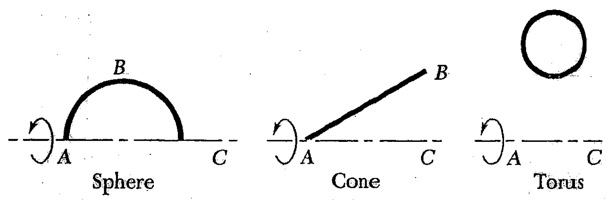
$$dL = (dx^2 + dy^2)^{½} = [1 + (dy/dx)^2]^{½}\, dx = [(dx/dy)^2 + 1]^{½}\, dy$$

$$dL = [(r\, d\theta)^2 + dr^2]^{½} = [r^2 + (dr/d\theta)^2]^{½}\, d\theta$$

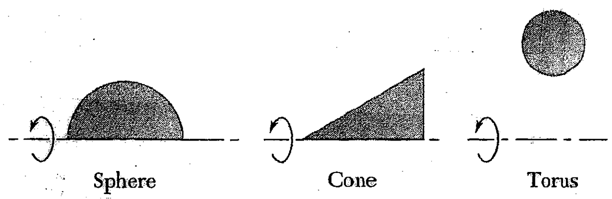
it is also reduced the double integral to a single integral

5.7 Theorems of Pappus-Guldinus

a surface of revolution is a surface which may be generated by rotating a plane curve about a fixed axis, e.g.

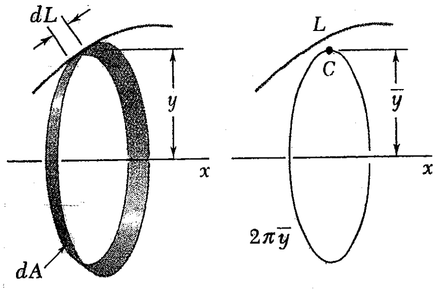


a body of revolution is a body which may be generated by rotating a plane area about a fixed axis, e.g.



Theorem I : the area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid of the curve while the surface is being generated, i.e.

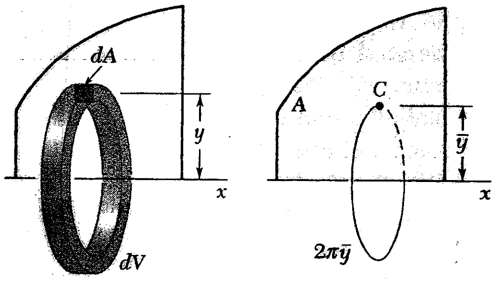
$$A = 2\pi \underline{y} L$$
$$dA = 2\pi y \, dL$$
$$A = \int dA = \int 2\pi y \, dL$$
$$= 2\pi \int y \, dL = 2\pi \underline{y} L$$



where $2\pi\underline{y}$ is the distance traveled by the centroid of L

Theorem II : the volume of a body of revolution is equal to the generating area times the distance traveled by the centroid of the area while the body is being generated, i.e.

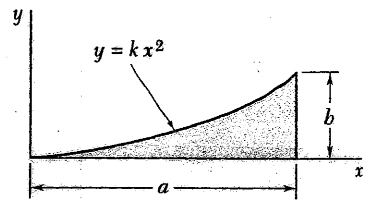
$$V = 2\pi \underline{y} A$$
$$dV = 2\pi y \, dA$$
$$V = \int dV = \int 2\pi y \, dV$$
$$= 2\pi \int y \, dA = 2\pi \underline{y} A$$



where $2\pi\underline{y}$ is the distance traveled by the centroid of A

Sample Problem 5.4

determine the centroid of the parabolic spandrel

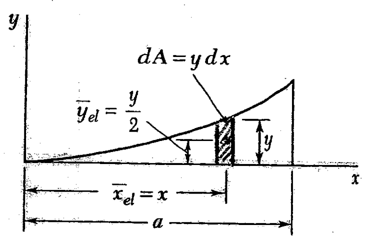


for the point $(x, y) = (a, b)$

$$b = k a^2 \quad => \quad k = b/a^2$$

then the equation of the curve is

$$y = (b/a^2)\, x^2 \qquad \text{or} \qquad x = (a/b^{1/2})\, y^{1/2}$$



vertical differential element

$$A = \int dA = \int y\,dx = \int (b/a^2)\,x^2\,dx = (b/a^2)\,[x^3/3]_0^a = ab/3$$

ΣM_y :

$$Q_y = \int \underline{x}_{el}\,dA = \int x\,y\,dx = \int x\,(b/a^2)\,x^2\,dx$$

$$= (b/a^2)\,[x^4/4]_0^a = a^2b/4$$

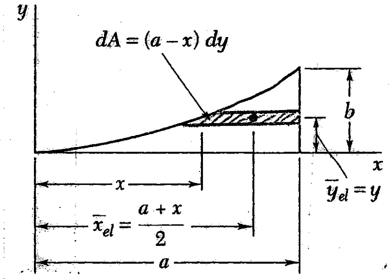
$$\underline{x}\,A = \int \underline{x}_{el}\,dA$$

$$\underline{x} = (a^2b/4)/(ab/3) = 3a/4$$

ΣM_x :

$$Q_x = \int \underline{y}_{el}\,dA = \int (y/2)\,y\,dx = \int \tfrac{1}{2}\,[(b/a^2)\,x^2]^2\,dx$$

$$= (b^2/2a^4)\,[x^5/5]_0^a = ab^2/10$$



$$\underline{y}\,A = \int \underline{y}_{el}\,dA$$

$$\underline{y} = (ab^2/10)/(ab/3) = 3b/10$$

horizontal differential element

ΣM_y :

$$Q_y = \int \underline{x}_{el}\,dA = \int \tfrac{1}{2}(a+x)\,(a-x)\,dy = \int \tfrac{1}{2}(a^2 - x^2)\,dy$$

$$= \tfrac{1}{2}\int_0^b [a^2 - a^2y/b)\,dy = a^2b/4$$

ΣM_x :

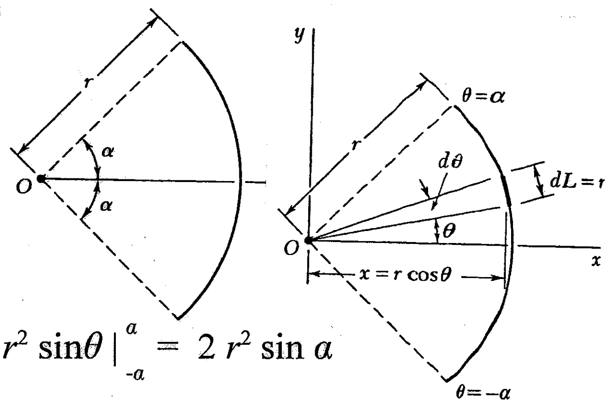
$$Q_x = \int \underline{y}_{el}\,dA = \int y\,(a-x)\,dy = \int y\,[a - (a/b^{1/2})\,y^{1/2}]\,dy$$

$$= \int_0^b [a\,y - (a/b^{1/2})\,y^{3/2}]\,dy = ab^2/10$$

$\underline{x}$ and $\underline{y}$ are the same as above

Sample Problem 5.5

determine the centroid of the arc



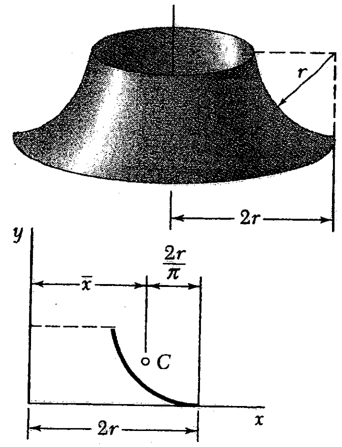
$$L = \int dL = \int_{-a}^{a} r\,d\theta = 2\,r\,a$$

$$Q_y = \int x_{el}\,dL = \int_{-a}^{a} r\cos\theta\,r\,d\theta = r^2\sin\theta\,\Big|_{-a}^{a} = 2\,r^2\sin a$$

$$\underline{x} = Q_y/L = 2\,r^2\sin a/(2\,r\,a) = r\sin a/a$$

$$\underline{y} = 0 \quad \text{(by symmetric)}$$

Sample Problem 5.6



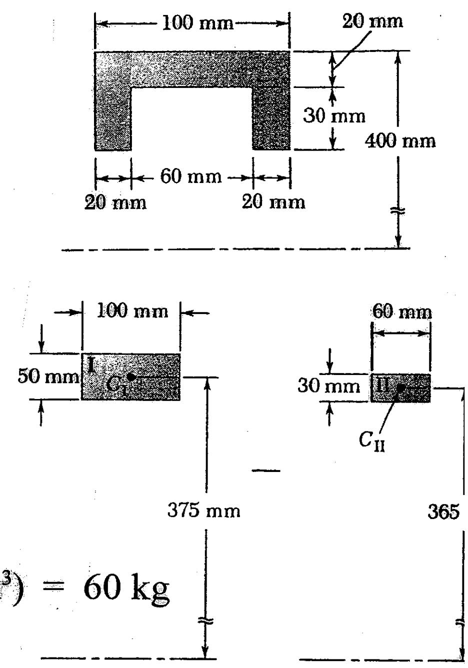
determine the area of the surface of revolution

$$\underline{x} = 2r - 2r/\pi = (1 - 1/\pi)\,2r$$

$$A = 2\pi\,\underline{x}\,L = 2\pi\,[2r(1 - 1/\pi)]\,(\pi r/2)$$

$$= 2\pi r^2(\pi - 1)$$

Sample Problem 5.7



determine the mass and the weight of the rim

i.e. fine the volume of the revolution body

$\rho = 7.85 \times 10^3$ kg/m^3

	A (mm^2)	$\underline{y}$ (mm)	$V = 2\pi\underline{y}\,A$ (mm^3)
I	5000	375	11.78×10^6
II	-1800	365	-4.13×10^6
		V of the rim =	7.65×10^6

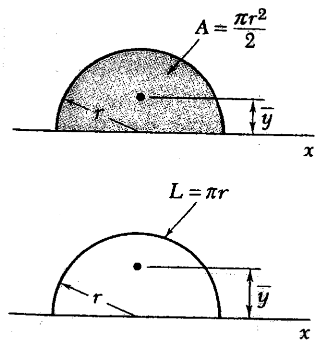
$$m = \rho V = (7.85 \times 10^3\ \text{kg/m}^3)(7.65 \times 10^{-3}\ \text{m}^3) = 60\ \text{kg}$$

$$W = m g = 60 \times 9.81 = 589\ \text{N}$$

Sample Problem 5.8

determine the centroids of

(a) semicircular area (b) semicircular arc



$$V = 2\pi\,\underline{y}\,A \qquad \frac{4}{3}\pi r^3 = 2\pi\,\underline{y}\,(\frac{1}{2}\pi r^2)$$

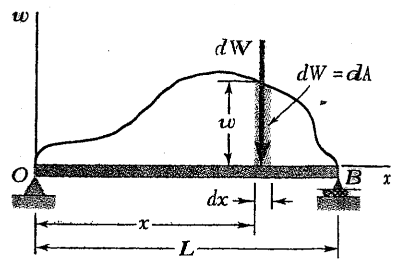
$$\underline{y} = 4r/3\pi$$

$$A = 2\pi\,\underline{y}\,L \qquad 4\pi r^2 = 2\pi\,\underline{y}\,(\pi r)$$

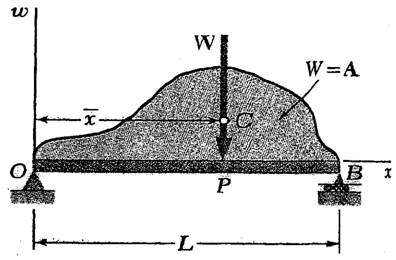
$$\underline{y} = 2r/\pi$$

5.8 Distributed Loads on Beams

the distributed load on beam can be represented by plotting the load w supported per unit length, N/m, the magnitude of the force exerted on the length dx is $dW = w\,dx$, then the total load on the beam is



$$W = \int_0^L w\,dx$$



$w\,dx$ is equal in magnitude to the element of area dA, then W is equal in magnitude to the total area A under the load curve

i.e. $\quad W = \int dA = A$

the point of application of the equivalent concentrated load W is obtained by expressing that the moment of W about point O is equal the sum of the moments of the elemental loads dW about O

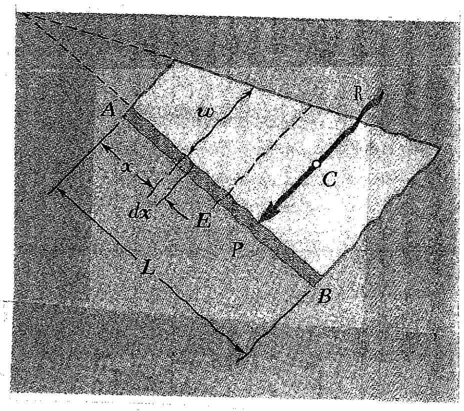
$$(OP)\,W = \int x\,dW$$

i.e. $\quad (OP)\,A = \int_0^L x\,dA$

a distributed load on a beam may thus be replaced by a concentrated load, the magnitude of this single load is equal to the area under load curve, and its line of action passes through the centroid of the area

5.9 Forces on Submerged Surfaces

consider a rectangular surface submerged in a liquid

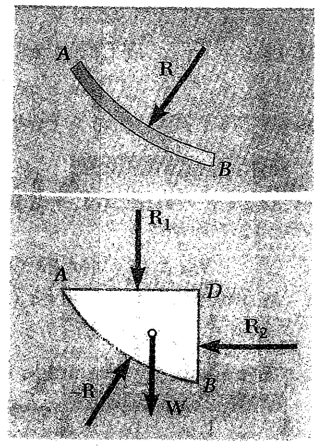


$$p = b\,p = b\,\rho\,g\,h$$

ρ : density of the liquid

h : vertical distance from the free surface

$$R = p_E \, b \, L$$



p_E : pressure at the center of the plate

the resultant R should not be applied at E

for a curved surface AB, the resultant R determinated by direct integration is not easy

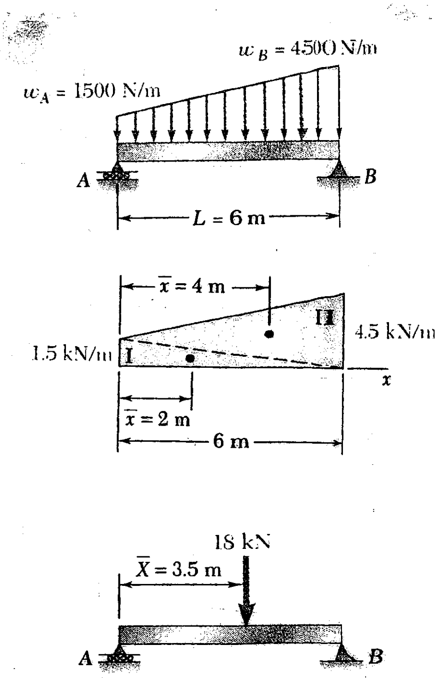
forces acting on body ABD are W, R_1, R_2, $-R$ are in equilibrium, then R can be determined

the resultants of forces on submerged surfaces of variable width will be determined in Chapter 9

Sample Problem 5.9

determine (a) equivalent concentrated load

(b) reactions at supports



	A (kN)	x (m)	$x\,A$ (kN-m)
I	4.5	2	9
II	13.5	4	54
	18		63

$x = 63 / 18 = 3.5$ m

$\Sigma F_x = 0 \qquad B_x = 0$

$\Sigma M_A = 0 \quad B_y \times 6 = 18 \times 3.5 \qquad B_y = 10.5$ kN

$\Sigma F_y = 0 \quad A_y + B_y = 18 \qquad A_y = 7.5$ kN

Sample Problem 5.10

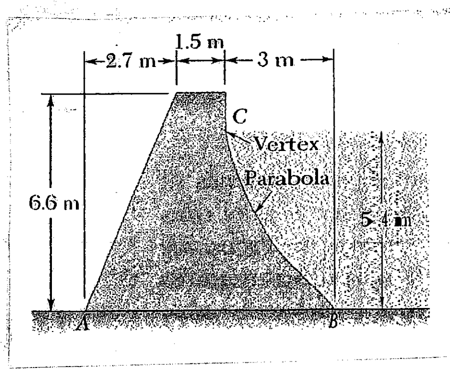
(a) determine the ground reaction

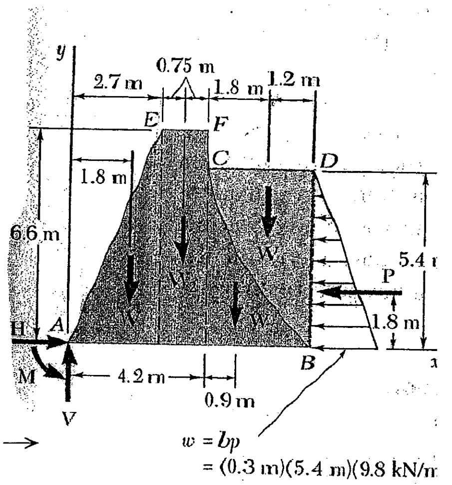
(b) determine the resultant force of water on BC

ρ of concrete : 2.4×10^3 kg/m^3

ρ of water : 10^3 kg/m^3

consider a 1 m thick section of the dam





$W_1 = ½ \times 2.7 \times 6.6 \times 2400 \times 9.81 = 62.93$ kN

$W_2 = 1.5 \times 6.6 \times 0.3 \times 2400 \times 9.81 = 69.9$ kN

$W_3 = ⅓ \times 3 \times 5.4 \times 0.3 \times 2400 \times 9.81 = 38.1$ kN

$W_4 = ⅔ \times 3 \times 5.4 \times 0.3 \times 1000 \times 9.81 = 31.8$ kN

$P = ½ \times 5.4 \times 0.3 \times 5.4 \times 1000 \times 9.81 = 42.9$ kN

$\Sigma F_x = 0 \quad H - 42.9 = 0 \qquad H = 42.9 \text{ kN} \rightarrow$

$\Sigma F_y = 0 \quad V - 62.93 - 69.9 - 38.1 - 31.8 = 0$

$V = 202.7 \text{ kN} \uparrow$

$\Sigma M_A = 0 \quad -62.93 \times 1.8 - 69.9 \times 3.45 - 38.1 \times 5.1 - 31.8 \times 6$

$+ 42.9 \times 1.8 + M = 0$

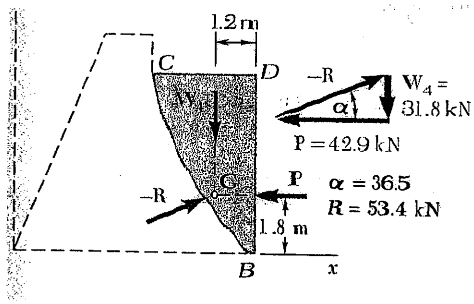
$M = 662$ kN-m

replace M and V by a single force

$d = 622 / 202.7 = 3.27$ m

the single force acting at a distance d to the right of A

resultant $\boldsymbol{R}$ of water force



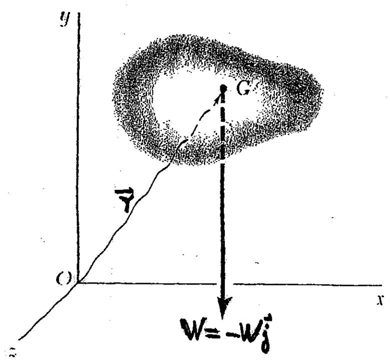
$\boldsymbol{R} = \boldsymbol{W}_4 + \boldsymbol{P}$

$\boldsymbol{R} = 1.98 \text{ MN} \angle 36.5^\circ$

VOLUMES

5.10 Center of Gravity of a Three-Dimensional Body, Centroid of a Volume

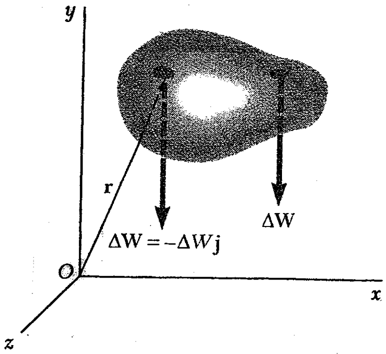
the center of gravity G of a body can be obtained



$\Sigma \boldsymbol{F}: \qquad -W\boldsymbol{j} = \Sigma(-\Delta W\boldsymbol{j})$

$\Sigma \boldsymbol{M}_o: \qquad \boldsymbol{r} \times (-W\boldsymbol{j}) = \Sigma [\boldsymbol{r} \times (-\Delta W\boldsymbol{j})]$

if the number of elements is increased



$$W = \int dW$$

$$\underline{\boldsymbol{r}}\, W = \int \boldsymbol{r}\, dW$$

or $\underline{x}\, W = \int x\, dW \qquad \underline{y}\, W = \int y\, dW$

$$\underline{z}\, W = \int z\, dW$$

if the body made by homogeneous material of density ρ

$$dW = \rho\, \mathrm{g}\, dV \qquad\qquad W = \rho\, \mathrm{g}\, V$$

and $\underline{\boldsymbol{r}}\, V = \int \boldsymbol{r}\, dV$

or $\underline{x}\, V = \int x\, dV \qquad \underline{y}\, V = \int y\, dV \qquad \underline{z}\, V = \int z\, dV$

the integrals $\int x\, dV$, $\int y\, dV$ and $\int z\, dV$ are the first moments of the volume with respect to the yz plane, xz plane and xy plane, respectively

centroids of common shapes and volumes are listed in figure 5.21 on page 249

5.11 Composite Bodies

divided the body into several of common shapes, the center of gravity can be determined as

$$\underline{X}\, W = \Sigma\, \underline{x}\, W \qquad \underline{Y}\, W = \Sigma\, \underline{y}\, W \qquad \underline{Z}\, W = \Sigma\, \underline{z}\, W$$

if the body is made of a homogeneous material, the centroid of the body can be obtained

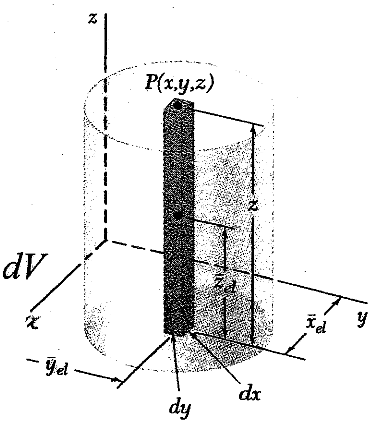
$$\underline{X}\, V = \Sigma\, \underline{x}\, V \qquad \underline{Y}\, V = \Sigma\, \underline{y}\, V \qquad \underline{Z}\, V = \Sigma\, \underline{z}\, V$$

5.12 Determination of Centroids of Volumes by Integration

$$\underline{x}\, V = \int x\, dV \qquad \underline{y}\, V = \int y\, dV \qquad \underline{z}\, V = \int z\, dV$$

if dV is chosen $dV = dx\, dy\, dz$

the integrals are triple integrations



if dV is chosen as a thin filament

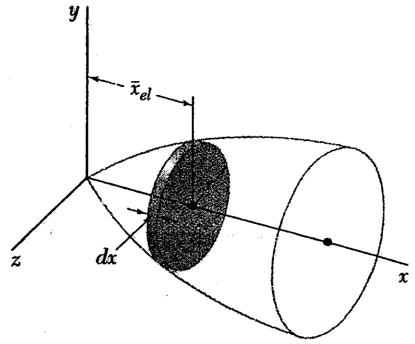
$$dV = z\,dx\,dy$$

$$x_{el} = x \qquad y_{el} = y \qquad z_{el} = z/2$$

$$\underline{x}\,V = \int x_{el}\,dV \qquad \underline{y}\,V = \int y_{el}\,dV \qquad \underline{z}\,V = \int z_{el}\,dV$$

it can reduced to double integration

if the volume possesses two planes of symmetry



$$\underline{y} = \underline{z} = 0$$

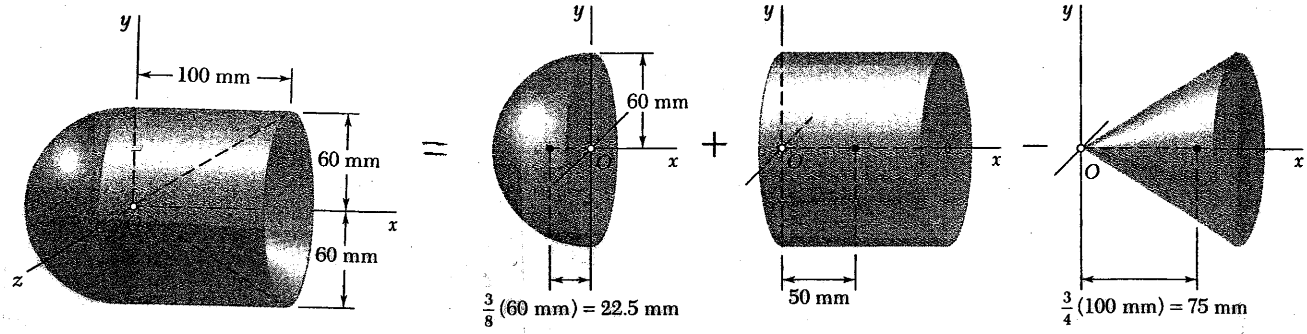
this can be done with a single integration

$$dV = \pi r^2\,dx \qquad x_{el} = x$$

$$\underline{x}\,V = \int x_{el}\,dV$$

Sample Problem 5.11

locate the centroid of the body



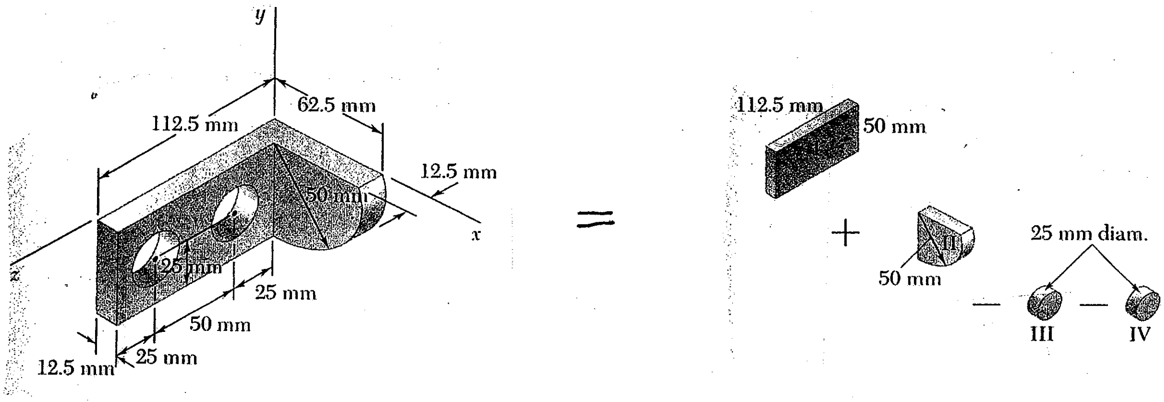
component	volume, mm^3	$\underline{x}$, mm	$\underline{x}\,V$, mm3
hemisphere	$\frac{1}{2}\frac{4\pi}{3}(60)^3 = 0.4524 \times 10^6$	- 22.5	$- 10.18 \times 10^6$
cylinder	$\pi\, 60^2 \times 100 = 1.131 \times 10^6$	50	56.55×10^6
cone	$- ⅓\, 60^2 \times 100 = -0.377 \times 10^6$	75	$- 28.28 \times 10^6$
Σ	1.206×10^6		18.09×10^6

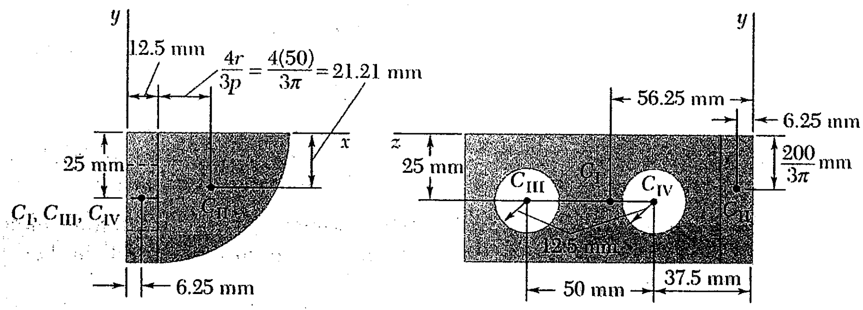
$$\underline{X} = 18.09 \times 10^6 / 1.206 \times 10^6 = 15 \text{ mm}$$

$\underline{Y} = \underline{Z} = 0$ by symmetry

Sample Problem 5.12

locate the centroid of the steel element





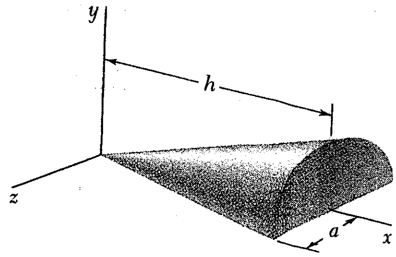
	V, mm^3	x, mm	y, mm	z, mm	$x\,V$, mm^4	$y\,V$, mm^4	$z\,V$, mm^4
I	73,102.5	6.25	-25	56.25	439,453	-1,757813	3,955,078
II	24,553.6	33.71	-21.21	6.25	827,702	-520,782	153,460
III	-6,138.4	6.25	-25	87.5	-38,365	153,460	-537,110
IV	-6,138.4	6.25	-25	37.5	-38,365	153,460	-239,190
Σ	82,589.3				1,200,425	-1,971,662	3,341,238

X = 1,200,425 /82,589.3 = 14.53 mm

Y = -1,971,662 /82,589.3 = -23.9 mm

Z = 3,341,238 /82,589.3 = 40.5 mm

Sample Problem 5.13

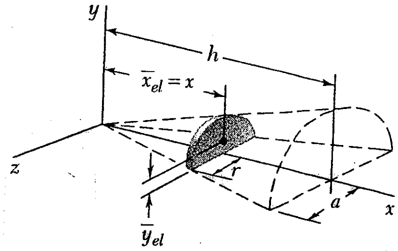


determine the centroid of the volume

since xy plane is the plane of symmetry

$\underline{z} = 0$

chosen a slab element



$dV = \frac{1}{2}\pi r^2\, dx \qquad \underline{x}_{el} = x \qquad \underline{y}_{el} = 4r / 3\pi$

$\because \quad r / x = a / h \qquad$ then $\qquad r = a\,x / h$

the volume of the body is

$$V = \int dV = \int_0^h \tfrac{1}{2}\pi r^2\, dx = \int_0^h \tfrac{1}{2}\pi\,(ax/h)^2\, dx = \frac{\pi\, a^2\, h}{6}$$

total moment of the body w. r. t. yz plane is

$$\int \underline{x}_{el}\, dV = \int_0^h x\,[\tfrac{1}{2}\pi\,(ax/h)^2]\, dx = \frac{\pi\, a^2\, h^2}{8}$$

$\underline{x}\, V = \int \underline{x}_{el}\, dV$

$\underline{x} = [\pi a^2 h^2/8] / [\pi a^2 h/6] = 3\, h / 4$

total moment of the body w. r. t. xz plane is

$$\int \underline{y}_{el}\, dV = \int_0^h (4r/3\pi)\,(\tfrac{1}{2}\pi r^2)\, dx = \tfrac{2}{3}\int_0^h (ax/h)^3\, dx = \frac{a^3\, h}{6}$$

$\underline{y}\, V = \int \underline{x}_{el}\, dV$

$\underline{y} = [a^3 h/6] / [\pi a^2 h/6] = a / \pi$

$\underline{z} = 0 \qquad$ by symmetric with respect to xy plane