Chapter 4 Equilibrium of Rigid Bodies

4.1 Introduction

a body is said to be in equilibrium when the external forces acting on it from a system of force equivalent is zero

the necessary and sufficient conditions for the equilibrium of the rigid body is

$$\Sigma \boldsymbol{F} = 0 \qquad \Sigma \boldsymbol{M}_0 = \Sigma (\boldsymbol{r} \times \boldsymbol{F}) = 0$$

i.e.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

4.2 Free-Body Diagram

the following steps which must be followed in drawing a free-body diagram

1. a clear decision is made regarding the choice of the free body to be used
2. all external forces are then indicated, note : the weight of free body should also be included among the external force
3. the magnitude and direction of the known external forces should be clearly marked on the free-body diagram
4. unknown external forces usually consist of the reaction
5. the free-body diagram should also indicate dimensions

4.3 Reactions at Supports and Connections for a Two Dimensional Structure

the reactions exerted on a two-dimensional structure may be divided into three groups

1. reactions equivalent to forces with known line of action

e.g. rollers, rockers, smooth surface, short links and cables,

collars on smooth rods, pins in smooth slots etc.

the line of action of the reaction is known and should be indicated clearly in the free-body diagram

2. reactions equivalent to a force with unknown direction

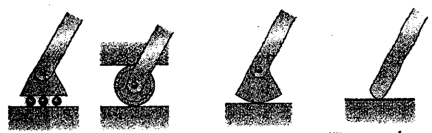
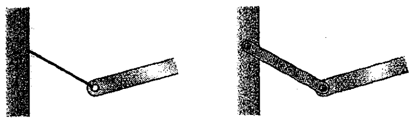
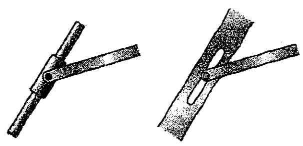
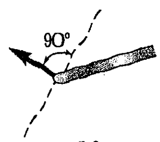
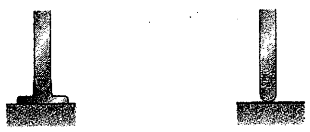
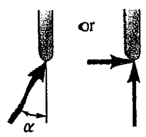
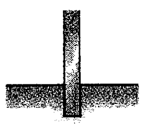
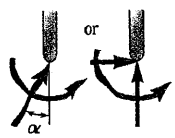
e.g. smooth pins, hinges, rough surface etc.

reaction in the group involve two unknowns, which are usually represented by x and y components

3. reaction equivalent to a force and couple

e.g. fixed supports

reaction of this group involve three unknowns, two components of force and the moment

Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge, Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

4.4 Equilibrium of a Rigid Body in Two Dimension

in this case, forces are all in x-y plane

thus $\quad F_z = 0 \quad M_x = M_y = 0 \quad$ are automatic satisfied

equations of equilibrium are reduced from 6 to 3, i.e.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \;\; (\Sigma M_z)$$

where A is any point in the plane of the structures

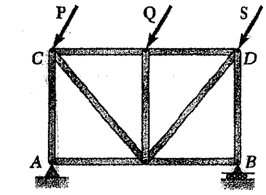
3 equations may be solved for no more than three unknowns

e.g. for a truss supported to loading

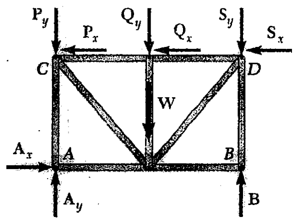
$\Sigma M_A = 0 \quad \Rightarrow \quad B$

$\Sigma F_x = 0 \quad \Rightarrow \quad A_x$

$\Sigma F_y = 0 \quad \Rightarrow \quad A_y$



(a)



(b)

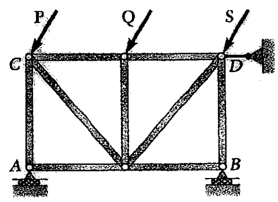
for additional equation $\Sigma M_B = 0$ is not independent and can not be used to determine the fourth unknown (check the solution)

any of the three equations may be replaced by another equation to solve three unknowns

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

e.g. for a truss supported by two rollers and a short link

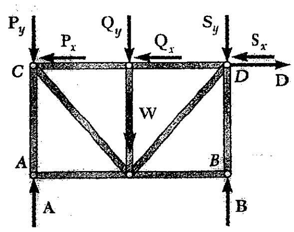


(a)

$\Sigma F_x = 0 \quad \Rightarrow \quad D$

$\Sigma M_C = 0 \quad \Rightarrow \quad B$

$\Sigma M_D = 0 \quad \Rightarrow \quad A$



(b)

each equation contains only one unknown

4.5 Statically Indeterminate Reactions, Partial Constraints

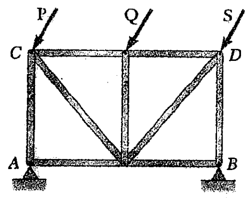
in last section, three unknowns are solved by three equations, it is said to be statically determinate, and the rigid body is said to be completely constrained

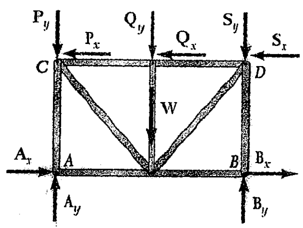
for a truss supported by two hinges

$\Sigma M_A = 0 \quad \Rightarrow \quad B_y$

$\Sigma M_B = 0 \quad \Rightarrow \quad A_y$

$\Sigma F_x = 0 \quad \Rightarrow \quad A_x + B_x$





the components A_x and B_x are said to be statically indeterminate, it may be solved by the method of mechanics of materials

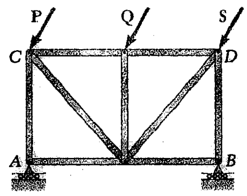
for a truss supported by two rollers

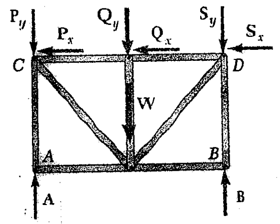
$\Sigma M_A = 0 \quad \Rightarrow \quad B$

$\Sigma M_B = 0 \quad \Rightarrow \quad A$

but $\quad \Sigma F_x \neq 0$

unless the sum of the horizontal component equal zero





thus equilibrium cannot be maintained under general loading conditions, it is said to be only partially constrained

for statically determinate, there must be as many unknowns as there are equations of equilibrium, it is necessary but not sufficient condition

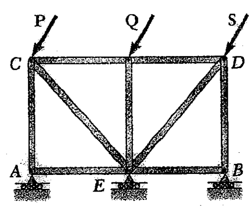
for a truss supported by three rollers

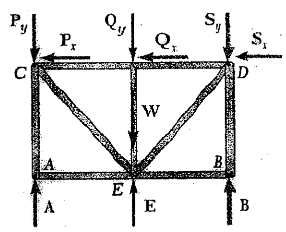
3 equations for 3 unknowns

but $\quad \Sigma F_x \neq 0 \quad$ (unstable)

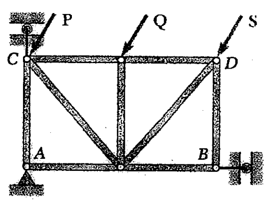
it is said to be improperly constrained

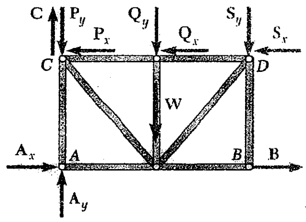
$\therefore \quad$ 3 unknowns for only 2 equations





thus the improperly constrained also produce static indeterminacy for a truss supported by a hinge and two links





3 equations for 4 unknowns

but $\quad \Sigma M_A \neq 0 \quad$ (unstable)

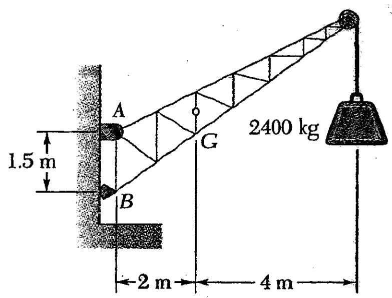
thus it is also improperly constrained

a rigid body is improperly constrained whenever the supports are arranged in such a way that the reactions must be either concurrent or parallel

Sample Problem 4.1

mass of truss = 1000 kg

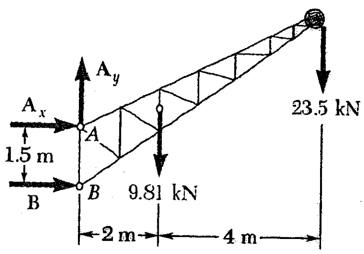
mass of weight = 2400 kg



$\Sigma M_A = 0$

$$B_x \times 1.5 - 9.81 \times 2 - 23.5 \times 6 = 0$$

$$B_x = 107.1 \text{ kN} \quad \rightarrow$$



$\Sigma F_x = 0$

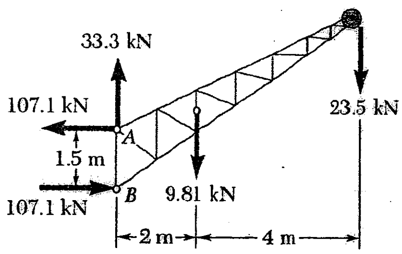
$$A_x + B_x = 0$$

$$A_x = -107.1 \text{ kN} \quad \leftarrow$$

$\Sigma F_y = 0$

$$A_y - 9.81 - 23.5 = 0$$

$$A_y = 33.3 \text{ kN} \quad \uparrow$$



check

$$\Sigma M_B = -9.81 \times 2 - 23.5 \times 6 + 107.1 \times 1.5 = 0 \qquad \text{(O.K.)}$$

Sample Problem 4.2

$P = 70$ kN

neglecting the weight of the beam

$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma M_A = 0 \quad -70 \times 0.9 + B_y \times 2.7 - 27 \times 3.3 - 27 \times 3.9 = 0$

$B_y = 95.3$ kN $\uparrow$

$\Sigma M_B = 0 \quad -A \times 2.7 + 70 \times 1.8 - 27 \times 0.6 - 27 \times 1.2 = 0$

$A = 28.7$ kN $\uparrow$

check $\Sigma F_y = 0 \quad 28.7 - 70 + 95.3 - 27 - 27 = 0$ (O.K.)

Sample Problem 4.3

weight of the loading car : $W = 25$ kN

determine : (1) the tension in cable

(2) reaction at wheels

$W_x = W \cos 25^\circ = 22.65$ kN

$W_y = -W \sin 25^\circ = -10.5$ kN

$\Sigma F_x = 0 \quad 22.65 - T = 0$

$T = 22.65$ kN $\nwarrow$

$\Sigma M_A = 0$

$-10.5 \times 625 - 22.65 \times 150 + R_2 \times 1250 = 0$

$R_2 = 8$ kN $\nearrow$

$\Sigma M_B = 0$

$-10.5 \times 625 - 22.65 \times 150 - R_1 \times 1250 = 0$

$R_1 = 2.5$ kN $\nearrow$

check : $\Sigma F_y = 0$

$2.5 + 8 - 10.5 = 0$ (O.K.)

Sample Problem 4.4

tension in cables : $T_1 = 600$ N $T_2 = 375$ N

weight of the pole = 1600 N

determine the reaction at A

$\Sigma F_x = 0 \quad A_x + 375\cos 20^\circ - 600\cos 10^\circ = 0$

$A_x = 238.50 \text{ N} \rightarrow$

$\Sigma F_y = 0 \quad A_y - 1600 - 600\sin 10^\circ - 375\sin 20^\circ = 0$

$A_y = 1832.45 \text{ N} \uparrow$

$\Sigma M_A = 0 \quad M_A + 600 \text{ x } \cos 10^\circ \text{ x } 6 - 375\cos 20^\circ \text{ x } 6 = 0$

$M_A = 1431$ N-m

Sample Problem 4.5

$W = 400$ N $\quad k = 12.5$ kN/m

spring is unstretched at $\theta = 0^\circ$

determine θ for equilibrium

$$F = ks = kr\theta$$

$\Sigma M_0 = 0 \quad Wl\sin\theta - r(kr\theta) = 0$

$$\sin\theta = \frac{kr^2\theta}{Wl} = \frac{12500\,(0.06)^2\,\theta}{400 \text{ x } 0.16} = 0.703\,\theta$$

solving by trial and error $\quad \theta = 0^\circ \quad \theta = 80.3^\circ$

4.6 Equilibrium of a Two-Force Body

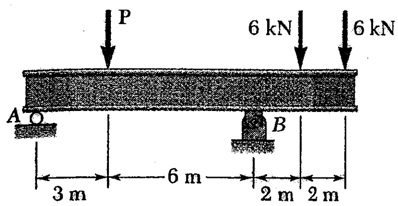
two-force body : a rigid body subjected to two forces only

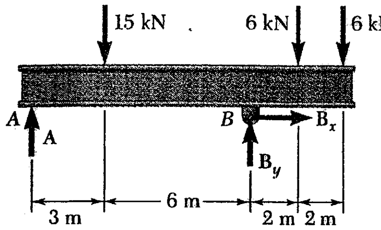
if the two-force body is in equilibrium, the two forces must have the same magnitude, same line of action, and opposite direction

Sample Problem 4.2

$P = 15$ kN

neglecting the weight of the beam





$\Sigma F_x = 0 \quad B_x = 0$

$\Sigma M_A = 0 \quad -15 \times 3 + B_y \times 9 - 6 \times 11 - 6 \times 13 = 0$

$B_y = 21$ kN $\uparrow$

$\Sigma M_B = 0 \quad -A \times 9 + 15 \times 6 - 6 \times 2 - 6 \times 4 = 0$

$A = 6$ kN $\uparrow$

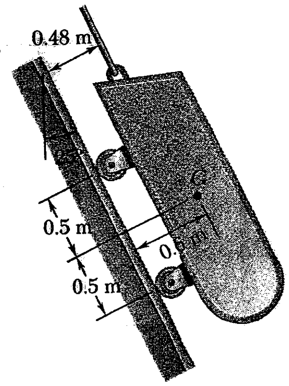
check $\Sigma F_y = 0 \quad 6 - 15 + 21 - 6 - 6 = 0$ (O.K.)

Sample Problem 4.3

weight of the loading car : 5500 N

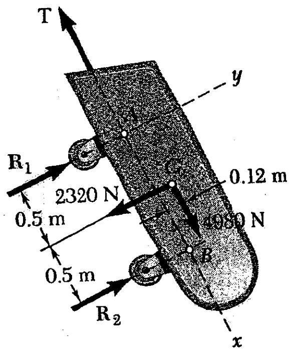
determine : (1) the tension in cable

(2) reaction at wheels



$W_x = W \cos 25^\circ = 4980$ N

$W_y = -W \sin 25^\circ = -2320$ N



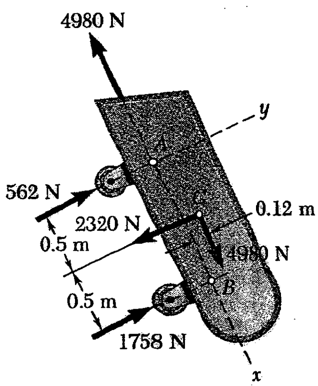
$\Sigma F_x = 0 \quad 4980 - T = 0$

$T = 4980$ N $\nwarrow$

$\Sigma M_A = 0$

$-2320 \times 0.5 - 4980 \times 0.12 + R_2 \times 1 = 0$

$R_2 = 1758$ N $\nearrow$



$\Sigma F_y = 0$

$R_1 + 1758 - 2320 = 0$

$R_1 = 562$ N $\nearrow$

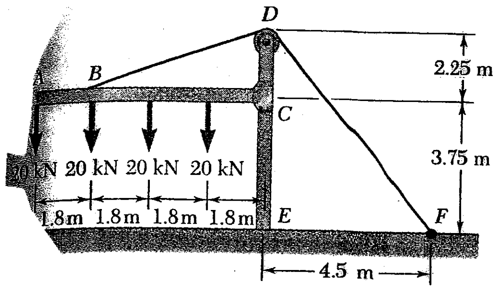
check : $\Sigma M_B = 0$

$2320 \times 0.5 - 4980 \times 0.12 - 562 \times 1 = 0$ (O.K.)

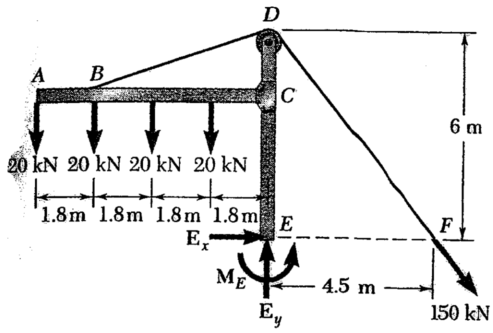
Sample Problem 4.4

tension in cable $= 150$ kN

determine the reaction at E



$$DF = (4.5^2 + 6^2)^{½} = 7.5 \text{ m}$$



$\Sigma F_x = 0 \quad E_x + 150 \text{ x } (4.5/7.5) = 0$

$E_x = -90 \text{ kN} \leftarrow$

$\Sigma F_y = 0 \quad E_y - 4 \text{ x } 20 - 150 \text{ x } (6/7.5) = 0$

$E_y = 200 \text{ kN} \uparrow$

$\Sigma M_E = 0 \quad M_E + 20 \text{ x } (7.2 + 5.4 + 3.6 + 1.8) - 150 \text{ x } (4.5/7.5) \text{ x } 6 = 0$

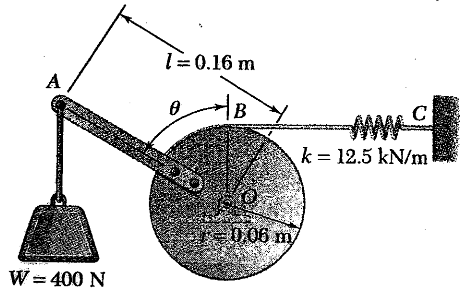
$M_E = 180 \text{ kN-m}$

Sample Problem 4.5

$W = 400$ N $\qquad k = 12.5$ kN/m

spring is unstretched at $\theta = 0°$

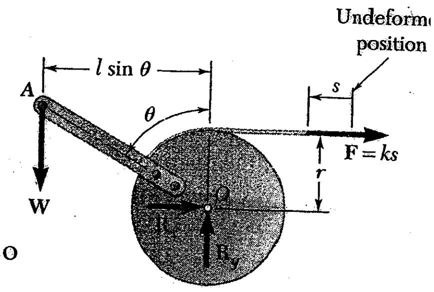
determine θ for equilibrium



$F = k s = k r \theta$

$\Sigma M_O = 0 \quad W l \sin\theta - r (k r \theta) = 0$

$$\sin\theta = \frac{k r^2 \theta}{W l} = \frac{12500\,(0.06)^2\,\theta}{400 \text{ x } 0.16} = 0.703\,\theta$$

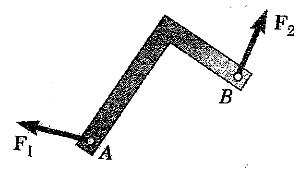


solving by trial and error $\qquad \theta = 0° \qquad \theta = 80.3°$

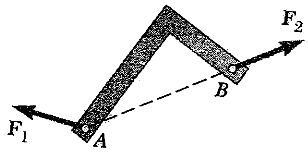
4.6 Equilibrium of a Two-Force Body

two-force body : a rigid body subjected to two forces only

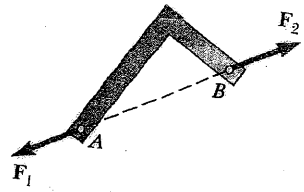
if the two-force body is in equilibrium, the two forces must have the same magnitude, same line of action, and opposite direction



$\Sigma M_A, \Sigma M_B \neq 0$



$\Sigma M_A = 0, \Sigma M_B \neq 0$



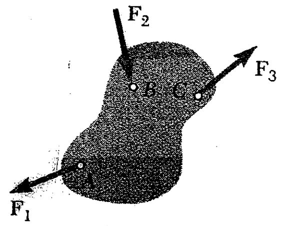
$\Sigma M_A, \Sigma M_B \neq 0$
$\Sigma F = 0$ (if $F_1 = F_2$)

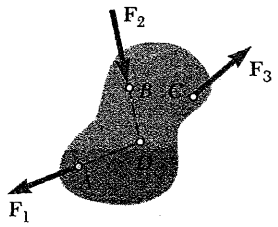
two-force body may be more general defined as a rigid body subjected to forces acting only at two points

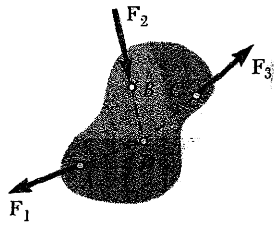
4.7 Equilibrium of a Three-Force Body

a rigid body subjected to forces acting at only three points is called

if the three-force body is in equilibrium, the line of action of the three forces must be either concurrent or parallel



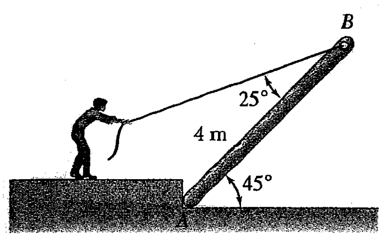




$\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ intersect at point D, $\Sigma M_D = 0$, thus $\boldsymbol{F}_3$ must pass through point D, 3 forces are concurrent

the only exception occurs when none of lines intersect, thus the line of action of forces must then be parallel

Sample Problem 4.6

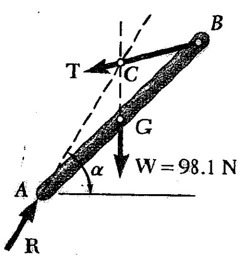


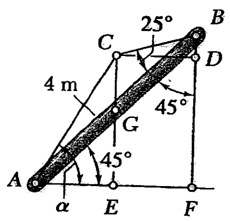
$m = 10$ kg $\qquad W = 98.1$ N

find the tension T and reaction at A

AB is a three-force body, three forces must intersect at point C

$$AF = BF = 4 \cos 45^\circ = 2.828 \text{ m}$$



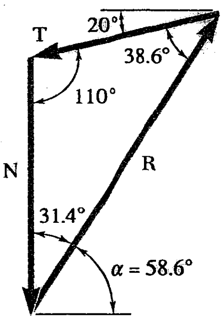


$$AE = CD = 1.414 \text{ m}$$

$$BD = CD \cot(45^\circ + 25^\circ) = 0.515 \text{ m}$$

$$\therefore \quad CE = DF = BF - BD = 2.313 \text{ m}$$

$$\tan \alpha = \frac{CE}{AE} = \frac{2.313}{1.414} \qquad \alpha = 58.6^\circ$$



using the law of sine

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1}{\sin 38.6^\circ}$$

$$T = 81.9 \text{ N} \swarrow \qquad\qquad R = 147.8 \text{ N} \measuredangle\ 58.6^\circ$$

EQUILIBRIUM IN THREE DIMENSIONS

4.8 Equilibrium of a Rigid Body in Three Dimensions

equations of equilibrium

$$\Sigma \boldsymbol{F} = 0 \qquad\qquad \Sigma \boldsymbol{M}_0 = \Sigma(\boldsymbol{r} \times \boldsymbol{F}) = 0$$

i.e. $\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$ six equations

if reactions involve more than 6 unknowns, there are statically indeterminate

if reaction involve less than 6 unknowns, there are partially constrained (some of the equation cannot be satisfied)

if the force of reaction are either parallel of concurrent, of pass through an axis, the rigid body is then improperly constrained (unstable)

4.9 Reaction at Supports and Connections for a Three-Dimensional Structures

1. force with known line of action : 1 unknown

   e.g. ball support, smooth surface, cable

2. two force components : 2 unknowns

   e.g. roller on rough surface, wheel on rails

3. three force components : 3 unknowns

   e.g. rough surface, ball and socket

4. two force components and two couples : 4 unknowns

   e.g. hinge and bearing supporting radial load only

5. three force components and one couple : 4 unknowns

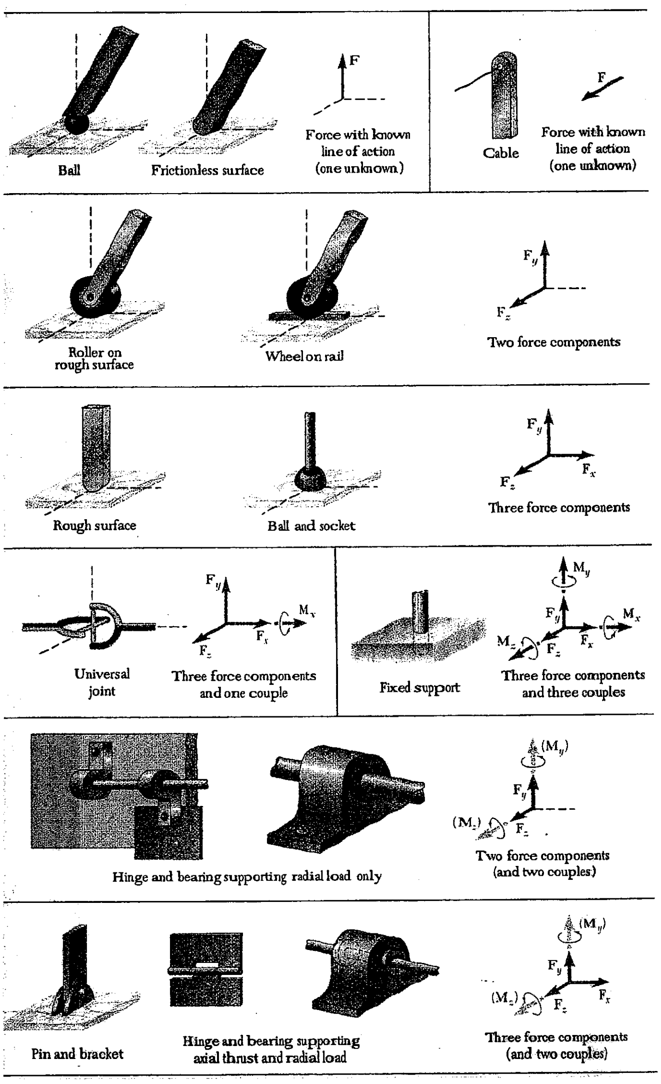
   e.g. universal joint

6. three force components and two couples : 5 unknowns

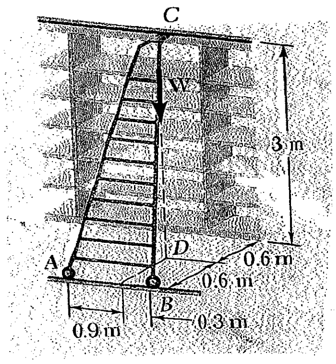
   e.g. pin and bracket, hinge and bearing supporting axial thrust and radial load

7. three force components and three couples : 6 unknowns

   e.g. fixed end



Sample Problem 4.7



mass of ladder $=$ 20 kg

mass of man $=$ 80 kg

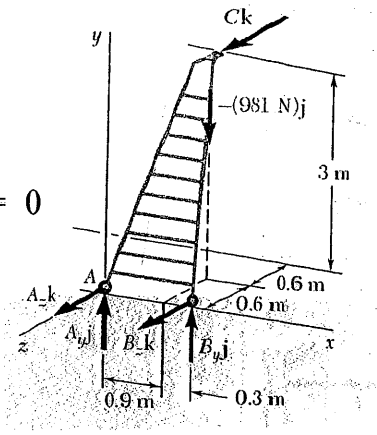
determine the reactions at A, B, and C

$W = (20 + 80) \times 9.81 = 981$ N

$\boldsymbol{W} = -981\,\boldsymbol{j}$ (N)

$A(0, 0, 0)$ $\quad$ $B(1.2, 0, 0,)$

$C(0.6, 3, -1.2)$ $\quad$ $D(0.9, 0, -0.6)$



$\Sigma\,\boldsymbol{F} = 0 \quad (A_y + B_y - 981)\,\boldsymbol{j} + (A_z + B_z + C)\,\boldsymbol{k} = 0$

$\Sigma\,\boldsymbol{M}_A = 0 \quad 1.2\,\boldsymbol{i} \times (B_y\boldsymbol{j} + B_z\boldsymbol{k}) + (0.9\,\boldsymbol{i} - 0.6\,\boldsymbol{k})$

$\quad \times (-981\,\boldsymbol{j}) + (0.6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 1.2\,\boldsymbol{k}) \times C\,\boldsymbol{k} = 0$

i.e. $\quad (3\,C - 588.6)\,\boldsymbol{i} - (1.2\,B_z + 0.6\,C)\,\boldsymbol{j}$

$\quad + (1.2\,B_y - 882.9)\,\boldsymbol{k} = 0$

5 equations solve for 5 unknowns

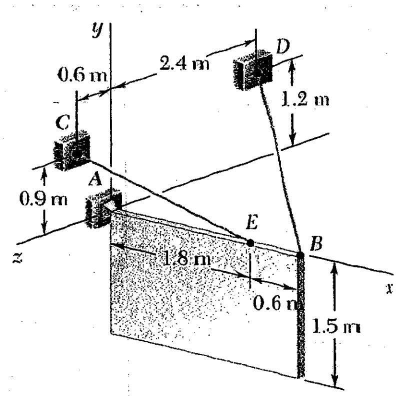
$\boldsymbol{A} = 245\,\boldsymbol{j} - 98.1\,\boldsymbol{k}$ (N)

$\boldsymbol{B} = 736\,\boldsymbol{j} - 98.1\,\boldsymbol{k}$ (N)

$\boldsymbol{C} = 196.2\,\boldsymbol{k}$ (N)

$\Sigma\,F_x = 0$ $\quad$ does not used, partially constrained

Sample Problem 4.8



A is a ball-and-socket joint

weight of the plate is 1200 N

determine the tension in cables and reaction A

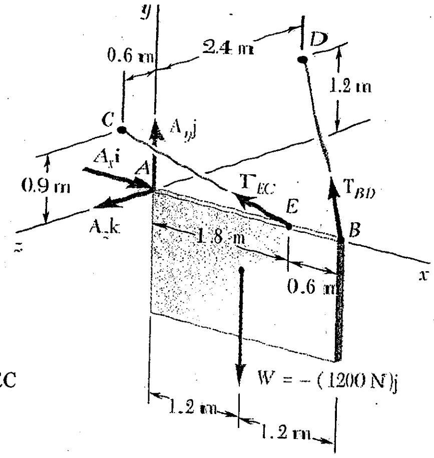
$B(2.4, 0, 0)$ $\ C(0, 0.9, 0.6)$ $\ D(0, 1.2, -2.4)$ $\ E(1.8, 0, 0)$

$\boldsymbol{BD} = -2.4\,\boldsymbol{i} + 1.2\,\boldsymbol{j} - 2.4\,\boldsymbol{k} \qquad BD = 3.6$ m

$\boldsymbol{EC} = -1.8\,\boldsymbol{i} + 0.9\,\boldsymbol{j} + 0.6\,\boldsymbol{k} \qquad EC = 2.1$ m

$$T_{BD} = T_{BD}\frac{\boldsymbol{BD}}{BD} = T_{BD}\left(-\frac{2}{3}\boldsymbol{i} + \frac{1}{3}\boldsymbol{j} - \frac{2}{3}\boldsymbol{k}\right)$$

$$T_{EC} = T_{EC}\frac{\boldsymbol{EC}}{EC} = T_{EC}\left(-\frac{6}{7}\boldsymbol{i} + \frac{3}{7}\boldsymbol{j} - \frac{2}{3}\boldsymbol{k}\right)$$



$$\Sigma \boldsymbol{F} = 0 \qquad A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k} + \boldsymbol{T}_{BD} + \boldsymbol{T}_{EC} - 1200\boldsymbol{j} = 0$$

$$\left(A_x - \frac{2}{3}T_{BD} - \frac{6}{7}T_{BD}\right)\boldsymbol{i} + \left(A_y + \frac{1}{3}T_{BD} + \frac{3}{7}T_{EC} - 12\right)\boldsymbol{j} + \left(A_z - \frac{2}{3}T_{BD} + \frac{2}{7}T_{EC}\right)\boldsymbol{k} = 0$$

$$\Sigma \boldsymbol{M}_A = \Sigma(\boldsymbol{r} \times \boldsymbol{F}) = 0$$

$$2.4\,\boldsymbol{i} \times T_{BD}\left(-\frac{2}{3}\boldsymbol{i} + \frac{1}{3}\boldsymbol{j} - \frac{2}{3}\boldsymbol{k}\right) + 6\,\boldsymbol{i} \times T_{EC}\left(-\frac{6}{7}\boldsymbol{i} + \frac{3}{7}j - \frac{2}{7}k\right) + 1.2\,\boldsymbol{i} \times (-1200)\boldsymbol{j} = 0$$

$$(1.6\,T_{BD} - 0.514\,T_{EC})\boldsymbol{j} + (0.8\,T_{BD} + 0.771\,T_{EC} - 1440)\boldsymbol{k} = 0$$

5 equations solve for 5 unknowns

$$\boldsymbol{A} = 1502\,\boldsymbol{i} + 449\,\boldsymbol{j} - 100.1\,\boldsymbol{k} \text{ (N)}$$

$$T_{BD} = 451 \text{ N} \qquad T_{EC} = 1402 \text{ N}$$

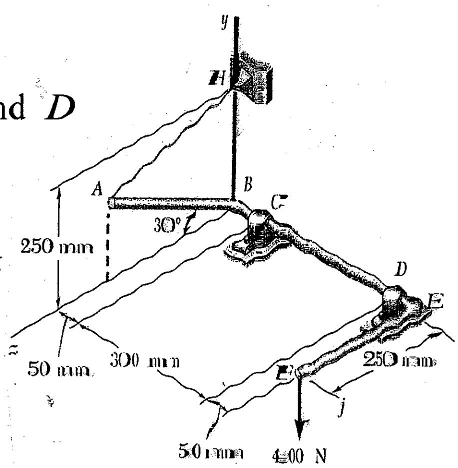
$\Sigma M_x = 0$ does not used, partially constrained

Sample Problem 4.9

$AB = 250$ mm

determine the tension in cable and reactions at C and D

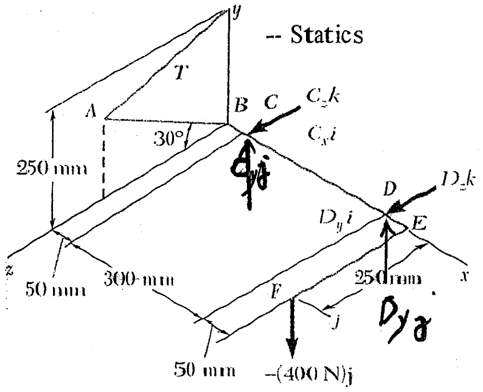
bearing D does not exert any thrust



$$\boldsymbol{T} = T\,\boldsymbol{e}_{AH} \qquad \boldsymbol{e}_{AH} = \sin 30^\circ\,\boldsymbol{j} - \cos 30^\circ\,\boldsymbol{k}$$

$$\boldsymbol{T} = T(0.5\,\boldsymbol{j} - 0.866\,\boldsymbol{k})$$

$$\boldsymbol{r}_{H/C} = -50\,\boldsymbol{i} + 250\,\boldsymbol{j} \qquad \boldsymbol{r}_{D/C} = 300\,\boldsymbol{i}$$



$\boldsymbol{r}_{F/C} = 350\,\boldsymbol{i} + 250\,\boldsymbol{k}$

$\Sigma M_C = \Sigma(\boldsymbol{r} \times \boldsymbol{F}) = 0$

$\boldsymbol{r}_{H/C} \times \boldsymbol{T} + \boldsymbol{r}_{D/C} \times \boldsymbol{D} + \boldsymbol{r}_{F/C} \times (-400\boldsymbol{j}) = 0$

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -.886 \end{vmatrix} T + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

$\boldsymbol{i}$ comp.: $-216.5\,T + 100 \times 10^3 = 0$ $\quad T = 462$ N

$\boldsymbol{j}$ comp.: $-43.3\,T - 300\,D_z = 0$ $\quad D_z = -66.67$ N

$\boldsymbol{k}$ comp.: $-25\,T + 300\,D_y - 140 \times 10^3 = 0$ $\quad D_y = 505.1$ N

$\boldsymbol{D} = 505.1\,\boldsymbol{j} - 66.67\,\boldsymbol{k}$ (N)

$\Sigma \boldsymbol{F} = 0$ $\quad \boldsymbol{C} + \boldsymbol{D} + \boldsymbol{T} - 400\,\boldsymbol{j} = 0$

$\boldsymbol{i}$ comp.: $C_x = 0$

$\boldsymbol{j}$ comp.: $-C_y + 416.9 \times 0.5 + 505.1 - 400 = 0$ $\quad C_y = -336$ N

$\boldsymbol{k}$ comp.: $C_z + 461.9 \times 0.886 - 66.67 = 0$ $\quad C_y = 467$ N

$\boldsymbol{C} = -336\,\boldsymbol{j} + 467\,\boldsymbol{k}$ (N)

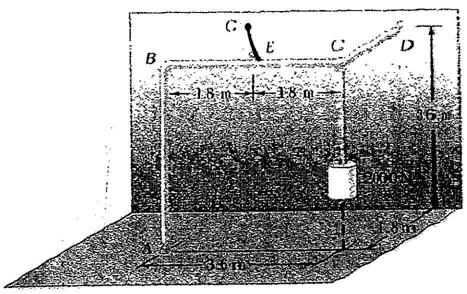
Sample Problem 4.10

A and D are ball-and-socket joints

EG : cable

find the position of G for T = minimum

and the corresponding tension force



$\boldsymbol{W} = -mg\,\boldsymbol{j} = -2000\,\boldsymbol{j}$ (N)

C(3.6, 3.6, 1.8) $\quad D$(3.6, 3.6, 0) $\quad E$(1.8, 3.6, 1.8)

to eliminate the reaction at A and D, take $\Sigma M_{AD} = 0$

$$\lambda \cdot (\boldsymbol{AE} \times \boldsymbol{T}) + \lambda \cdot (\boldsymbol{AC} \times \boldsymbol{W}) = 0$$

$$\lambda = \frac{\boldsymbol{AD}}{AD} = \frac{3.6\,\boldsymbol{i} + 3.6\,\boldsymbol{j} - 1.8\,\boldsymbol{k}}{5.4} = \frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}$$

$$\lambda \cdot (\boldsymbol{AC} \times \boldsymbol{W}) = (\frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}) \cdot [(3.6\,\boldsymbol{i} + 3.6\,\boldsymbol{j}) \times (-2000\,\boldsymbol{j})]$$

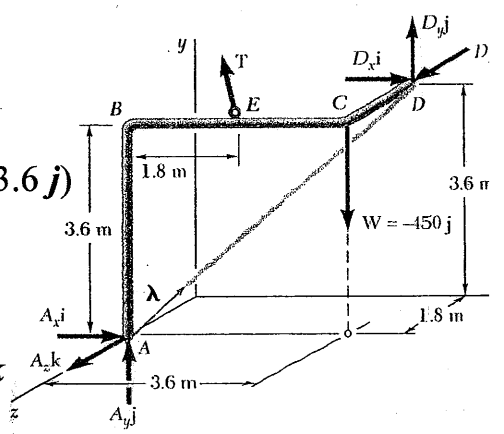
$$= 2400 \text{ N-m}$$

i.e. $\lambda \cdot (\boldsymbol{AE} \times \boldsymbol{T}) = -2400$ N-m

or $\boldsymbol{T} \cdot (\lambda \times \boldsymbol{AE}) = -2400$ N-m

thus the projection of $\boldsymbol{T}$ on $(\lambda \times \boldsymbol{AE})$ direction is constant

i.e. for $T = T_{\min}$, $\boldsymbol{T}$ must // $(\lambda \times \boldsymbol{AE})$



$$\lambda \times \boldsymbol{AE} = (\frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}) \times (1.8\,\boldsymbol{i} + 3.6\,\boldsymbol{j})$$

$$= 1.2\,\boldsymbol{i} - 0.6\,\boldsymbol{j} + 1.2\,\boldsymbol{k}$$

the corresponding unit vector is $\frac{2}{3}\boldsymbol{i} - \frac{1}{3}\boldsymbol{j} + \frac{2}{3}\boldsymbol{k}$

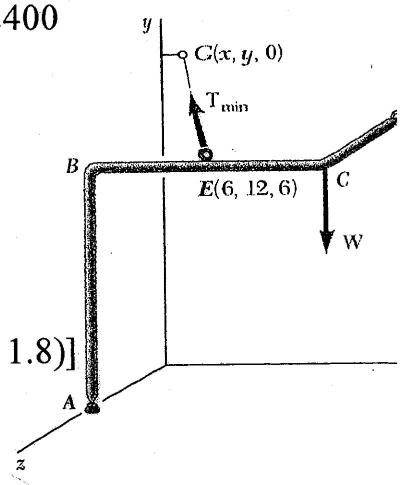
$\therefore$ $\boldsymbol{T}_{\min} = T(\frac{2}{3}\boldsymbol{i} - \frac{1}{3}\boldsymbol{j} + \frac{2}{3}\boldsymbol{k})$

$$\boldsymbol{T} \cdot (\lambda \times \boldsymbol{AE}) = -2400$$

$$T(\tfrac{2}{3}\boldsymbol{i} - \tfrac{1}{3}\boldsymbol{j} + \tfrac{2}{3}\boldsymbol{k}) \cdot \tfrac{2}{3}(4\,\boldsymbol{i} - 2\,\boldsymbol{j} + 4\,\boldsymbol{k}) = -2400$$

$$T_{\min} = -1333 \text{ N}$$

or $\boldsymbol{T}_{\min} = -888.8\,\boldsymbol{i} + 444.4\,\boldsymbol{j} - 888.8\,\boldsymbol{k}$ (N)



the location of G is $(x, y, 0)$, $\boldsymbol{EG}$ // $\boldsymbol{T}$

$$\frac{x - 1.8}{-888.8} = \frac{y - 3.6}{444.4} = \frac{0 - 1.8}{-888.8} \qquad [E(1.8, 3.6, 1.8)]$$

thus we have $x = 0$ $y = 4.5$ m

in this problem, we have 7 unknowns [A : 3 unknowns, D : 3 unknowns, T : 1 unknown], it is a statically indeterminate problem

without T, $\Sigma M_{AD} \neq 0$, [A and D pass through AD] : improperly constrained