Chapter 2 Statics of Particles

2.1 Introduction

two or more forces acting on particles ==> resultant

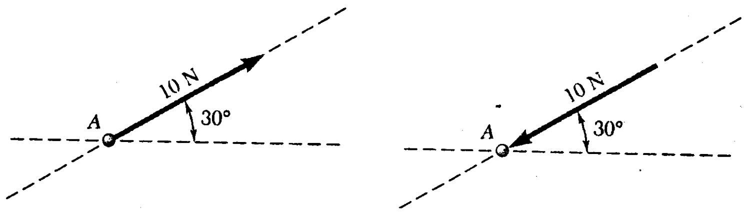
equilibrium

force in plane (sections 2.2 - 2.11)

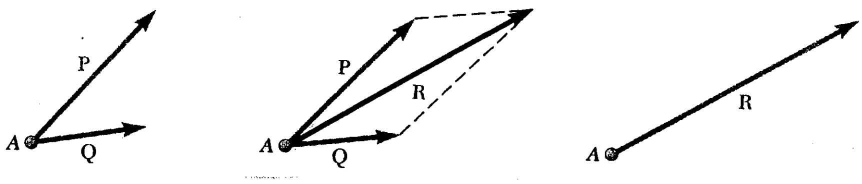
force in space (sections 2.12 - 14)

2.2 Force on a Particle, Resultant of Two Forces

force : point of application, magnitude and direction



resultant :



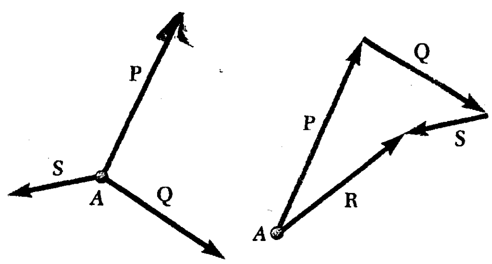
2.3 Vectors

2.4 Addition of Vectors

2.5 Resultant of Several Concurrent Forces

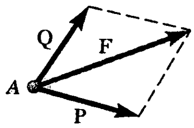
concurrent force : all forces pass through a point

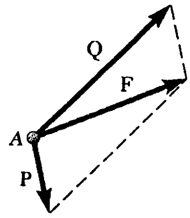
coplanar force : forces contained in the same plane



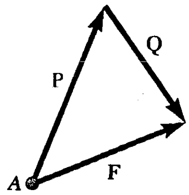
2.6 Resolution of a Force into Components

for each force ***F***, there exist an infinite number of possible sets of components, two cases are particular interest :

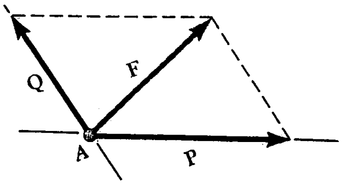




1. one of the two components ***P*** is known

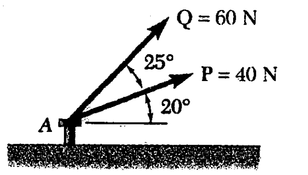


2. the line of action of each component is known



Sample Problem 2.1

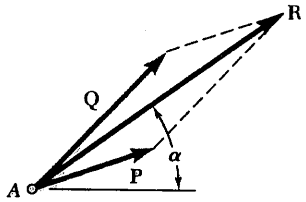
determine the resultant of two forces ***P*** and ***Q***



graphical solution

$R = 98$ N $\qquad \alpha = 35°$

$\boldsymbol{R} = 98$ N $\angle 35°$

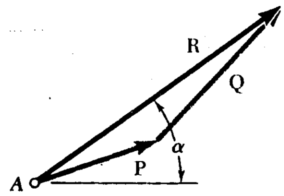


trigonometric solution

$R^2 = P^2 + Q^2 - 2\,P\,Q \cos \beta$

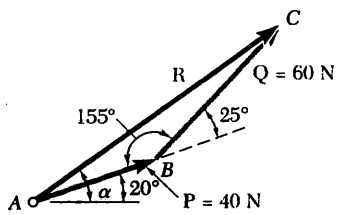
$= 40^2 + 60^2 - 2 \times 40 \times 60 \cos 155°$

$R = 97.73$ N



sine law

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60} = \frac{\sin 155°}{97.73}$$



$A = 15.04°$ $\qquad \alpha = A + 20° = 35.04°$

$\boldsymbol{R} = 97.73$ N $\angle 35.04°$

alternative method

$CD = 60 \times \sin 25° = 25.36$ N

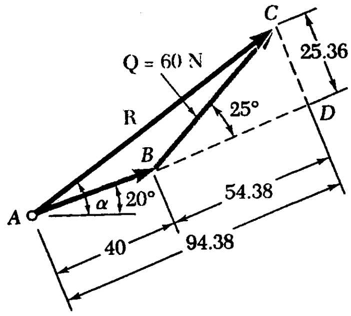
$BD = 60 \text{ x} \cos 25^{o} = 54.38 \text{ N}$

$$\tan A = \frac{CD}{AD} = \frac{25.36}{54.38 + 40}$$

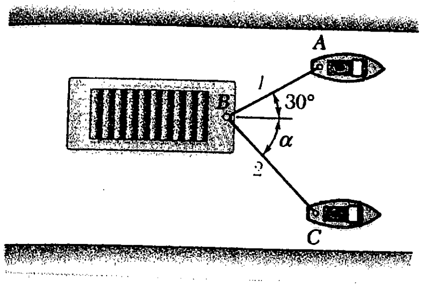
$A = 15.04^{o} \qquad \alpha = A + 20^{o} = 35.04^{o}$

$R = 25.36 / \sin \text{A} = 97.73\ N$

$\boldsymbol{R} = 97.73 \text{ N} \ \angle 35.04^{o}$



Sample Problem 2.2



for $\boldsymbol{R} = 25 \text{ kN} \ \rightarrow$

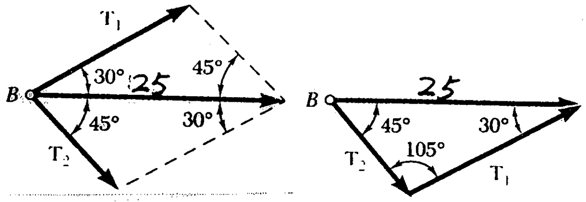
(a) fine T_1 and T_2 for $\alpha = 45^{o}$

(b) determine α for T_2 = minimum

(a) sine law

$$\frac{T_1}{\sin 45^{o}} = \frac{T_2}{\sin 30^{o}} = \frac{25}{\sin 105^{o}}$$

$T_1 = 18.5 \text{ kN} \qquad T_2 = 13 \text{ kN}$

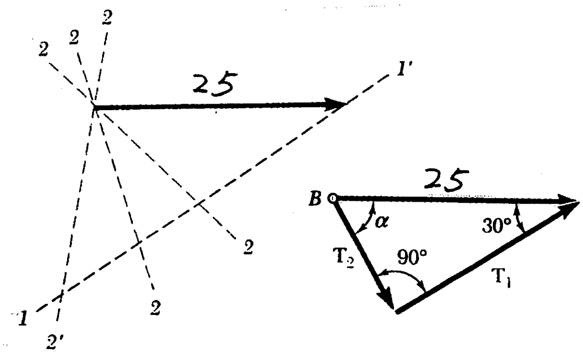


(b) for T_2 = minimum, $T_2 \perp T_1$

then $\alpha = 60^{o}$

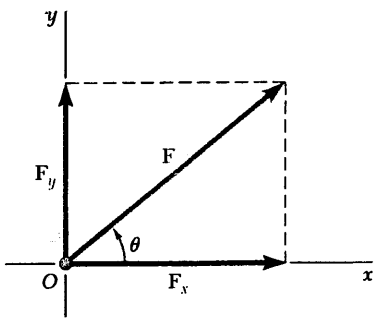
$T_2 = 25 \sin 30^{o} = 12.5 \text{ kN}$

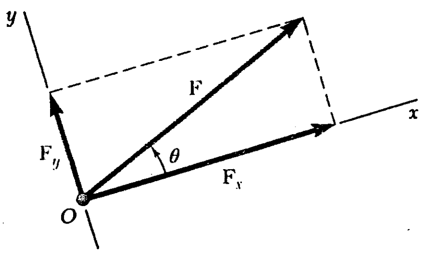
$T_1 = 25 \cos 30^{o} = 21.7 \text{ kN}$



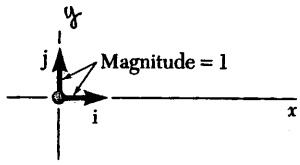
2.7 Rectangular Components of a Force, Unit Vectors

$\boldsymbol{F}$ may be resolved into a component F_x along x-axis and a component F_y along y-axis, x and y axes are usually chosen horizontal and vertical, respectively, they may chosen in any two perpendicular directions

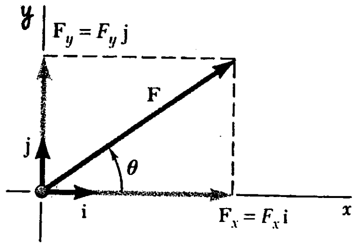




unit vectors : vectors has magnitude equal 1, directed respectively along the positive x and y axes, they may denoted by $\boldsymbol{i}$ and $\boldsymbol{j}$, respectively



the force $\boldsymbol{F}$ may resolved into $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$



$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y$$

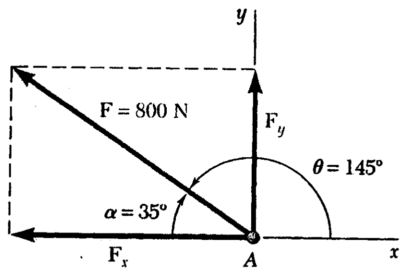
$$\boldsymbol{F}_x = F_x \boldsymbol{i} \qquad \boldsymbol{F}_y = F_y \boldsymbol{j}$$

the scalars F_x and F_y are called scalars components of the force $\boldsymbol{F}$

denoting by F the magnitude of the force $\boldsymbol{F}$ and by θ the angle between $\boldsymbol{F}$ and the x-axis, measured counterclockwise from the positive x-axis, then

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

example 1



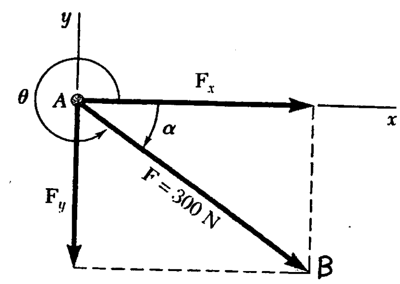
$$F = 800\text{ N} \qquad \alpha = 35^{\circ} \qquad \theta = 145^{\circ}$$

$$F_x = F\cos\theta = 800\cos 145^{\circ} = -655\text{ N}$$

$$F_y = F\sin\theta = 800\sin 145^{\circ} = 459\text{ N}$$

$$\boldsymbol{F} = -655\,\boldsymbol{i} + 459\,\boldsymbol{j} \quad \text{(N)}$$

example 2



$$F = 300\text{ N} \qquad AB = (6^2 + 8^2)^{1/2} = 10\text{ m}$$

$$\sin\alpha = 6/10 = 0.6 \qquad \cos\alpha = 8/10 = 0.8$$

$$F_x = 300\cos\alpha = 240\text{ N}$$

$$F_y = -300\sin\alpha = -180\text{ N}$$

$$\boldsymbol{F} = 240\,\boldsymbol{i} - 180\,\boldsymbol{j} \quad \text{(N)}$$

when a force $\boldsymbol{F}$ is defined by its rectangular components F_x and F_y, the angle θ can be obtained as

$$\tan\theta = F_y / F_x$$

the magnitude of $\boldsymbol{F}$ can be written as

$$F = (F_x^2 + F_y^2)^{1/2}$$

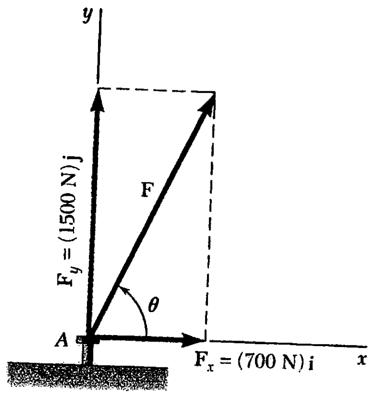
example 3

$$F = 700\,\boldsymbol{i} + 1500\,\boldsymbol{j} \quad \text{(N)}$$

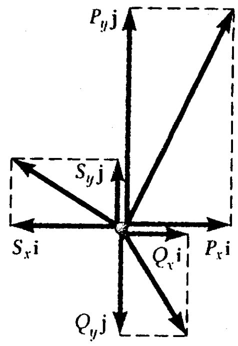
$$\tan\theta = 1500/700 = 2 \quad \Rightarrow \quad \theta = 65^\circ$$

$$\sin\theta = F_y / F$$

$$F = F_y / \sin\theta = 1500 / \sin 65^\circ = 1655 \text{ N}$$



2.8 Addition of Forces by Summing x and y Components



$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{S}$$

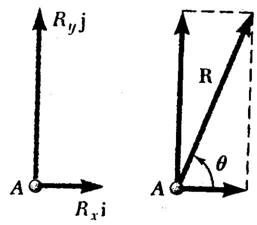
resolving each force into its rectangular components

$$R_x\,\boldsymbol{i} + R_y\,\boldsymbol{j} = (P_x\,\boldsymbol{i} + P_y\,\boldsymbol{j}) + (Q_x\,\boldsymbol{i} + Q_y\,\boldsymbol{j}) + (S_x\,\boldsymbol{i} + S_y\,\boldsymbol{j})$$

$$= (P_x + Q_x + S_x)\,\boldsymbol{i} + (P_y + Q_y + S_y)\,\boldsymbol{j}$$

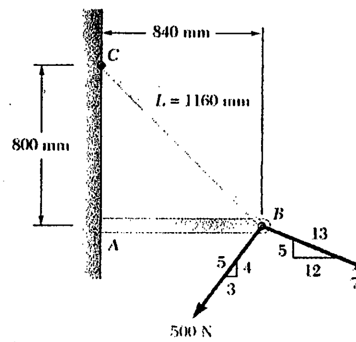
or $\quad R_x = P_x + Q_x + S_x = \Sigma F_x$

$$R_y = P_y + Q_y + S_y = \Sigma F_y$$



Sample Problem 2.3

$T_{BC} = 725$ N, determine the resultant force



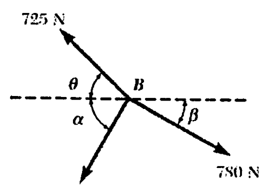
$\cos\theta = 840/1160 = 21/29 \qquad \sin\theta = 20/29$

$\cos\alpha = 3/5$, $\sin\alpha = 4/5$, $\cos\beta = 12/13$, $\sin\beta = 5/13$

magnitude (N)	x-comp. (N)	y-comp. (N)
725	- 525	500
500	- 300	- 400
780	720	- 300

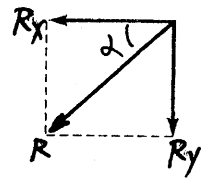
$$R_x = -105 \qquad R_y = -200$$

$$\boldsymbol{R} = R_x\,\boldsymbol{i} + R_y\,\boldsymbol{j} = -105\,\boldsymbol{i} - 200\,\boldsymbol{j} \quad \text{(N)}$$



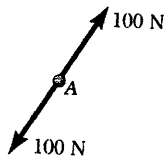
$$R = (105^2 + 200^2)^{½} = 255.9 \text{ N}$$

$$\theta = \tan^{-1}(-R_y / -R_x) = \tan^{-1}(200/105) = 62.3^{\circ}$$

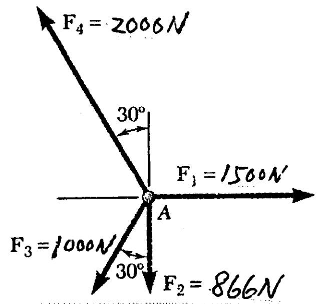


2.9 Equilibrium of a Particle

when the resultant of the forces acting on a particle is zero, the particle is in equilibrium

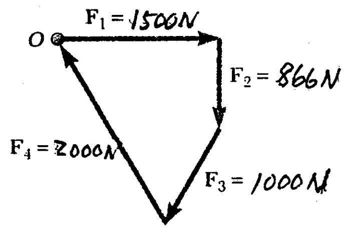


two forces acting on a particle, it will be in equilibrium if the two forces have the same magnitude, same line of action, and opposite direction



when three or more forces acting on a particle, the particle will be in equilibrium if the force polygon is closed, the closed polygon provides a graphical expression of the equilibrium of the point

the express algebraically the conditions for the equilibrium of a particle, we write



$$\boldsymbol{R} = \Sigma \boldsymbol{F} = 0$$

$$\Sigma (F_x \boldsymbol{i} + F_y \boldsymbol{j}) = 0$$

$$\text{i.e.} \quad \Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0$$

we conclude that the necessary and sufficient conditions for the equilibrium of a particle are

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \qquad \text{[equations of equilibrium]}$$

for example

$$\Sigma F_x = 2000 - 1000 \sin 30^{\circ} - 2000 \sin 30^{\circ} = 0$$

$$\Sigma F_y = -866 - 1000 \cos 30^{\circ} + 2000 \cos 30^{\circ} = 0$$

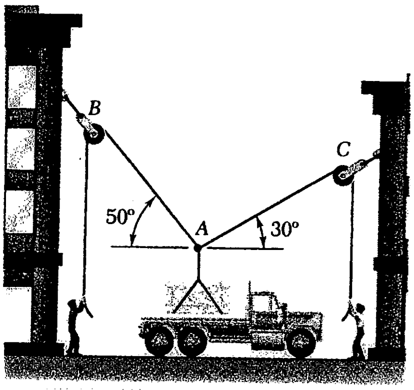
these equations may be solved for no more than two unknowns

2.10 Newton's First Law of Motion

if the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest), or will move with constant speed in straight line (if originally in motion)

2.11 Problem Involving the Equilibrium of Particle, Free-Body Diagram

free-body diagram : for particle system these diagram isolate each particle to which forces are applied and shown every force exerted on the particle

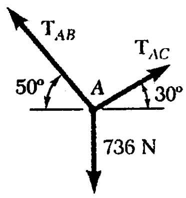


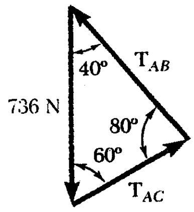
take A as a free body

$$W = \text{m g} = 75 \times 9.81 = 736 \text{ N}$$

$$\frac{T_{AB}}{\sin 60^o} = \frac{T_{AC}}{\sin 40^o} = \frac{736}{\sin 80^o}$$

$$T_{AB} = 647 \text{ N} \qquad T_{AC} = 480 \text{ N}$$





equations of equilibrium

$$\Sigma F_x = 0 \qquad T_{AC} \cos 30^o - T_{AB} \cos 50^o = 0$$

$$\Sigma F_y = 0 \qquad T_{AC} \sin 30^o + T_{AB} \sin 50^o - 736 = 0$$

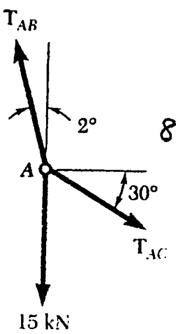
two equations can be solved for two unknowns

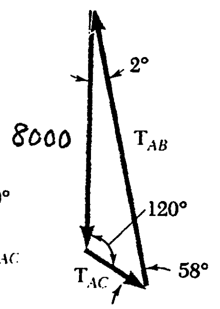
$$T_{AB} = 647 \text{ N} \qquad T_{AC} = 480 \text{ N}$$

Sample Problem 2.4

$W = 8,000$ N, find T_{AB} and T_{AC}

take free body at A

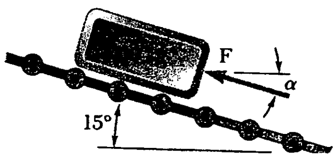




$$\frac{T_{AB}}{\sin 120^o} = \frac{T_{AC}}{\sin 2^o} = \frac{8,000}{\sin 58^o}$$

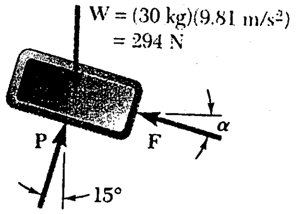
$$T_{AB} = 8,710 \text{ N} \qquad T_{AC} = 329 \text{ N}$$

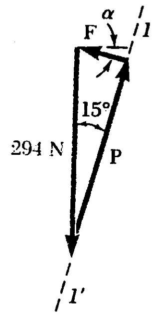
Sample Problem 2.5



determine the magnitude and direction of the smallest force F which will remain the package shown in equilibrium

free-body diagram ($\boldsymbol{P}$ must $\perp$ $\boldsymbol{F}$)



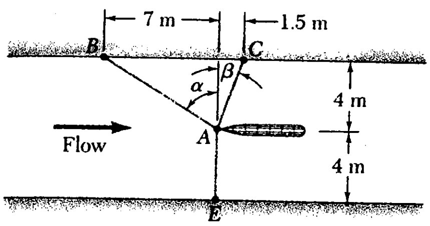


W = m g = 30 x 9.81 = 294 N

F = 294 sin 15° = 76.1 N $\qquad \alpha = 15^\circ$

$\boldsymbol{F}$ = 76.1 N $\searrow$ 15°

Sample Problem 2.6

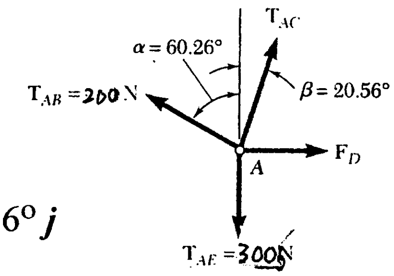


T_{AB} = 200 N $\qquad T_{AE}$ = 300 N

determine the drag force and T_{AC}

$\tan\alpha = 7/4 = 1.75 \qquad \alpha = 60.26^\circ$

$\tan\beta = 1.5/4 = 0.375 \qquad \beta = 20.56^\circ$



take free body at A, equilibrium condition

$$\boldsymbol{R} = \boldsymbol{T}_{AB} + \boldsymbol{T}_{AC} + \boldsymbol{T}_{AE} + \boldsymbol{F}_D = 0$$

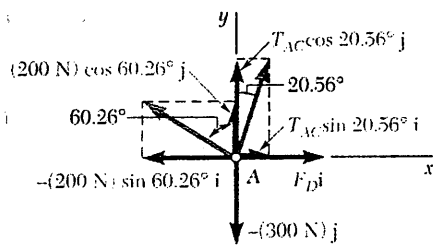
$$\boldsymbol{T}_{AB} = -200\sin 60.26^\circ\,\boldsymbol{i} + 200\cos 60.26^\circ\,\boldsymbol{j}$$

$$= -173.66\,\boldsymbol{i} + 99.21\,\boldsymbol{j} \quad \text{(N)}$$

$$\boldsymbol{T}_{AC} = T_{AC}\sin 20.56^\circ\,\boldsymbol{i} + T_{AC}\cos 20.56^\circ\,\boldsymbol{j}$$

$$= 0.3512\,T_{AC}\,\boldsymbol{i} + 0.9363\,T_{AC}\,\boldsymbol{j}$$

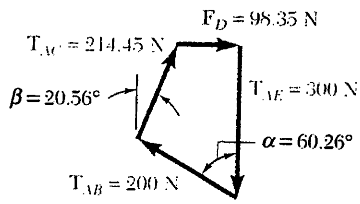
$$\boldsymbol{T}_{AE} = -300\,\boldsymbol{j} \quad \text{(N)} \qquad \boldsymbol{F}_D = F_D\,\boldsymbol{i}$$



$$\Sigma F_x = 0 \qquad -173.66 + 0.3512\,T_{AC} + F_D = 0$$

$$\Sigma F_y = 0 \qquad 99.21 + 0.9363\,T_{AC} - 300 = 0$$

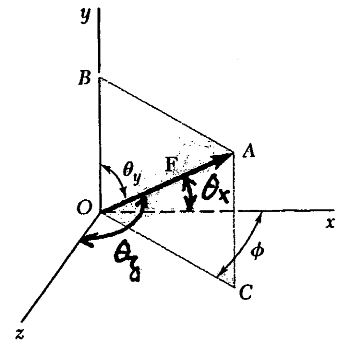
two equation solve for two unknowns



T_{AC} = 214.45 N $\qquad F_D$ = 98.35 N

FORCES IN SPACE

2.12 Rectangular Components of a Force in Space

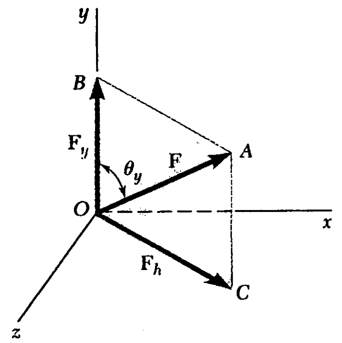


$$\boldsymbol{F} = \boldsymbol{F}_h + \boldsymbol{F}_y$$

$\boldsymbol{F}_y$: vertical component of $\boldsymbol{F}$

$\boldsymbol{F}_h$: horizontal component of $\boldsymbol{F}$

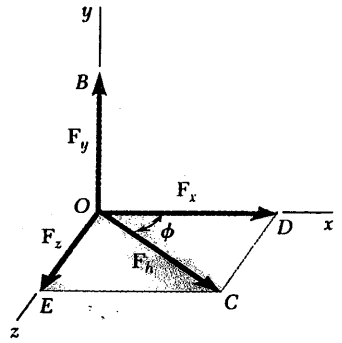
$$F_y = F\cos\theta_y \qquad F_h = F\sin\theta_y$$



$\boldsymbol{F}_h$ may be solved in $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$

$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

$$F_y = F_h\sin\phi = F\sin\theta_y\sin\phi$$



$$\therefore \quad \boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z$$

$$= F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k}$$

$$F^2 = (OA)^2 = (OB)^2 + (OC)^2 = F_y^2 + F_h^2$$

$$F_h^2 = (OD)^2 + (OE)^2 = F_x^2 + F_z^2$$

$$\therefore \quad F = [F_x^2 + F_y^2 + F_z^2]^{1/2}$$

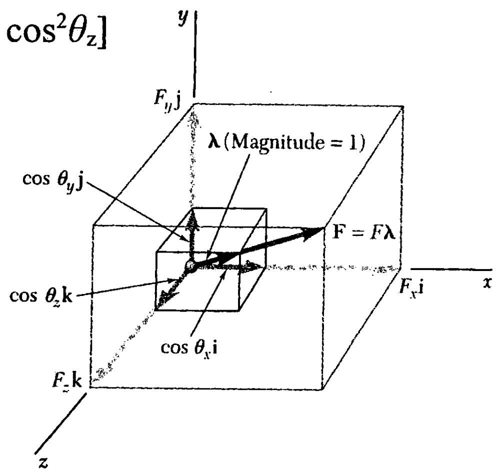
$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

θ_x, θ_y, θ_z are the angles between the force $\boldsymbol{F}$ and the x, y, z axes, respectively, the cosines of θ_x, θ_y, θ_z are known as the direction cosines of the force $\boldsymbol{F}$

$$F_x^2 + F_y^2 + F_z^2 = F^2\cos^2\theta_x + F^2\cos^2\theta_y + F^2\cos^2\theta_z$$

$$F^2 = F^2\,[\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z]$$

thus $\qquad \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$



$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k}$$

$$= F(\cos\theta_x\,\boldsymbol{i} + \cos\theta_y\,\boldsymbol{j} + \cos\theta_z\,\boldsymbol{k})$$

$$= F\,\boldsymbol{\lambda}$$

$\boldsymbol{\lambda}$ is a vector of magnitude equal 1 and of the same direction as $\boldsymbol{F}$ with

$$\lambda_x = \cos\theta_x \qquad \lambda_y = \cos\theta_y \qquad \lambda_z = \cos\theta_z$$

and $$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$$

example 1

$$F = 500 \text{ N} \qquad \theta_x = 60^\circ \quad \theta_y = 45^\circ \quad \theta_z = 120^\circ$$

$$F_x = F\cos\theta_x = 500\cos 60^\circ = 250 \text{ N}$$

$$F_y = F\cos\theta_y = 500\cos 45^\circ = 354 \text{ N}$$

$$F_z = F\cos\theta_z = 500\cos 120^\circ = -250 \text{ N}$$

$$\therefore \quad \boldsymbol{F} = 250\,\boldsymbol{i} + 354\,\boldsymbol{j} - 250\,\boldsymbol{k} \quad \text{(N)}$$

example 2

$$F_x = 20 \text{ N} \qquad F_y = -30 \text{ N} \qquad F_x = 60 \text{ N}$$

$$F = [F_x^2 + F_y^2 + F_z^2]^{1/2} = [(20)^2 + (-30)^2 + (60)^2]^{1/2} = 70 \text{ N}$$

$$\cos\theta_x = F_x/F = 20/70 \quad \Rightarrow \quad \theta_x = 73.4^\circ \quad [\because F_x > 0 \therefore \theta_x < 90^\circ]$$

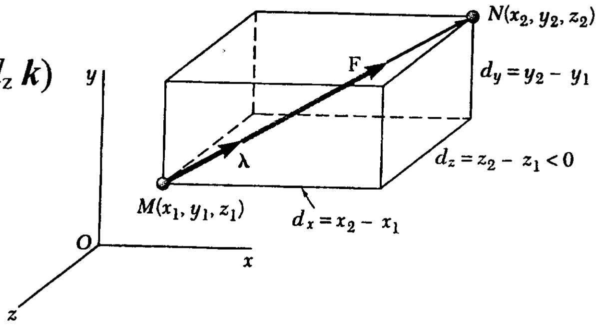
$$\cos\theta_y = F_y/F = -30/70 \quad \Rightarrow \quad \theta_x = 115.4^\circ \quad [\because F_y < 0 \therefore \theta_y > 90^\circ]$$

$$\cos\theta_x = F_x/F = 20/70 \quad \Rightarrow \quad \theta_x = 31.0^\circ \quad [\because F_z > 0 \therefore \theta_z < 90^\circ]$$

2.13 Force Defined by its Magnitude and Two Points on its Line of Action

$$\boldsymbol{MN} = d_x\,\boldsymbol{i} + d_y\,\boldsymbol{j} + d_z\,\boldsymbol{k}$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{MN}}{MN} = \frac{1}{d}(d_x\,\boldsymbol{i} + d_y\,\boldsymbol{j} + d_z\,\boldsymbol{k})$$



where d is the magnitude of $\boldsymbol{MN}$

$$d = (d_x^2 + d_y^2 + d_z^2)^{1/2}$$

$$\boldsymbol{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\,\boldsymbol{i} + d_y\,\boldsymbol{j} + d_z\,\boldsymbol{k})$$

$$F_x = F\frac{d_x}{d} \qquad F_x = F\frac{d_y}{d} \qquad F_x = F\frac{d_z}{d}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{d_x}{d} \qquad \cos\theta_y = \frac{d_y}{d} \qquad \cos\theta_z = \frac{d_z}{d}$$

2.14 Addition of Concurrent Force in Space

$$\boldsymbol{R} = \Sigma \boldsymbol{F}$$

$$R_x \boldsymbol{i} + R_y \boldsymbol{j} + R_z \boldsymbol{k} = \Sigma (F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k})$$

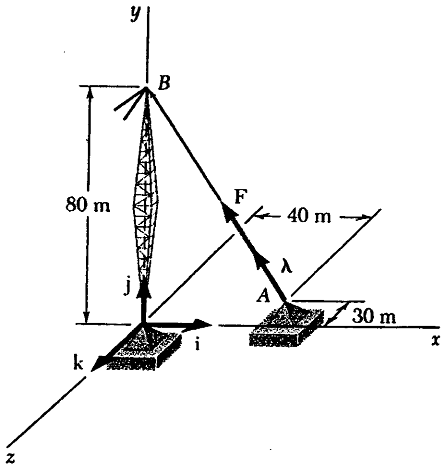
$$= \Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} + \Sigma F_z \boldsymbol{k}$$

$$\therefore \quad R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

and $\quad R = (R_x^2 + R_y^2 + R_z^2)^{1/2}$

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

Sample Problem 2.7

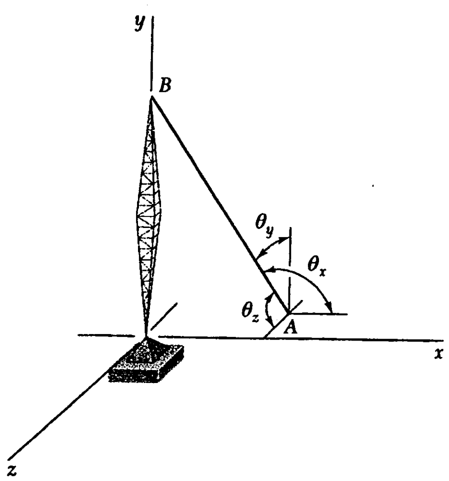


tension force on AB is 2500 N

determine F_x, F_y, F_z and θ_x, θ_y, θ_z at A

$A(40, 0, -30) \qquad B(0, 80, 0)$

the components of vector $\boldsymbol{AB}$ which has the same direction as the force $\boldsymbol{F}$



$$\boldsymbol{AB} = (0 - 40)\,\boldsymbol{i} + (80 - 0\,)\,\boldsymbol{j} + [0 - (-30)]\,\boldsymbol{k}$$

$$= -40\,\boldsymbol{i} + 80\,\boldsymbol{j} + 30\,\boldsymbol{k} \quad \text{(m)}$$

$$AB = [40^2 + 80^2 + 30^2]^{1/2} = 94.3 \text{ m}$$

$$\lambda = \frac{\boldsymbol{AB}}{AB} = \frac{1}{94.3}(-40\,\boldsymbol{i} + 80\,\boldsymbol{j} + 30\,\boldsymbol{k})$$

$$\boldsymbol{F} = F\lambda = \frac{2500}{94.3}(-40\,\boldsymbol{i} + 80\,\boldsymbol{j} + 30\,\boldsymbol{k})$$

$$\boldsymbol{F} = -1060\,\boldsymbol{i} + 2120\,\boldsymbol{j} + 795\,\boldsymbol{k} \qquad \text{(N)}$$

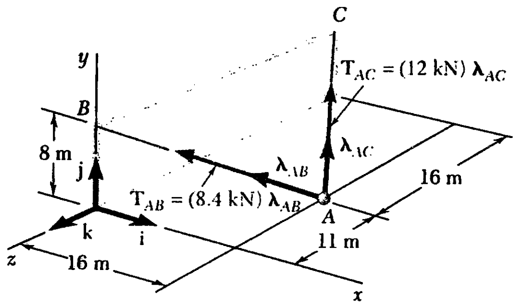
i.e. $F_x = -1060$ N $\quad F_y = -2120$ N $\quad F_z = -795$ N

$$\cos\theta_x = \frac{F_x}{F} = -\frac{1060}{2500} \quad \Rightarrow \quad \theta_x = 115.1^\circ$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{2120}{2500} \quad \Rightarrow \quad \theta_y = 32^\circ$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{795}{2500} \quad \Rightarrow \quad \theta_z = 71.5^\circ$$

Sample Problem 2.8



$T_{AB} = 8.4$ kN $\qquad T_{AC} = 12$ kN

determine $\boldsymbol{R}$ at A

$A(16, 0\ {-11})\quad B(0, 8, 0)\quad C(0, 8, -27)$

$$\boldsymbol{AB} = -16\,\boldsymbol{i} + 8\,\boldsymbol{j} + 11\,\boldsymbol{k} \text{ (m)} \qquad AB = 21 \text{ m}$$

$$\boldsymbol{AC} = -16\,\boldsymbol{i} + 8\,\boldsymbol{j} - 16\,\boldsymbol{k} \text{ (m)}\, AC = 24 \text{ m}$$

$$\boldsymbol{T}_{AB} = T_{AB}\,\lambda_{AB} = T_{AB}\frac{\boldsymbol{AB}}{AB} = \frac{8.4}{21}(-16\,\boldsymbol{i} + 8\,\boldsymbol{j} + 11\,\boldsymbol{k})$$

$$= -6.4\,\boldsymbol{i} + 3.2\,\boldsymbol{j} + 4.4\,\boldsymbol{k} \text{ (kN)}$$

$$\boldsymbol{T}_{AC} = T_{AC}\,\lambda_{AC} = T_{AC}\frac{\boldsymbol{AC}}{AC} = \frac{12}{24}(-16\,\boldsymbol{i} + 8\,\boldsymbol{j} - 16\,\boldsymbol{k})$$

$$= -8\,\boldsymbol{i} + 4\,\boldsymbol{j} - 8\,\boldsymbol{k} \text{ (kN)}$$

$$\boldsymbol{R} = \boldsymbol{T}_{AB} + \boldsymbol{T}_{AB} = -14.4\,\boldsymbol{i} + 7.2\,\boldsymbol{j} - 3.6\,\boldsymbol{k} \text{ (kN)}$$

$$R = [(-14.4)^2 + 7.2^2 + (-3.6)^2]^{1/2} = 16.5 \text{ kN}$$

the direction cosines of the resultant are

$$\cos\theta_x = \frac{R_x}{R} = -\frac{14.4}{16.5} \quad \Rightarrow \quad \theta_x = 150.8^\circ$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{7.2}{16.5} \quad \Rightarrow \quad \theta_y = 64.1^\circ$$

$$F = -1060\,\boldsymbol{i} + 2120\,\boldsymbol{j} + 795\,\boldsymbol{k} \qquad \text{(N)}$$

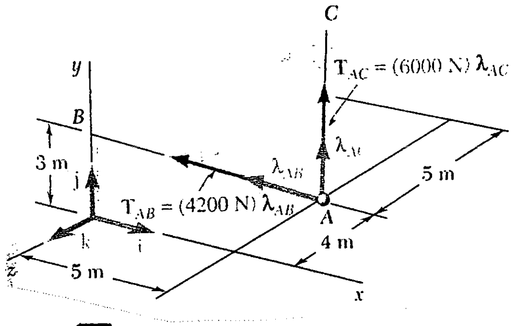
i.e. $F_x = -1060$ N $\quad F_y = -2120$ N $\quad F_z = -795$ N

$$\cos\theta_x = \frac{F_x}{F} = -\frac{1060}{2500} \quad => \quad \theta_x = 115.1^o$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{2120}{2500} \quad => \quad \theta_y = 32^o$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{795}{2500} \quad => \quad \theta_z = 71.5^o$$

Sample Problem 2.8



$T_{AB} = 4{,}200$ N $\qquad T_{AC} = 6{,}000$ N

determine $\boldsymbol{R}$ at A

$A(5, 0, -4) \qquad B(0, 3, 0) \quad C(0, 3, -9)$

$$\boldsymbol{AB} = -5\,\boldsymbol{i} + 3\,\boldsymbol{j} + 4\,\boldsymbol{k} \text{ (m)} \quad AB = 5\sqrt{2}\text{ m}$$

$$\boldsymbol{AC} = -5\,\boldsymbol{i} + 3\,\boldsymbol{j} - 5\,\boldsymbol{k} \text{ (m)} \quad AC = \sqrt{59}\text{ m}$$

$$\boldsymbol{T}_{AB} = T_{AB}\,\lambda_{AB} = T_{AB}\frac{\boldsymbol{AB}}{AB} = \frac{4{,}200}{5\sqrt{2}}(-5\,\boldsymbol{i} + 3\,\boldsymbol{j} + 4\,\boldsymbol{k})$$

$$= -2969.9\,\boldsymbol{i} + 1781.9\,\boldsymbol{j} + 42375.9\,\boldsymbol{k} \text{ (N)}$$

$$\boldsymbol{T}_{AC} = T_{AC}\,\lambda_{AC} = T_{AC}\frac{\boldsymbol{AC}}{AC} = \frac{6000}{\sqrt{59}}(-5\,\boldsymbol{i} + 3\,\boldsymbol{j} - 5\,\boldsymbol{k})$$

$$= -3905.7\,\boldsymbol{i} + 2343.4\,\boldsymbol{j} - 3905.7\,\boldsymbol{k} \quad \text{(N)}$$

$$\boldsymbol{R} = \boldsymbol{T}_{AB} + \boldsymbol{T}_{AB} = -6875.6\,\boldsymbol{i} + 4125.3\,\boldsymbol{j} - 1529.8\,\boldsymbol{k} \qquad \text{(N)}$$

$$R = [R_x^2 + R_y^2 + R_z^2]^{1/2} = 8162.9 \text{ N}$$

the direction cosines of the resultant are

$$\cos\theta_x = \frac{R_x}{R} = -\frac{6875.6}{8162.9} \quad => \quad \theta_x = 147.4^o$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{4125.3}{8162.9} \quad => \quad \theta_y = 59.6^o$$

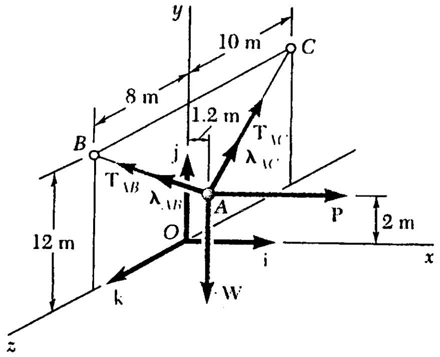
$$\cos\theta_z = \frac{R_z}{R} = -\frac{1529.8}{8162.9} \Rightarrow \theta_z = 100.8^\circ$$

2.15 Equilibrium of a Particle in Space

a particle A is in equilibrium if the resultant of all the forces acting on A is zero, i.e.

$$\boldsymbol{R} = \Sigma \boldsymbol{F} = 0 \quad \Rightarrow \quad \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

Sample Problem 2.9



m = 200 kg $\qquad W = 1962$ N

determine P, T_{AB} and T_{AC}

$A(1.2, 2, 0) \quad B(0,12, 8) \quad C(0,12, -10)$

$$\boldsymbol{AB} = -1.2\,\boldsymbol{i} + 10\boldsymbol{j} + 8\,\boldsymbol{k} \text{ (m)} \qquad AB = 12.86 \text{ m}$$

$$\lambda_{AB} = \boldsymbol{AB} / AB = -0.0933\,\boldsymbol{i} + 0.778\boldsymbol{j} + 0.622\,\boldsymbol{k}$$

$$\boldsymbol{T}_{AB} = T_{AB}\,\lambda_{AB} = -0.0933\,T_{AB}\,\boldsymbol{i} + 0.778\,T_{AB}\boldsymbol{j} + 0.622\,T_{AB}\,\boldsymbol{k}$$

$$\boldsymbol{AC} = -1.2\,\boldsymbol{i} + 10\boldsymbol{j} - 10\,\boldsymbol{k} \text{ (m)} \qquad AC = 14.19 \text{ m}$$

$$\lambda_{AC} = \boldsymbol{AC} / AC = -0.0846\,\boldsymbol{i} + 0.705\boldsymbol{j} - 0.705\,\boldsymbol{k}$$

$$\boldsymbol{T}_{AC} = T_{AC}\,\lambda_{AC} = -0.0846\,T_{AC}\,\boldsymbol{i} + 0.705\,T_{AC}\boldsymbol{j} + 0.705\,T_{AC}\,\boldsymbol{k}$$

$$\boldsymbol{P} = P\,\boldsymbol{i} \qquad \boldsymbol{W} = -W\boldsymbol{j}$$

equilibrium condition on particle A

$$\Sigma \boldsymbol{F} = 0 \qquad \boldsymbol{T}_{AB} + \boldsymbol{T}_{AC} + \boldsymbol{P} + \boldsymbol{W} = 0$$

$$\Sigma F_x = 0 \qquad -0.0933\,T_{AB} - 0.0846\,T_{AC} + P = 0$$

$$\Sigma F_y = 0 \qquad 0.778\,T_{AB} + 0.705\,T_{AC} - 1962 = 0$$

$$\Sigma F_z = 0 \qquad 0.622\,T_{AB} - 0.705\,T_{AC} = 0$$

three equations for three unknowns

$$P = 235 \text{ N} \qquad T_{AB} = 1402 \text{ N} \qquad T_{AC} = 1238 \text{ N}$$