

Viscoelasticity.

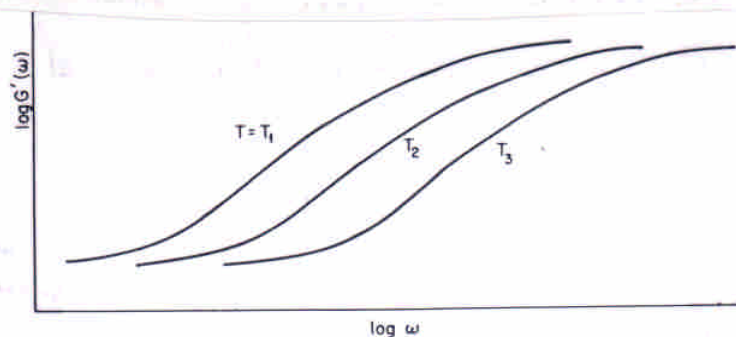
in most of materials the mechanical response in the heat conduction is very small when compared to the thermal response, may be neglected, then the heat conduction equation may be reduced to an non-coupled equation

$$\frac{k_{ij}}{T_0} \theta_{,ij} = \frac{\partial}{\partial t} \int_0^t m(t-\tau) \frac{\partial \theta(\tau)}{\partial \tau} d\tau$$

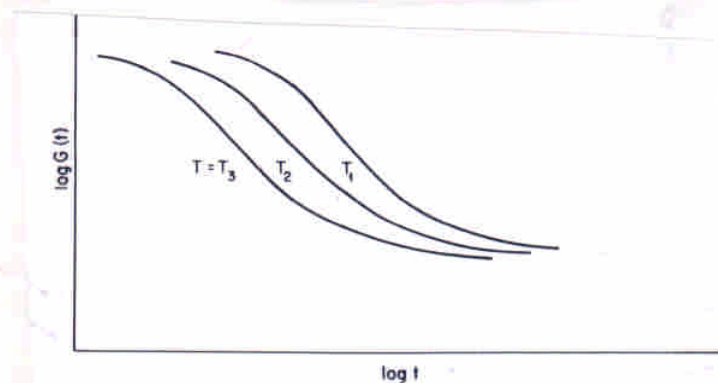
Temperature Dependence of Mechanical Properties

some of mechanical properties for typical polymeric viscoelastic materials have a very strong dependence upon temperature

temperature dependence of complex moduli and relaxation functions are schematically shown in figures below



Temperature dependence of complex moduli.

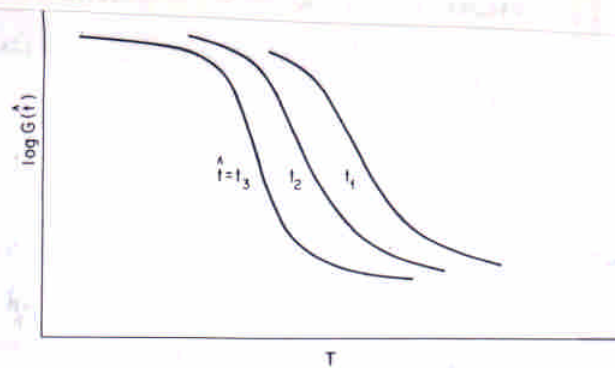


Temperature dependence of relaxation functions.

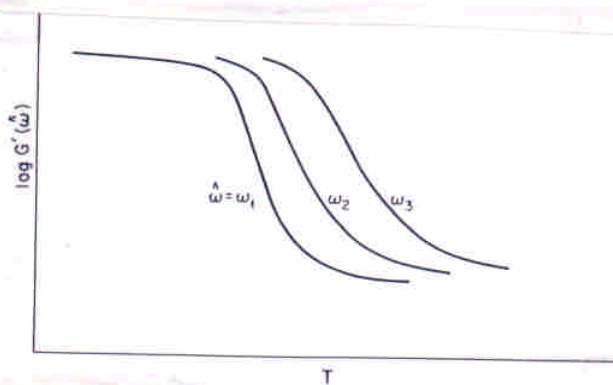
Viscoelasticity

high frequency or short-time range is referred to as the glassy region

low frequency or long-time range is referred to as the rubbery region

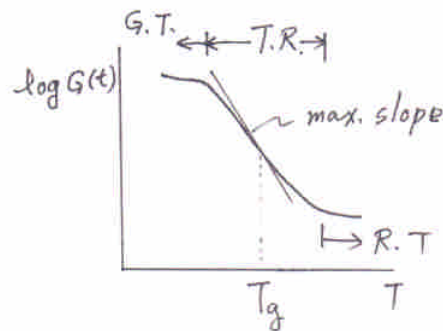
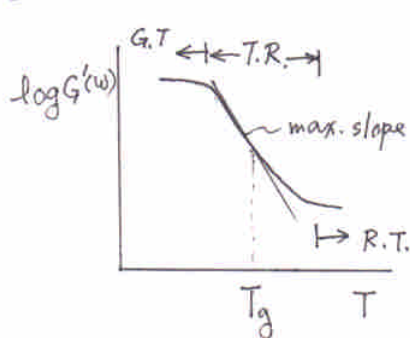


Relaxation function versus temperature.



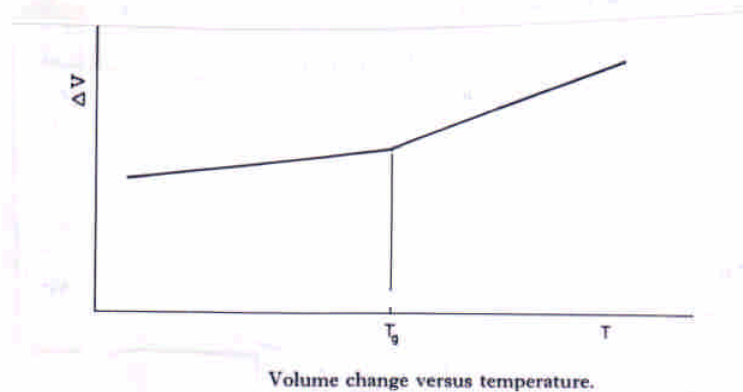
Complex modulus versus temperature.

for a fixed ω or time, the region where the modulus has the greatest dependence upon temperature (maximum magnitude of slope), is called the transition region, the corresponding temperature, or narrow temperature range, is designated as the glass transition temperature T_g , it must be noted that T_g necessarily depends upon the time of measurement t of the relaxation function, or upon the frequency of measurement ω of the complex modulus



two further means of defining T_g

(i) for a stress free sample or the material is subjected to a uniform and changing temperature field, a measurement of the volume change ΔV from some initial volume, the temperature at which the slope has a discontinuity, is defined as T_g



at high temperature, volume expansion has given the individual molecules great mobility with comparatively little constraint (low value of the modulus)

at low temperature, due to volumetric shrinking, great constraints for the motion of molecules (high value of the modulus)

(ii) for a fixed temperature, T_g defines the greatest variation of G' with ω , that is the maximum ratio of the imaginary to the real parts of the complex modulus $G''(\omega) / G'(\omega)$

the projection of T_g upon the temperature-frequency plane defines a curve which can be taken as that the glass transition temperature as a function of frequency

$$T_g = f(\omega)$$

Viscoelasticity

for a fixed ω , if $T < T_g$, material is in glassy manner, if $T > T_g$, material is in rubbery manner

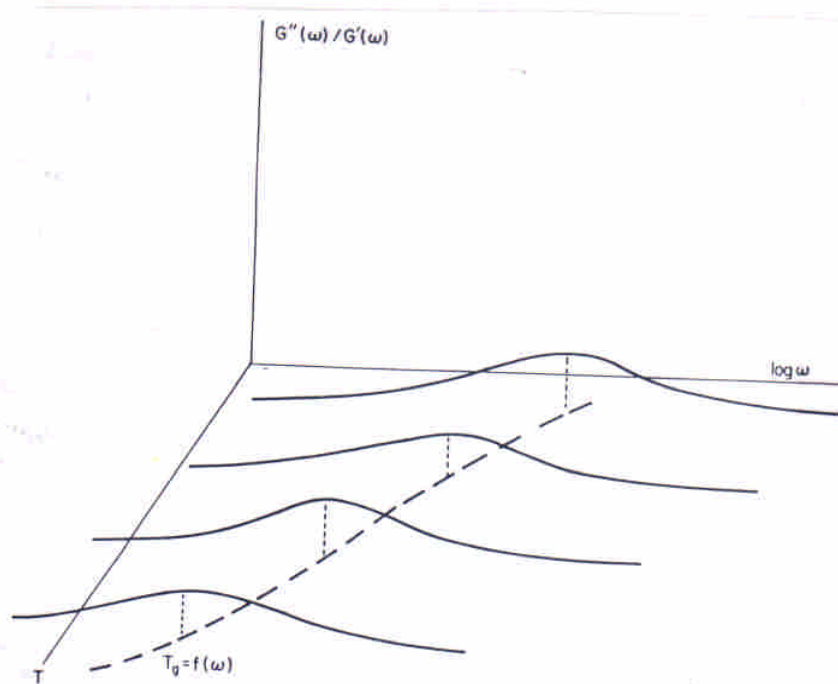


Fig. 3.6. Temperature dependence of $G''(\omega)/G'(\omega)$.

Thermorheologically Simple Materials (temperature-time equivalence)

Constant temperature state

this is a special type of temperature dependence of mechanical properties, designate the isotropic relaxation and creep functions at the base temperature $T = T_0$ by

$$G_\alpha(t) \text{ and } J_\alpha(t) \quad \text{for } T = T_0$$

where $\alpha = 1, 2$

Viscoelasticity

if the temperature field change uniformly (independent of space coordinate), the corresponding relaxation function will be

$$G_a(t, T) \leftarrow \text{function of time and temperature}$$

$$\text{with } G_a(t, T_0) = G_a(t)$$

where T denotes absolute temperature

$$\text{let } G_a(t) = L_a(\log t)$$

$$\text{then } G_a(t, T) = L_a[\log t + \phi(T)]$$

$$\text{with } \phi(T_0) = 0 \quad \text{and} \quad d\phi(T) / dT > 0$$

introduce a shift function $\chi(T)$ by setting

$$\phi(T) = \log \chi(T)$$

$$\text{with } \chi(T_0) = 1 \quad \text{and} \quad d\chi(T) / dT > 0$$

$$\therefore G_a(t, T) = L_a[\log t \chi(T)] = G_a(\xi)$$

$$\text{with } \xi = t \chi(T)$$

the relaxation function $G_a(t, T)$ at any temperature can directly be obtained from $G_a(t)$ at base temperature T_0 by replacing t with $[t \chi(T)]$

$\chi(T)$ is a basic property of materials, determined experimentally, same $\chi(T)$ for G_1 and G_2

the corresponding temperature dependent complex modulus is seen to be given by

$$G_a^*(i\omega, T) = G_a^*[i(\omega/\chi(T))]$$

i.e. the temperature dependent complex modulus at various temperature is obtained from the complex modulus $G_a^*(i\omega)$ at the base temperature through replacing ω by $\omega/\chi(T)$

proof $G_a^*(i\omega) = i\omega \mathcal{L}[G(t)]$

or $G'(\omega) = \omega \int_0^\infty G(t) \sin \omega t dt$
 $G''(\omega) = \omega \int_0^\infty G(t) \cos \omega t dt$

then $G'(\omega, T) = \omega \int_0^\infty G(t, T) \sin \omega t dt$
 $= \omega \int_0^\infty G[t \chi(T)] \sin \omega t dt$

let $\xi = t \chi(T)$

then $dt = d\xi / \chi(T)$

$\therefore G'(\omega, T) = \omega / \chi(T) \int_0^\infty G(\xi) \sin [\omega \xi / \chi(T)] d\xi$
 $= G'[\omega / \chi(T)]$