Chapter 17 Plane Motion of Rigid Bodies : Energy and Momentum Methods

17.1 Introduction

two additional methods are introduced :

1. method of work and energy
2. method of impulse and momentum

problems of eccentric impact of rigid body are also considered

17.2 Principle of Work and Energy for a Rigid Body

for a system of particles

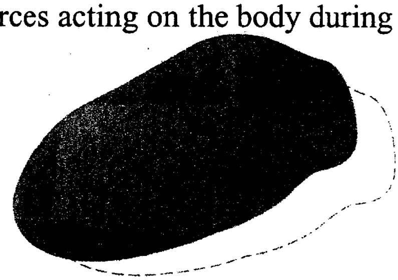
$$T_1 + U_{1\to2} = T_2$$

consider a rigid body is made of large number of particles

T_1, T_2 : initial and final values of total kinetic energy of the particles forming the rigid body

$U_{1\to2}$: work of all forces acting on the various particles of the rigid body

total work due to internal forces acting on the particle of the rigid body is zero, thus $U_{1\to2}$ is the work of external forces acting on the body during the displacement considered



the kinetic energy can be expressed

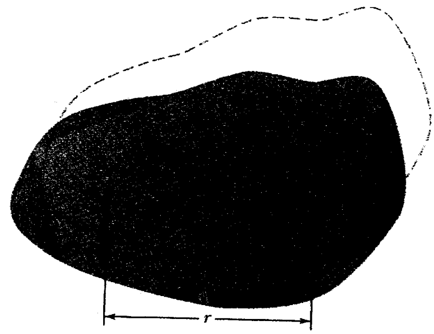
$$T = \tfrac{1}{2}\sum_{i=1}^{n} (\triangle m_i)\, v_i^2$$

17.3 Work of Forces Acting on a Rigid Body

the work done of a force $\boldsymbol{F}$ during a displacement from A_1 to A_2 is

$$U_{1\to2} = \int_{A_1}^{A_2} \boldsymbol{F} \cdot d\boldsymbol{r}$$

or $\quad U_{1\to2} = \int_{s_1}^{s_2}(F\cos a)\,ds$



if $F\cos a$ = constant, then

$$U_{1\to2} = F\cos a\,(s_2 - s_1)$$

the work done of a moment M during the rotation from θ_1 to θ_2 is

$$U_{1\to2} = \int_{\theta_1}^{\theta_2} M\,d\theta$$

if M = constant, then $\quad U_{1\to2} = M(\theta_2 - \theta_1)$

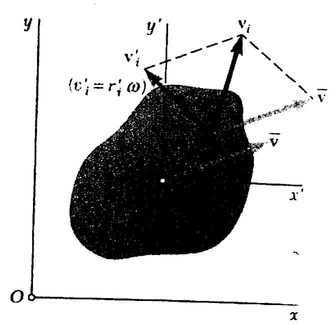
forces do no work ;

1. acting on a fixed point
2. force $\perp$ displacement
3. in rolling motion, friction force do no work

17.4 Kinetic Energy of a Rigid Body in Plane Motion

consider a rigid body of mass m in plane motion

$\because \quad \boldsymbol{v}_i = \underline{v} + \boldsymbol{v}_i'$



$\boldsymbol{v}_i'$ is the relative velocity of P_i to its mass center

the kinetic energy is

$$T = \sum_{i=1}^{n} \tfrac{1}{2}\Delta m_i v_i^2 = \sum_{i=1}^{n} \tfrac{1}{2}\Delta m_i (\underline{v} + v_i')^2$$

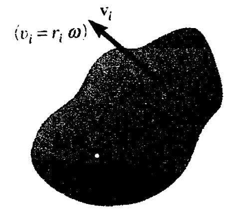
$$= \tfrac{1}{2} m \underline{v}^2 + \underline{v}\sum_{i=1}^{n}\Delta m_i v_i' + \sum_{i=1}^{n}\tfrac{1}{2}\Delta m_i v_i'^2$$

$$v_i' = r_i'\,\omega$$

$$\therefore \quad T = \tfrac{1}{2} m \underline{v}^2 + \tfrac{1}{2}\Big(\sum_{i=1}^{n}\Delta m_i r_i'^2\Big)\omega^2$$

$$= \tfrac{1}{2} m \underline{v}^2 + \tfrac{1}{2}\underline{I}\,\omega^2 = T_{tr} + T_{rot}$$

for a body in noncentroidal rotation



$$T = \sum_{i=1}^{n} \tfrac{1}{2}\,\Delta m_i\, v_i^2 = \sum_{i=1}^{n} \tfrac{1}{2}\,\Delta m_i\,(r_i'\,\omega)^2$$

$$= \sum_{i=1}^{n} \tfrac{1}{2}\,(\Delta m_i\, r_i'^2)\,\omega^2 = \tfrac{1}{2}\, I_0\,\omega^2$$

where I_0 is the moment of inertia with respect to O

or $\quad T = \tfrac{1}{2}\, m\, \underline{v}^2 + \tfrac{1}{2}\, \underline{I}\,\omega^2 = \tfrac{1}{2}\,(m\,\underline{r}^2 + \underline{I})\,\omega^2$

$= \tfrac{1}{2}\, I_0\,\omega^2$ [same result]

17.5 Systems of Rigid Bodies

each rigid body may be considered separately, the principle of work and energy may be applied to each body, then we have

$$T_1 + U_{1\to2} = T_2$$

T_1, T_2 : the sum of the kinetic energies of the rigid bodies forming the system

$U_{1\to2}$: work of all forces acting on the various bodies

17.6 Conservation of Energy

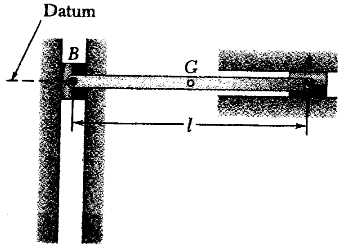
the work of conservative force may be expressed as a change in potential energy, then

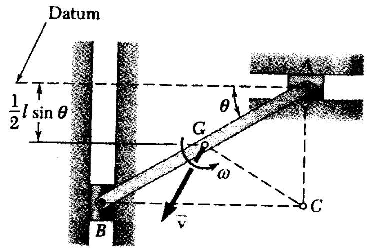
$$T_1 + V_1 = T_2 + V_2$$

the sum of kinetic energy and potential energy of a system remains constant

note that the kinetic energy should include both translational term $\tfrac{1}{2}\, m\, \underline{v}^2$ and the rotational term $\tfrac{1}{2}\, \underline{I}\,\omega^2$

consider a slender rod AB of length l, and mass m, released from rest





$$T_1 = 0 \qquad V_1 = 0$$

$$T_2 = \tfrac{1}{2} m \underline{v}_2^2 + \tfrac{1}{2} \underline{I} \omega^2 \qquad [\underline{v}_2 = CG\,\omega = \tfrac{1}{2} l \omega]$$

$$= \tfrac{1}{2} m (\tfrac{1}{2} l \omega)^2 + \tfrac{1}{2} \underline{I} \omega^2$$

$$= \tfrac{1}{8} m l^2 \omega^2 + (1/24) m l^2 \omega^2$$

$$= (1/6) m l^2 \omega^2$$

$$V_2 = -m g (\tfrac{1}{2} l \sin\theta) = -\tfrac{1}{2} m g l \sin\theta$$

$$T_2 + V_2 = 0$$

$$(1/6) m l^2 \omega^2 - \tfrac{1}{2} m g l \sin\theta = 0$$

$$\omega = (\frac{3g}{l} \sin\theta)^{1/2} \qquad \text{and} \qquad \underline{v} = (\frac{3g}{4} l \sin\theta)^{1/2}$$

17.7 Power

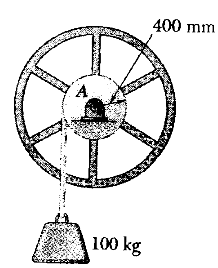
the time rate at which work is done is called power

$$\text{Power} = dU / dt = \boldsymbol{F} \bullet \boldsymbol{v}$$

if the work done is due to M and ω

$$\text{Power} = M\, d\theta / dt = M \omega$$

Sample Problem 17.1

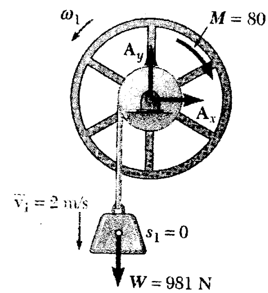


$m = 100$ kg $\qquad \underline{v} = 2$ m/s

$\underline{I}$ of the flywheel $= 15$ kg-m^2

$\boldsymbol{M}$ due to poor lubrication $= 80$ N-m

determine $\underline{v}$ after it moved 1.2 m downward at position 1

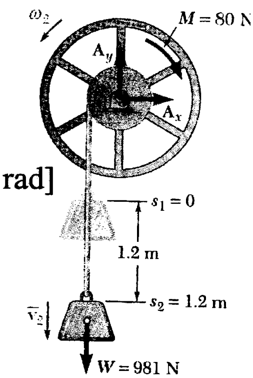


$$\underline{v}_1 = 2 \text{ m/s} \qquad \omega_1 = \underline{v}_1 / r = 2 / 0.4 = 5 \text{ rad/s}$$

$$T_1 = \tfrac{1}{2} \, m \, \underline{v}_1^2 + \tfrac{1}{2} \, \underline{I} \, \omega_1^2$$

$$= \tfrac{1}{2} \, 100 \times 2^2 + \tfrac{1}{2} \, 15 \times 5^2 = 388 \text{ J}$$

at position 2



$$\omega_2 = \underline{v}_2 / 0.4$$

$$T_2 = \tfrac{1}{2} \, m \, \underline{v}_2^2 + \tfrac{1}{2} \, \underline{I} \, \omega_2^2$$

$$= \tfrac{1}{2} \, 100 \, \underline{v}_2^2 + \tfrac{1}{2} \, 15 \, (\underline{v}_2/0.4)^2 = 96.9 \, \underline{v}_2^2$$

$$U_{1\to2} = 100 \, g \, 1.2 - 80 \, \theta \qquad [\theta = 1.2 / 0.4 = 3 \text{ rad}]$$

$$= 937 \text{ J}$$

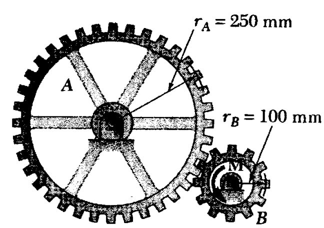
principle of work and energy

$$T_1 + U_{1\to2} = T_2$$

$$388 + 937 = 96.9 \, \underline{v}_2^2$$

$$\underline{v}_2 = 3.7 \text{ m/s} \downarrow$$

Sample Problem 17.2



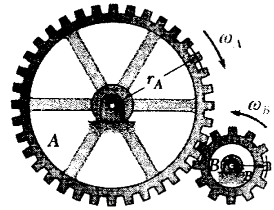
$$m_A = 10 \text{ kg} \qquad k_A = 0.2 \text{ m}$$

$$m_B = 3 \text{ kg} \qquad k_B = 0.08 \text{ m}$$

M = 6 N-m, no friction

determine (1) number of revolutions for ω_B = 600 rpm

(2) tangential force exerts on gear A



$$r_A \, \omega_A = r_B \, \omega_B$$

$$\omega_A = r_B \, \omega_B / r_A = (100/250) \, \omega_B = 0.4 \, \omega_B$$

for ω_B = 600 rpm = 62.8 rad/s

$$\omega_A = 25.1 \text{ rad/s}$$

$$\underline{I}_A = 10 \times 0.2^2 = 0.4 \text{ kg-m}^2$$

$\underline{I}_B = 3 \times 0.08^2 = 0.0192 \text{ kg-m}^2$

$T_1 = 0$

$T_2 = \frac{1}{2} \underline{I}_A \, \omega_A^2 + \frac{1}{2} \underline{I}_B \, \omega_B^2 = 163.9 \text{ J}$

$U_{1\to2} = M \, \theta_B = 6 \, \theta_B$

principle of work and energy for the system

$T_1 + U_{1\to2} = T_2$

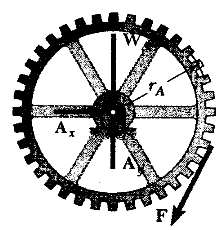
$0 + 6 \, \theta_B = 163.9 \qquad \theta_B = 27.32 \text{ rad} = 4.35 \text{ rev.}$

for gear A

$T_1 = 0$

$T_2 = \frac{1}{2} \underline{I}_A \, \omega_A^2 = 126 \text{ J}$

$U_{1\to2} = F \, (r_B \, \theta_B) = F \times 0.1 \times 27.32 = 2.73 \, F$



principle of work and energy for gear A

$T_1 + U_{1\to2} = T_2$

$0 + 2.73 \, F = 126 \qquad F = 46.2 \text{ N} \swarrow$

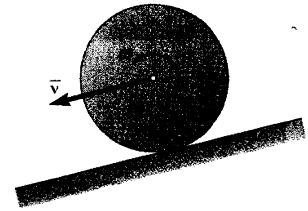
Sample Problem 17.3

a sphere, a cylinder and a hoop

each having the same mass and radius

determine $\underline{v}$ after its rolls for the center change in elevation h

rolling without sliding



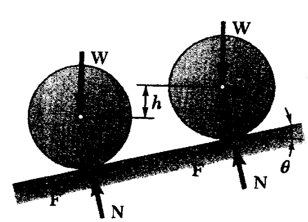
$\underline{v} = r \, \omega$

$T_1 = 0$

$T_2 = \frac{1}{2} m \, \underline{v}^2 + \frac{1}{2} \underline{I} \, \omega^2$

$\quad = \frac{1}{2} m \, \underline{v}^2 + \frac{1}{2} \underline{I} \, (\underline{v}/r)^2$

$\quad = \frac{1}{2} (m + \underline{I}/r^2) \, \underline{v}^2$



$$U_{1\to2} = W\,h = m\,g\,h$$

principle of work and energy

$$m\,g\,h = \tfrac{1}{2}\,(m + I/r^2)\,v^2$$

$$v^2 = \frac{2\,m\,g\,h}{m + I/r^2} = \frac{2\,g\,h}{1 + I/mr^2}$$

sphere $\quad I = (2/5)\,m\,r^2 \qquad v = 0.845\,(2\,g\,h)^{1/2}$

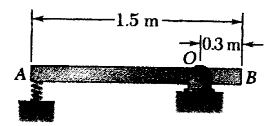
cylinder $\quad I = \tfrac{1}{2}\,m\,r^2 \qquad v = 0.816\,(2\,g\,h)^{1/2}$

hoop $\quad I = m\,r^2 \qquad v = 0.707\,(2\,g\,h)^{1/2}$

if no friction, $\omega = 0$, $\quad v = (2\,g\,h)^{1/2}$ is the same for each body

Sample Problem 17.4

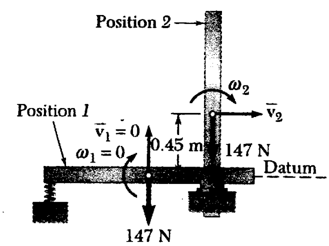
$m = 15$ kg $\qquad W = m\,g = 147.2$ N



k of the spring $= 300$ N/m

the spring is compressed 25 mm initially

determine ω and reaction at O when the rod at vertical position



$$T_1 = 0$$

$$V_1 = (V_1)_e = \tfrac{1}{2}\,300 \times 0.0252 = 93.8 \text{ J}$$

$$T_2 = \tfrac{1}{2}\,m\,v_2^2 + \tfrac{1}{2}\,I\,\omega_2^2$$

$$v_2 = 0.45\,\omega_2$$

$$I = (1/12)\,m\,l^2 = (1/12)\,15 \times 1.5^2 = 2.81 \text{ kg-m}^2$$

$$T_2 = \tfrac{1}{2}\,15\,(0.45\,\omega_2)^2 + \tfrac{1}{2}\,2.81\,\omega_2^2 = 2.92\,\omega_2^2$$

$$V_2 = (V_2)_g = m\,g\,h = 147.2 \times 0.45 = 66.2 \text{ J}$$

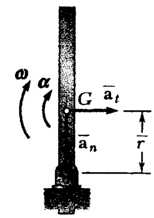
conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 93.8 = 2.92\,\omega_2^2 + 66.2$$

$\omega_2 = 3.07$ rad/s clockwise

reaction at position 2



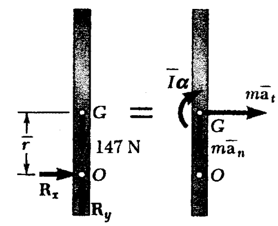
$a_t = 0.45\, a \rightarrow$

$a_n = \underline{r}\, \omega_2^2 = 0.45 \times 3.072 = 4.24 \text{ m/s}^2 \downarrow$

$\Sigma M_O = (\Sigma M_O)_{eff}$

$0 = m\, \underline{a_t}\, \underline{r} + I\, a = (\underline{I} + m\, \underline{r}^2)\, a$

$a = 0$ and $a_t = 0$



$\Sigma F_x = (\Sigma F_x)_{eff} \quad \Rightarrow \quad R_x = 0$

$\Sigma F_y = (\Sigma F_y)_{eff}$

$147.2 - R_y = m\, a_n = 15 \times 4.24$

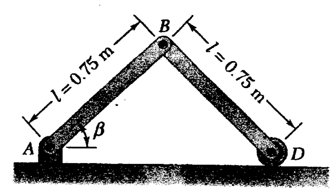
$R_y = 83.6 \text{ N} \uparrow$

Sample Problem 17.5

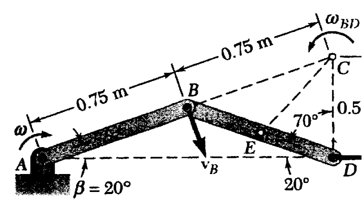
$m = 6$ kg

release from $\beta = 60°$

determine ω_{AB} and v_D for $\beta = 20°$



for $\beta = 20°$

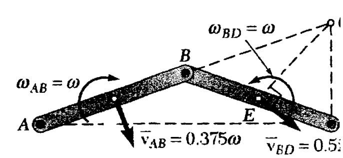


$BC = 0.75$ m

$CD = 2 \times 0.75 \sin 20° = 0.513$ m

$EC^2 = CD^2 + ED^2 - 2\, CD \times ED \cos 70°$

$EC = 0.522$ m



let $\omega_{AB} = \omega \qquad \underline{v}_{AB} = 0.375\, \omega \mid\searrow 20°$

$v_B = 0.75\, \omega \mid\searrow 20°$

$v_B = CB\, \omega_{BD} = 0.75\, \omega_{BD}$

$\omega_{BD} = \omega$ counterclockwise

$$\underline{v}_{BD} = 0.522\,\omega \searrow$$

at position 1

$$T_1 = 0$$

$$V_1 = 2\,m\,g\,(0.375 \sin 60^o) = 38.26\text{ J}$$

at position 2

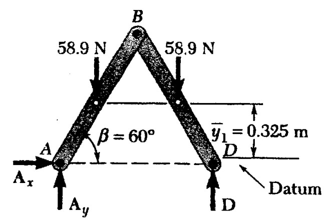
$$T_2 = \tfrac{1}{2}\,m_{AB}\,\underline{v}_{AB}^{\,2} + \tfrac{1}{2}\,\underline{I}_{AB}\,\omega^2 + \tfrac{1}{2}\,m_{BD}\,\underline{v}_{BD}^{\,2} + \tfrac{1}{2}\,\underline{I}_{BD}\,\omega^2$$

$$\underline{I}_{AB} = \underline{I}_{BD} = (1/12)\,6 \times 0.75^2 = 0.281\text{ kg-m}^2$$

$$T_2 = \tfrac{1}{2}\,(6)\,(0.375\omega)^2 + \tfrac{1}{2}\,(0.281)\,\omega^2 + \tfrac{1}{2}\,(6)\,(0.522\omega)^2 + \tfrac{1}{2}\,(0.281)\,\omega^2$$

$$= 1.52\,\omega^2$$

$$V_2 = 2\,m\,g\,(0.375 \sin 20^o) = 15.1\text{ J}$$



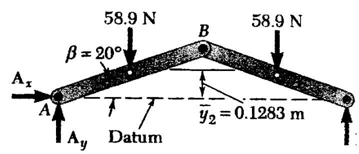
Position *1*

conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 38.26 = 1.52\,\omega^2 + 15.1$$

$$\omega = 3.9\text{ rad/s}$$



Position 2

$$\omega_{AB} = 3.9\text{ rad/s clockwise}$$

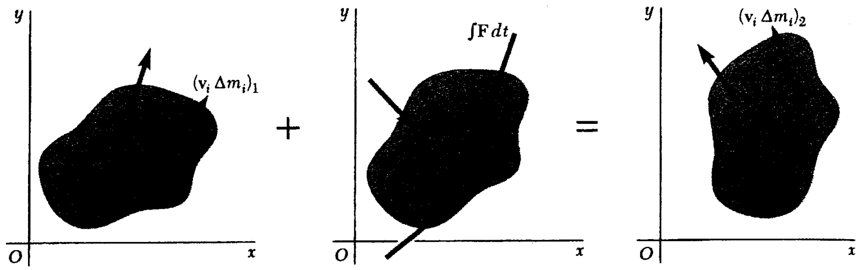
$$\omega_{BD} = 3.9\text{ rad/s counterclockwise}$$

$$v_D = CD\,\omega = 0.513 \times 3.9 = 2.0\text{ m/s} \rightarrow$$

17.8 Principle of Impulse and Momentum for the Plane Motion of a Rigid Body

a rigid body can be considered as a large number of particles, the principle of impulse and moment derived in chapter 14 can be used

consider again a rigid body as made of large number of particles P_i, the momenta at time t_1 and at time t_2 can be related



System Momenta$_1$ + System Ext Imp$_{1\to2}$ = System Momenta$_2$

the "equal" sign means equipollent for system of particles, but it means equivalent for rigid body

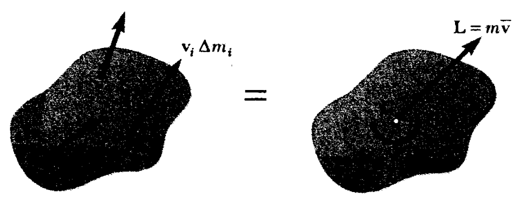
linear momentum :

$$\boldsymbol{L} = \sum_{i=1}^{n} \boldsymbol{v}_i \Delta m_i = m\,\underline{\boldsymbol{v}}$$

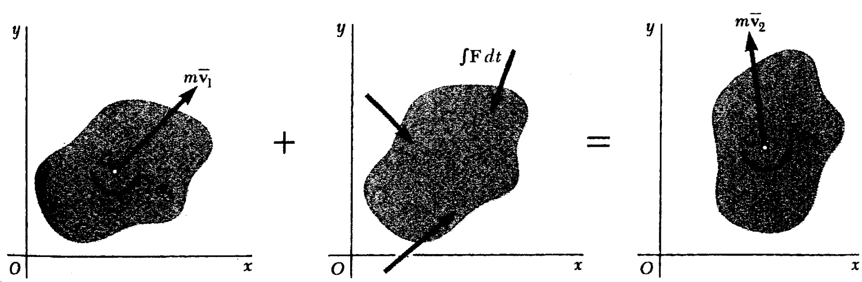
angular momentum :

$$H_G = \sum_{i=1}^{n} \boldsymbol{r}_i' \times \boldsymbol{v}_i \Delta m_i = I\,\omega$$

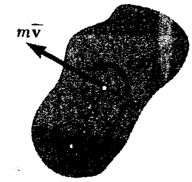
that means the linear moment and angular momentum can be expressed



and the principle of impulse and momentum can be shown in figure below, there are three equations, two linear and one angular



for noncentroidal rotation



$$H_0 = \underline{I}\,\omega + (m\,\underline{r}\,\omega)\,\underline{r}$$
$$= (\underline{I} + m\,\underline{r}^2)\,\omega = I_0\,\omega$$

thus the impulse-moment relation can be expressed

$$I_0\,\omega_1 + \Sigma \int_{t_1}^{t_2} M_0\,dt = I_0\,\omega_2$$

linear moment equations can also be used, but the reactions at O must be included

17.9 Systems of Rigid Bodies

system of rigid bodies may be analyzed by applying the principle of impulse and momentum to each body separately

17.10 Conservation of Angular Momentum

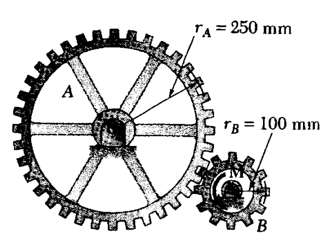
if no external force acts on a rigid body, the impulses of external force are zero, the momenta at t_1 is equipollent to momenta at t_2

the total linear momentum of the system is conserved in any direction and the total angular momentum is conserved about any point

if the external forces all pass through a point O, its linear momenta may not conversed, but its angular momentum is conserved

that is $(H_0)_1 = (H_0)_2$

Sample Problem 17.6



$m_A = 10$ kg $\quad k_A = 0.2$ m

$m_B = 3$ kg $\quad k_B = 0.08$ m

$M = 6$ N-m

determine (a) time required for $\omega_B = 600$ rpm

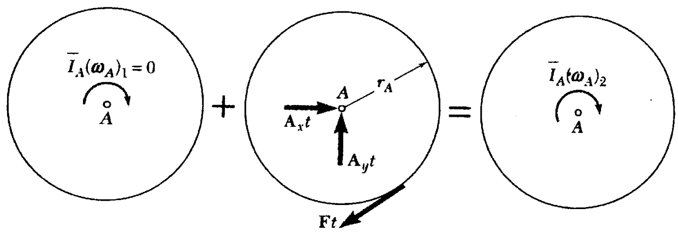
(b) tangential force

$I_A = m_A k_A^2 = 0.4 \text{ kg-m}^2 \qquad I_B = m_B k_B^2 = 0.0192 \text{ kg-m}^2$

for $\omega_B = 600 \text{ rpm} = 62.8 \text{ rad/s}$

$\omega_A = r_B \omega_B / r_A = 25.1 \text{ rad/s}$

principle of impulse and momentum for gear *A*



Syst Momenta$_1$ + Syst Ext Imp$_{1\to2}$ = Syst Momenta$_2$

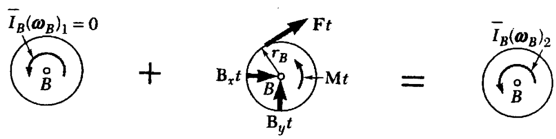
take moment about *A*

$$0 + F t r_A = I_A (\omega_A)_2$$

$$F t \times 0.25 = 0.4 \times 25.1$$

$$F t = 40.2 \text{ N-s}$$

principle of impulse and momentum for gear *B*



Syst Momenta$_1$ + Syst Ext Imp$_{1\to2}$ = Syst Momenta$_2$

take moment about *B*

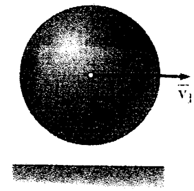
$$0 + M t - F t r_B = I_B (\omega_B)_2$$

$$6\, t - 40.2 \times 0.1 = 0.0192 \times 62.8$$

$$t = 0.871 \text{ s}$$

and $\qquad F = 46.2 \text{ N}$

Sample Problem 17.7



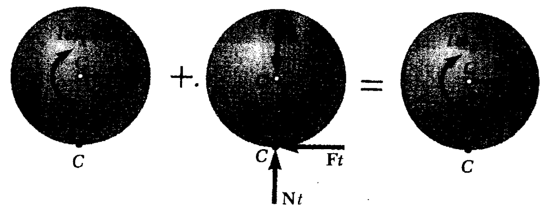
initial linear velocity $\underline{v}_1$, no angular velocity

coefficient of kinetic friction μ_k

determine (a) time t_2 for the sphere start rolling without sliding

(b) $\underline{v}_2$ and ω_2 at t_2

principle of impulse and momentum



x- : $m\,\underline{v}_1 - F\,t = m\,\underline{v}_2$ (1)

y- : $N\,t - W\,t = 0$ (2)

M_G : $0 + F\,t\,r = \underline{I}\,\omega_2$ (3)

(2) => $N = W$

$F = \mu_k N = \mu_k m\,g \qquad F\,t = \mu_k m\,g\,t$

(1) => $\underline{v}_2 = \underline{v}_1 - \mu_k g\,t$

(3) => $\omega_2 = (5/2)\,\mu_k g\,t\,/\,r$

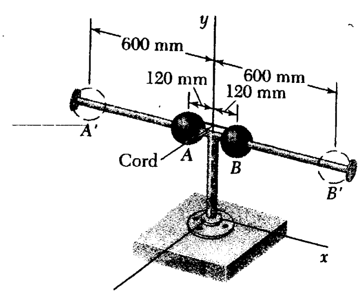
when rolling without sliding

$$\underline{v}_2 = r\,\omega_2 \quad \Rightarrow \quad t_2 = \frac{2}{7}\frac{\underline{v}_1}{\mu_k g}$$

at time t_2

$$\underline{v}_2 = \frac{5}{7}\underline{v}_1 \qquad \omega_2 = \frac{5}{7}\frac{\underline{v}_1}{r}$$

Sample Problem 17.8



r of sphere = 75 mm $\quad \underline{I}_{rod} = 0.4$ kg-m^2

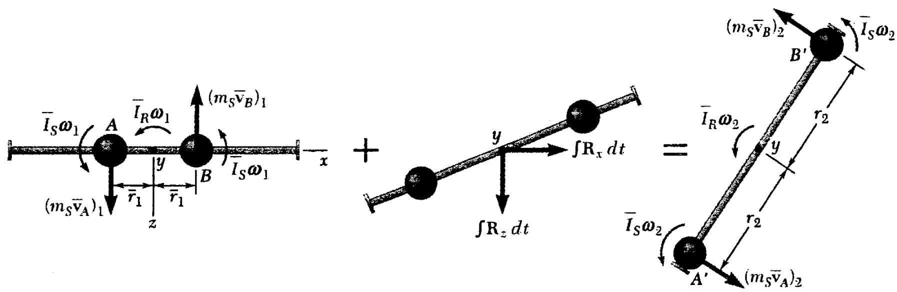
m of sphere = 1.5 kg $\quad \omega_1 = 6$ rad/s

when the cord is suddenly cut, determine

(a) ω_2 when spheres have moved to A' and B'

(b) energy lost due to plastic impact of the spheres and the stops at A' and B'

principle of impulse and momentum



Syst Momenta$_1$ + Syst Ext Imp$_{1\rightarrow 2}$ = Syst Momenta$_2$

moment about y-axis

$$2\,(m_s\,\underline{r}_1\,\omega_1)\,\underline{r}_1 + 2\,\underline{I}_s\,\omega_1 + \underline{I}_R\,\omega_1 + 0 = 2\,(m_s\,\underline{r}_2\,\omega_2)\,\underline{r}_2 + 2\,\underline{I}_s\,\omega_2 + \underline{I}_R\,\omega_2$$

$$(2\,m_s\,\underline{r}_1^2 + 2\,\underline{I}_s + \underline{I}_R)\,\omega_1 = (2\,m_s\,\underline{r}_2^2 + 2\,\underline{I}_s + \underline{I}_R)\,\omega_2$$

$$\underline{I}_s = (2/5)\,m_s\,r_s^2 = 3.38 \times 10^{-3}\text{ kg-m}^2$$

$$m_s\,\underline{r}_1^2 = 21.6 \times 10^{-3}\text{ kg-m}^2 \qquad m_s\,\underline{r}_2^2 = 540 \times 10^{-3}\text{ kg-m}^2$$

with $\omega_1 = 6$ rad/s $\quad$ => $\quad \omega_2 = 1.82$ rad/s

$$T_1 = 2\,(\tfrac{1}{2}\,m_s\,\underline{v}_1^2 + \tfrac{1}{2}\,\underline{I}_s\,\omega_1^2) + \tfrac{1}{2}\,\underline{I}_R\,\omega_1^2 = 8.1\text{ J}$$

$$T_2 = 2\,(\tfrac{1}{2}\,m_s\,\underline{v}_2^2 + \tfrac{1}{2}\,\underline{I}_s\,\omega_2^2) + \tfrac{1}{2}\,\underline{I}_R\,\omega_2^2 = 2.46\text{ J}$$

$$\Delta T = T_2 - T_1 = -5.64\text{ J}$$

energy lost $= (T_2 - T_1)\,/\,T_1 = -69.7\%$

17.11 Impulsive Motion

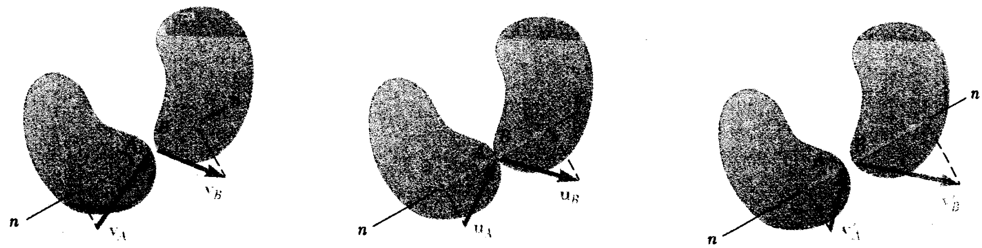
very large impulse force acting on a rigid at very short time interval

thus non-impulse force are not considered during this interval

17.12 Eccentric Impact

in chapter 13, central impact was introduced

in this chapter, eccentric impact are presented, consider two bodies which collide, and denote by $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ the velocities before impact of the two points of contact A and B



assume bodies are frictionless along t-direction

at maximum deformation, $\boldsymbol{u}_A$ and $\boldsymbol{u}_B$ have the equal component along n-n direction

denote $\int P\,dt$: deformation impulse

$\int R\,dt$: restitution impulse

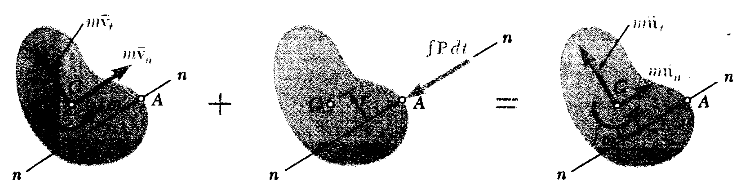
the coefficient of restitution e is defined

$$e = \int R\,dt \,/ \int P\,dt$$

now we propose to show

$$(v_B')_n - (v_A')_n = e\,[(v_A)_n - (v_B)_n]$$

assuming the bodies are frictionless, the forces exert on each other are directed along the line of impact



from body A

$$m\,\underline{v}_n - \int P\,dt = m\,\underline{u}_n \qquad (1)$$

$$\underline{I}\,\omega - r\int P\,dt = \underline{I}\,\omega^* \qquad (2)$$

similarly

$$m\,\underline{u}_n - \int R\,dt = m\,\underline{u}_n' \qquad (3)$$

$$\underline{I}\,\omega^* - r\int R\,dt = \underline{I}\,\omega' \qquad (4)$$

equations (1) and (3), we obtain

$$e = \frac{u_n - v_n'}{v_n - u_n} \qquad (5)$$

equations (2) and (4), we obtain

$$e = \frac{\omega^* - \omega'}{\omega - \omega^*} \qquad (6)$$

multiplying by r the numerator and denominator of equation (6) and adding respectively to the numerator and denominator of equation (5), and let $(u_A)_n = u_n + r\,\omega^*$ etc., it is obtained

$$e = \frac{u_n + r\,\omega^* - (v_n' + r\,\omega')}{v_n + r\,\omega - (u_n + r\,\omega^*)} = \frac{(u_A)_n - (v_A')_n}{(v_A)_n - (u_A)_n} \qquad (a)$$

from body B, it is also obtained

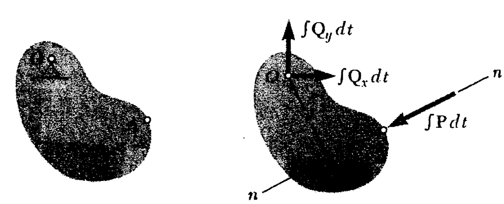
$$e = \frac{(v_B')_n - (u_B)_n}{(u_B)_n - (v_B)_n} \qquad (b)$$

knowing that $\quad (u_A)_n = (u_B)_n$

adding respectively the numerator and denominator of equations (a) and (b), we obtain

$$e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

if the body is constrained to rotate about a fixed point O, the similarly expression can be obtained



$$I_0\,\omega - r \int P\,dt = I_0\,\omega^*$$

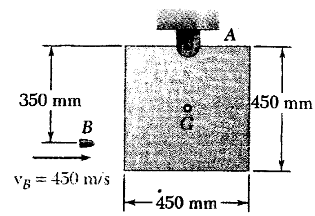
$$I_0\,\omega^* - r \int R\,dt = I_0\,\omega'$$

$$e = \frac{\omega^* - \omega'}{\omega - \omega^*} = \frac{r\,\omega^* - r\,\omega'}{r\,\omega - r\,\omega^*} = \frac{(u_A)_n - (v_A')_n}{(v_A)_n - (u_A)_n}$$

similarly for body B, the same expression for e can be obtained

$$e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

Sample Problem 17.9



$m_B = 20$ g

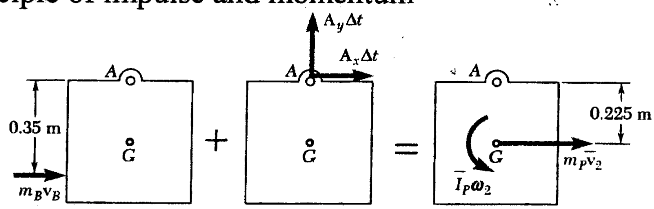
$v_B = 450$ m/s

$m_p = 10$ kg

determine (a) ω after the bullet becomes embedded

(b) reaction at A if time of embedding is $t = 0.6$ ms

principle of impulse and momentum



Syst Momenta$_1$ + Syst Ext Imp$_{1\to2}$ = Syst Momenta$_2$

x- : $\quad m_B v_B + A_x \Delta t = m_p \underline{v}_2 \qquad (1)$

y- : $\quad - W \Delta t + A_y \Delta t = 0 \qquad (2)$

M_A: $\quad m_B v_B \times 0.35 = (m_p + m_B) \underline{v}_2 \times 0.225 + \underline{I}_p \omega_2 \qquad (3)$

$\underline{I}_p = m_p (a^2 + b^2) / 12 = 0.3375$ kg-m^2

$\because \quad \underline{v}_2 = 0.225\, \omega_2$

(3) => $\quad \omega_2 = 3.73$ rad/s (counterclockwise)

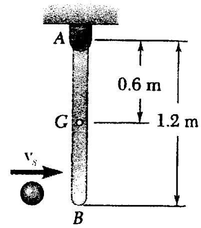
$\underline{v}_2 = 0.839$ m/s

for $\Delta t = 0.0006$ s

(1) => $\quad A_x = -1017$ N $\leftarrow$

(2) => $\quad A_y = W = 98.1$ N $\uparrow$

Sample Problem 17.10

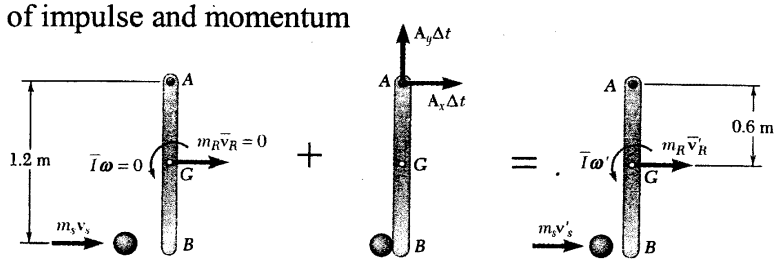


$m_s = 2$ kg $\qquad m_R = 5$ kg

$v_s = 5$ m/s $\qquad e = 0.8$

determine ω' and v_s'

principle of impulse and momentum



Syst Momenta$_1$ + Syst Ext Imp$_{1\to2}$ = Syst Momenta$_2$

$v_R' = 0.6\,\omega' \qquad v_B' = 1.2\,\omega' \qquad I = m\,l^2/12 = 0.96$ kg-m^2

ΣM_A: $\qquad m_s\,v_s \times 1.2 = m_s\,v_s' \times 1.2 + m_R\,v_R' \times 0.6 + I\,\omega'$

$2.4\,v_s' + 3.84\,\omega' = 12 \qquad (1)$

relative velocity

$v_B' - v_s' = e\,(v_s - v_B)$

$1.2\,\omega' - v_s' = 4 \qquad (2)$

(1) and (2) => $\quad v_s' = -\,0.143$ m/s $\leftarrow$

$\omega' = 3.21$ rad/s counterclockwise

x-comp. $\qquad m_s\,v_s + A_x\,\Delta t = m_s\,v_s' + m_R\,v_R'$

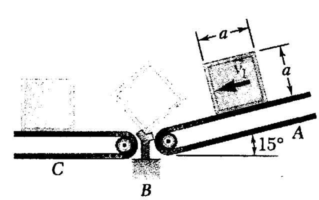
$A_x\,\Delta t = 5.122$ N-s

for $\quad \Delta t = 0.01$ sec $\qquad A_x = 512.2$ N

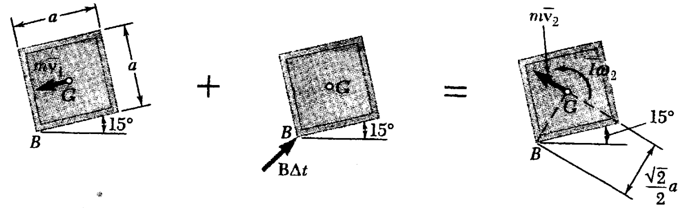
$\Delta t = 0.001$ sec $\qquad A_x = 5122$ N $\qquad$ [very large]

Sample Problem 17.11

determine minimum v_1 for which the package will rotate about B and reach conveyor belt C



principle of impulse and momentum



Syst Momenta$_1$ + Syst Ext Imp$_{1\to 2}$ = Syst Momenta$_2$

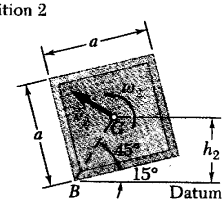
ΣM_B : $\quad m\,v_1\,(a/2) = m\,v_2\,a/(2)^{1/2} + I\,\omega_2$

$$I = m\,a^2/6 \qquad v_2 = a\,\omega_2\,/\,(2)^{1/2}$$

$$v_1 = (4/3)\,a\,\omega_2$$

conservation of energy

Position 2

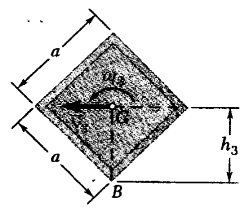


$GB = \frac{1}{2}\sqrt{2}a = 0.707a$

$h_2 = GB\sin(45° + 15°)$

$= 0.612a$

Position 3



$h_3 = GB = 0.707a$

$$h_2 = BG\sin 60^\circ = \frac{(2)^{1/2}}{2}a\,\frac{(3)^{1/2}}{2} = \frac{(6)^{1/2}}{4}a$$

$$h_3 = \frac{(2)^{1/2}}{2}a$$

for minimum $v_1,\ v_3 = \omega_3 = 0$

$$T_2 = \tfrac{1}{2}\,m\,v_2^2 + \tfrac{1}{2}\,I\,\omega_2^2$$

$$= \frac{1}{2}\,m\,[\frac{(2)^{1/2}}{2}\,a\,\omega_2]^2 + \frac{1}{2}\,\frac{1}{6}\,m\,a^2\,\omega_2^2$$

$$= \tfrac{1}{3}\,m\,a^2\,\omega_2^2$$

$$T_3 = 0$$

$$V_2 = W\,h_2 = \frac{(6)^{1/2}}{4}\,m\,g\,a$$

$$V_3 = W\,h_3 = \frac{(2)^{1/2}}{2}\,m\,g\,a$$

$$T_2 + V_2 = T_3 + V_3$$

$$\tfrac{1}{3}\,m\,a^2\,\omega_2^2 + \frac{(6)^{1/2}}{4}\,m\,g\,a = \frac{(2)^{1/2}}{2}\,m\,g\,a$$

$$\omega_2 = (0.284\,g\,/\,a)^{1/2}$$

$$v_1 = (4/3)\,a\,\omega_2 = 0.711\,(g\,a)^{1/2}$$