remark for the periodic time

$$\tau^2 = \frac{4\,\pi^2\,a^2\,b^2}{h^2} = \frac{4\,\pi^2\,a^2\,r_0\,r_1}{h^2}$$

$$\frac{1}{r} = \frac{G\,M}{h^2} + C\cos\theta$$

$$\theta = 0^\circ \qquad \frac{1}{r_0} = \frac{G\,M}{h^2} + C \qquad\qquad C = \frac{1}{r_0} - \frac{G\,M}{h^2}$$

$$\theta = 180^\circ \quad \frac{1}{r_1} = \frac{G\,M}{h^2} - C = \frac{2\,G\,M}{h^2} - \frac{1}{r_0}$$

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2\,G\,M}{h^2} = \frac{r_0 + r_1}{r_0\,r_1} = \frac{2\,a}{r_0\,r_1}$$

$$\therefore \qquad r_0\,r_1 = \frac{a\,h^2}{G\,M}$$

then

$$\tau^2 = \frac{4\,\pi^2\,a^2}{h^2}\,\frac{a\,h^2}{G\,M} = \left(\frac{4\,\pi^2}{G\,M}\right) a^3 \qquad \text{i.e. } \tau^2 \sim a^3$$

remark for sample problem 12-9

$G\,M = 3.98 \times 10^{14}\ \mathrm{m^3/s^2}$

$r_0 = 6.87 \times 10^6\ \mathrm{m}$

$v_{esc} = (2\,G\,M\,/\,r_0)^{1/2} = (2 \times 3.98 \times 10^{14}\,/\,6.87 \times 10^6)$

$= 10.76 \times 10^3\ \mathrm{m/s}$

$v_0 = 10.25 \times 10^3\ \mathrm{m/s}$

$v_0 < v_{esc}$ the orbit is an ellipse

for $v_0 < v_{\text{circ}} = (GM / r_0)^{1/2}$

$$\frac{1}{r} = \frac{GM}{h^2} + C \cos\theta$$

when $\theta = 0^\circ$

$$\frac{1}{r_0} = \frac{GM}{r_0^2 v_0^2} + C$$

$$C = \frac{1}{r_0} - \frac{GM}{r_0^2 v_0^2}$$

$\because\ v_0^2 < \dfrac{GM}{r_0}$ $\qquad\qquad \therefore\ \dfrac{1}{r_0} < \dfrac{GM}{r_0^2 v_0^2}$

then $\qquad C < 0$

when $\theta = 180^\circ$

$$\frac{1}{r_1} = \frac{GM}{r_0^2 v_0^2} - C = \frac{2GM}{r_0^2 v_0^2} - \frac{1}{r_0} = \frac{1}{r_0}\left[\frac{2GM}{r_0 v_0^2} - 1\right]$$

$\because\ GM > r_0 v_0^2$ $\qquad\qquad \therefore\ [\cdots] > 1$

i.e. $\quad \dfrac{1}{r_1} > \dfrac{1}{r_0} \quad$ or $r_1 < r_0$

$\because\ r_0 v_0 = r_1 v_1$ $\qquad\qquad$ i.e. $v_1 > v_0$

$$v_1 = \frac{r_0}{r_1} v_0 = \left[\frac{2GM}{r_0 v_0^2} - 1\right] v_0 = \frac{2GM - r_0 v_0^2}{r_0 v_0^2} v_0$$

$$= \frac{2GM - r_0 v_0^2}{r_0 v_0} > \frac{GM}{r_1 v_1} \qquad [GM > r_0 v_0^2]$$

then $\quad v_1^2 > GM / r_1$

i.e. $\quad v_1 > (GM / r_1)^{1/2} = v_{\text{circ}}$ at A"

$\therefore$ the orbit is an ellipse with apogee at A and perigee at A"