

1. The functions

$$y_1 = x^3 \quad \text{and} \quad y_2 = \begin{cases} x^3 & \text{when } x \geq 0 \\ -x^3 & \text{when } x \leq 0 \end{cases}$$

are two linearly independent solutions of the differential equation

$$\frac{d^2 y}{dx^2} - \frac{3}{x} \frac{dy}{dx} + \frac{3}{x^2} y = 0 \quad \text{for all } x.$$

(a) Find the Wronskian of y_1 and y_2 . (5%)

(b) Does the result of (a) contradict the linear independence of y_1 and y_2 ? Why? (10%)

2. We have the equation

$$\mathcal{F}\left[\frac{d^2 f}{dx^2}\right] = -\omega^2 \mathcal{F}[f]$$

for the Fourier transform of the second derivative of $f(x)$. The condition $f(x) \rightarrow 0$ for $x \rightarrow \pm\infty$ may be relaxed slightly. Find the least restrictive condition for the above equation to hold. (10%)

3. Don't need to show your solution procedures, just give your answer only.

(a) What is the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point

$P: (2, 1, 3)$ in the direction of the vector $\vec{a} = \vec{i} - 2\vec{k}$? (5 %)

(b) What is the unit normal vector of the cone of revolution $z^2 = 4(x^2 + y^2)$ at

the point $P: (1, 0, 2)$? (5 %)

(c) Evaluate $\iint_S (7x\vec{i} - z\vec{k}) \cdot \vec{n} dA$ over $S: x^2 + y^2 + z^2 = 4$ by the divergence

theorem. (5 %)

4. Don't need to show your solution procedures, just give your answer only.

(a) Any real square matrix A can be written as the sum of a symmetric R and a

skew-symmetric matrix S . Now A is given as below, what are R and S ?

$$A = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix} \quad (5 \%)$$

(b) What are the eigenvalues and corresponding eigenvectors of a matrix

$$A = \begin{bmatrix} a & -b \\ -b & a \end{bmatrix} ? (5 \%)$$

(背面仍有題目,請繼續作答)

5. Suppose $f(t)$ satisfies the difference-differential equation

$$\frac{df(t)}{dt} + f(t) - f(t-1) = 0, \quad t \geq 0,$$

and the initial condition, $f(t) = f_0(t)$, $-1 \leq t \leq 0$, where $f_0(t)$ is given. Show that the Laplace transform $F(s)$ of $f(t)$ satisfies

$$F(s) = \frac{f_0(0)}{1+s-e^{-s}} + \frac{e^{-s}}{1+s-e^{-s}} \int_{-1}^0 e^{-st} f_0(t) dt.$$

Find $f(t)$, $t \geq 0$, when $f_0(t) = 1$. Check the result. (13%)

6. Apply Cauchy's theorem to $f(z) = e^{-z^2}$ using the contour C consisting of the boundary of the sector formed from the positive real axis, the arc $|z| = R$, $0 \leq \theta \leq \pi/4$, and the line through the origin making an angle $\pi/4$ with the real axis. Let $R \rightarrow \infty$ to show that

$$\int_0^\infty \sin x^2 dx = \int_0^\infty \cos x^2 dx = \sqrt{\pi}/8.$$

[You may assume $\int_{-\infty}^\infty e^{-x^2} dx = \sqrt{\pi}$.] (12%)

7. Please find solution of the following partial differential equation:

$$\nabla^2 u = 0 \quad (0 < x < a, 0 < y < b)$$

$$u(x, 0) = u(x, b) = 0 \quad (0 < x < a)$$

$$u(0, y) = 0 \quad (0 < y < b)$$

$$u(a, y) = T \quad (0 < y < b) \quad (15\%)$$

8. Consider the regular Sturm-Liouville problem on $[0, 1]$:

$$y'' + \lambda y = 0; \quad y(0) = 0, \quad 2y(1) + y'(1) = 0$$

Discuss cases on λ .

(10%)

機械製造(共5大題,每題分數相等)

①請詳述

- (a) 切削時欲得較佳的表面粗糙度,應注意那些事項?
(b) 各種精密研拋法並比較其優缺。

② 機械加工製程可分為車銑鉋鑽磨等類別,一個工件在被完成前通常需經過其中數種製程,為何?並舉一例以說明這些加工製程的先後順序。

③ 在大小為 $30\text{ mm} \times 30\text{ mm}$, 厚度為 0.15 mm 之平面鋼板上,製造出 30 萬個小孔,其適合量產之製造方法應如何?為何?

④請詳述

- (a) 何謂切削性 (machinability), 用何種方法可定出某種金屬之切削性。
(b) 模具加工應先熱處理再研磨或先研磨再熱處理,為何?

⑤ 某產品所用材料的真實應力-應變曲線 (true stress-strain curve) 為

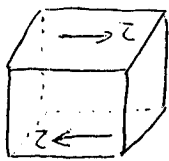
$$\sigma = 100,000 \epsilon^{0.5} \text{ psi}$$

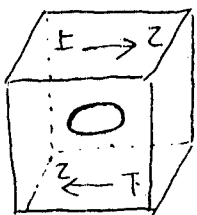
求該產品材料的工程最大拉力強度 (engineering ultimate tensile strength)?

(背面仍有題目,請繼續作答)

機械材料

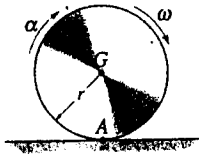
1. 何謂結晶材料之滑移系統？請以原子模型圖示體心立方、面心立方及密結晶格子之滑移系統。(10%)
2. 材料在拉伸時，工程應力(s)、工程應變(e)曲線中，有那些地方呈現 $ds/de = 0$ ，其物理意義為何？(10%)
3. 材料在拉伸時，工程應力(s)—工程應變(e)曲線轉換成真應力(σ)—真應變(ϵ)曲線時，其二者之相關性如何？並證明在最大負荷處(S_{max})時， $d\sigma/d\epsilon = \sigma$ 。(10%)

4.  左圖中，材料內部產生刃狀差排及螺旋差排時，二者之差異為何？(10%) (註“ τ ”為剪切應力)

5.  在左圖中，材料內部產生一個差排環 (dislocation loop)，請說明在差排環各處，是為刃狀差排、螺旋差排或混合差排？此差排環會如何運動？若所施加之外力(τ)之方向左右相反 ($\begin{smallmatrix} \tau & \leftarrow \\ \tau & \rightarrow \end{smallmatrix}$)，又將如何運動？(10%)

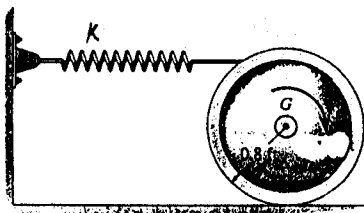
B1. At a given instant, the cylinder has an angular velocity ω and angular acceleration α .
Derive the velocity and acceleration of its center if it rolls without slipping.

(10%)



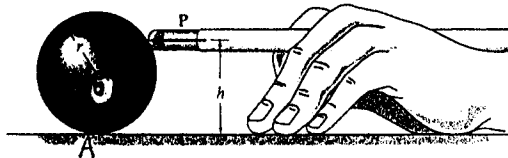
B2. The wheel weighs 40 lb, and has a radius of gyration $R = 0.6$ ft about its mass center G. It is subjected to a clockwise couple moment 150 lb-ft and rolls from rest without slipping. The spring has a stiffness k and is initially unstretched when the couple moment is applied. Determine the smallest stiffness k so that the angular velocity of the wheel will not exceed 6.5 rad/s after its center moves 0.5 ft.

(25%)



B3. The ball of mass m must be struck by the horizontal force P . Determine the height h so that no frictional force develops between it and the table at A. ($I_0 = 2mr^2/5$)

(15%)



(背面仍有題目,請繼續作答)

- A1. (a) What is the "conservative force"? (5%)
(b) If $\mathbf{F}$ is a conservative force, show that $\nabla \times \mathbf{F} = 0$, where $\nabla = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial z)\mathbf{k}$, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along x , y , and z axes, respectively. (10%)
- A2. The crate has a mass of 1000 kg and rests on a surface, as shown in Fig.2, for which the coefficients of static and kinetic friction are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. If the motor M supplies a cable force of $F = (60t^2 + 100)\text{N}$, where t is in second, determine the power that must be supplied to the motor when $t = 10$ sec. The motor has an efficiency $\epsilon = 0.8$. (20%)
- A3. The 200-kg roller-coaster car starts from rest on the track, as shown in Fig.3, having the shape of a cylindrical helix. If the helix descends 4 m for every one revolution, determine the time required for the car to attain a speed of 20 m/s. Neglect friction and the size of the car. (15%)

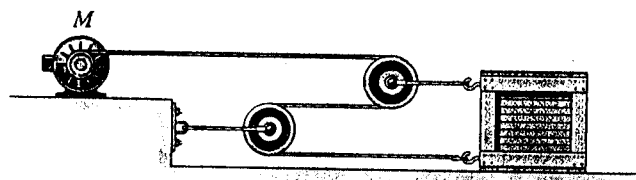


Fig.2

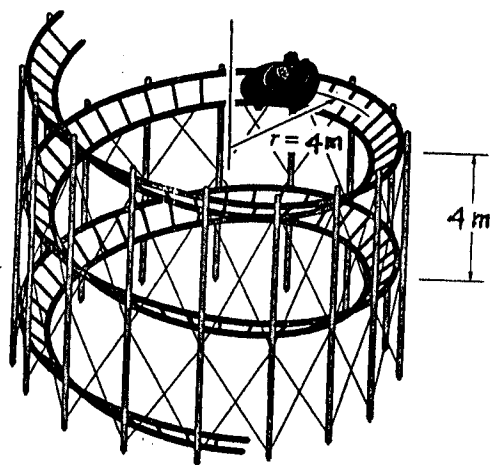


Fig.3

1. Define the following terminologies: (25%)
 - (a) Safety factor.
 - (b) von Mises stress.
 - (c) Endurance limit.
 - (d) Miner's rule.
 - (e) Soderberg and modified Goodman criterion.

2. A ball bearing is subjected to a radial load of 770N for 2.15×10^9 revolutions.
 - (a) What value of dynamic load rating should be used to enter the bearing catalog? (7%)
 - (b) The bore diameter is 25mm. Select suitable ball bearing from the following ball bearings. (6%)

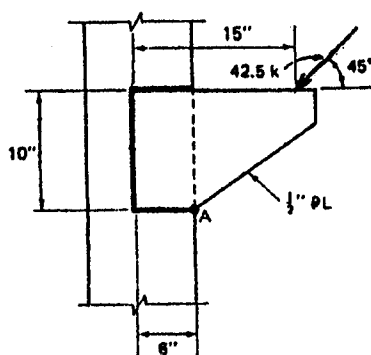
Bearing Number	*Dynamic Load Rating (C)*
6004	7200N
6005	8650N
6006	10200N
6204	9800N
6205	10800N
6206	15000N
6304	12200N

3. Design a single disk clutch to transmit 100N-m. of torque at 750 rpm using a molded lining with a maximum pressure of 1.0Mpa and the coefficient of friction $\mu=0.25$. Assume uniform wear.
 - (a) Find the outside and inside diameters (d_o and d_i) required if $d_i = 0.577d_o$. (6%)
 - (b) What is the actuating force? (6%)

(背面仍有題目,請繼續作答)

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4. For a single bracket welded to the face of a column, as shown in the figure, determine the size of the fillet welds required. Use the E70xx electrodes ($S_u=70\text{ksi}$, $S_y=57\text{ksi}$) and AISI 1018 steel ($S_u=58\text{ksi}$, $S_y=32\text{ksi}$). According to the AISC Code, the permissible stress for shear should be $<0.4S_y$, and for bending $<0.6S_y$. Weld size should be specified in the increment of $1/16"$. Calculate the final safety factor of your design based on the distortion energy theory. (30%)



5. A machine uses a pair of concentric helical compression springs to support a dynamic load of 0 to 2.25kN. Both springs are made of steel and have the same length when loaded and when unloaded. The outer spring has $D=45\text{ mm}$, $d=8\text{ mm}$, $N=5$; the inner spring $D=25\text{ mm}$, $d=5\text{ mm}$, and $N=10$. Calculate the deflection, and also the maximum stress in each spring. If the working frequency should be at least 20 times the natural frequency of the spring, what is the minimum working frequency the combined springs can be used for? (20%)

1. A fluid flows at a velocity V through a horizontal pipe of diameter D . An orifice plate containing a hole of diameter d is placed in the pipe. It is desired to investigate the pressure drop, Δp , across the plate. Assume that

$$\Delta p = f(D, d, \rho, V)$$

where ρ is the fluid density. Determine a suitable set of pi terms. (15%)

2. A soft drink with the properties of 10 °C water ($\rho = 999.7 \text{ kg/m}^3$, $\nu = 1.307 \times 10^{-6}$) is sucked through a 4-mm-diameter, 0.25-m-long straw at a rate of 4 cm^3/s . Is the flow at the outlet of the straw laminar? Is it fully developed? Explain. (15%)
3. Please define major head loss and minor head loss. (5%)
4. State the conditions under which prototype behavior can be predicted from model tests. (7%)
5. For fully-developed flow in a pipe determine the wall shear stress and the shear stress variation in the flow in terms of the pressure gradient. (8%)

(背面仍有題目,請繼續作答)

6. 回答下列問題 (請完整地將答案寫在答案紙上, 否則不予計分)
(10 %)

(a) The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

is valid for -----

- (b) If for a flow, a stream function ψ exists and satisfies the Laplace equation, then the flow is-----
(c) In two-dimensional flow the equation of a streamline is given as-----
(d) If a stream function ψ exists it implies that the potential function also exists. True or False
(e) In a steady flow, streamline, streakline and pathline can be different from each other. True or False

7. The stream function ψ for uniform flow past a circular cylinder of radius a in a polar coordinate is given by

$$\psi = U_{\infty} \left(1 - \frac{a^2}{r^2} \right) r \sin \theta$$

where U_{∞} is the free stream velocity parallel to the horizontal axis. Determine the velocity on the cylinder at $\theta = 90^\circ$. (10%)

8. By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$W_{\text{shaft}} = \frac{V_1^2 - V_2^2 + U_1^2 - U_2^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed (12%)

9. An infinitely long, solid, vertical cylinder of radius R is located in an infinite mass of an incompressible fluids. Start with the Navier-Stokes equation in the θ direction and derive an expression for the velocity distribution for the steady flow case in which the cylinder is rotating about a fixed axis with a constant angular velocity ω . You need not consider body forces. Assume that the flow is axisymmetric and the fluid is at rest at infinity. (18%)

Navier-Stokes equation in θ direction is given as

$$\begin{aligned} & \rho \left(\frac{\partial v_{\theta}}{\partial t} + v_r \frac{\partial v_{\theta}}{\partial r} + \frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_r v_{\theta}}{r} + v_z \frac{\partial v_{\theta}}{\partial z} \right) \\ &= -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_{\theta} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_{\theta}}{\partial r} \right) - \frac{v_{\theta}}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_{\theta}}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_{\theta}}{\partial z^2} \right] \end{aligned}$$

1. You are supposed to buy a plotter system, give and explain your decision making based on time-domain justification. (15%)

2. Use time-domain equation to express the condition of causality in input-output model. (10%)

3. Give the stability condition of the parameter set $\{a, b, c, d\}$ for the following dynamic system

$$ax''' + bx'' + cx' + dx = f(t)$$

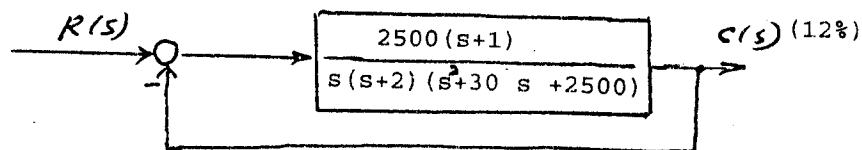
where $f(t)$ is a time function of D.C. and A.C. components. (10%)

4. A unity feedback system with transfer function $1/s(s+1)$ is controlled by a proportional controller. Plot the root locus and determine the condition when there is no oscillation in impulse response. (15%)

(背面仍有題目,請繼續作答)

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- 5.) (a) Try to plot a bode-plot of the following control system



- (b) According to the result of (a), can you decide whether the system stable or not? why? (8%)
(c) If you want to have phase margin 45° , how do you design? (5%)

- 6.) (a) Try to plot the complete Nyquist diagram of the following transfer function of a plant? (9%)
(b) Using the Nyquist criterion to judge and explain whether the control system is stable or not? why? (9%)

$$G(s) = \frac{11.7}{s(0.05s+1)(0.1s+1)}$$

- 7.) In the following diagram, if the force input to the mass spring system, is $f(t)$, the output is $y(t)$, d and k are the damping coefficient and spring constant, respectively.

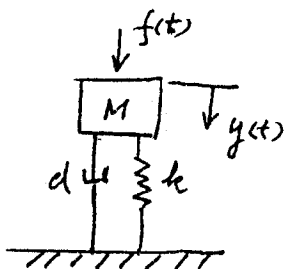
(a) express the system with state space model,

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} \underline{U}$$

$$Y = \underline{C} \underline{X} + \underline{D} \underline{U}$$

find the constant matrix $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$? (5%)

- (b) using the above state space model, find the characteristic equation of the system? (2%)



A1. (5%)

試解釋機構(Mechanism)與機器(Machine)的異同。

A2. (10%)

試說明一個圓球與一個圓筒相接觸的運動對(Kinematic pair)特性。

A3. (10%)

有一個RSSR空間四連桿機構，如圖A1所示，試：

1. 計算這個機構的自由度(Degrees of freedom)，
2. 說明這個機構是否做拘束運動(Constrained motion)。

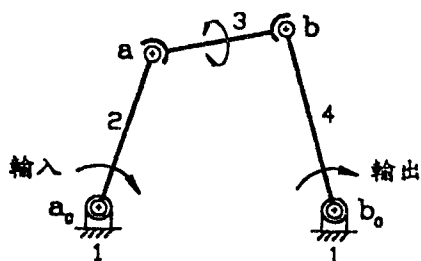
A4. (5%)

針對滑件曲柄機構(Slider-crank mechanism)而言，試定義其傳力角(Transmission angle)。

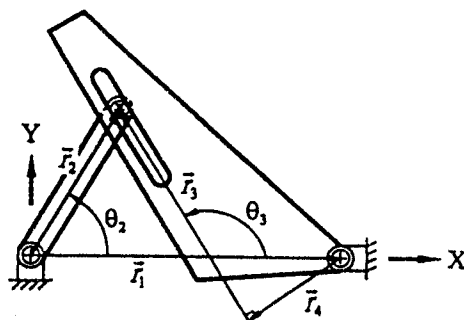
A5. (20%)

有一個三桿機構，如圖 A2 所示， $r_1=14\text{cm}$ 、 $r_2=8\text{cm}$ 、 $r_3=10\text{cm}$ 、 $r_4=5.196\text{cm}$ ， $\theta_2=60^\circ$ 、 $\theta_3=120^\circ$ ，桿 2 為輸入桿，以等角速度 $\omega_2=1\text{ rad/sec}$ 順時針方向旋轉。當 $\theta_2=50^\circ$ 時，試利用向量迴路法(Vector loop approach)：

- (a) 導出位移方程式(Displacement equation)，
- (b) 導出速度方程式(Velocity equation)，
- (c) 導出加速度方程式(Acceleration equation)，
- (d) 以數值法(Numerical method)進行二次疊代，求解未知的位置變數。



圖A1



圖A2

(背面仍有題目,請繼續作答)

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- B1. A cam with a translating flat-faced follower is shown in Figure 1. This figure depicts an inversion of the cam-follower mechanism where the cam is fixed and the follower moves relative to it. In normal operation, the cam would rotate and the follower would translate in a guideway along the y axis. The cam, having base circle radius r_b , is assumed to rotate in the clockwise direction under normal operation. Thus, for a cam rotation θ , the follower will rotate counterclockwise relative to the cam through angle θ while experiencing a translational displacement s , as shown in the figure. It is assumed that the follower displacement is a known function of the cam angle. Point P is the intersection of the face of the follower and its axis. Point Q is the cam-follower contact point for cam angle θ . The distance l between points P and Q is the perpendicular distance from the follower centerline to the contact point.
- Derive expressions for the cam profile coordinates.(10%)
 - Show that the expression for the distance l can be written as v/ω which is the translational follower velocity divided by the rotational cam velocity.(10%)

- B2. (a) Design a gear train similar to Figure 2 so that $\omega_c = 100$ rad/s ccw for $\omega_s = 300$ rad/s cw when ring gear is fixed. Let the sun gear pitch diameter d_s be 80 mm and the module 4 mm. All gears are standard gears with full-depth involute teeth. Determine the tooth number N_r of ring gear and find the possible range of tooth number N_p of planet gears. Hint: Consider the addendum of teeth.(10%)
- (b) Find the speed of each gear after selecting $N_{p1} = N_{p2} = N_s$, and then check the speed ratio using the formula method.(10%)

- B3. (a) State the fundamental law of gearing.(5%)
- (b) Describe the necessary conditions for parallel helical gears to mesh properly. (5%)

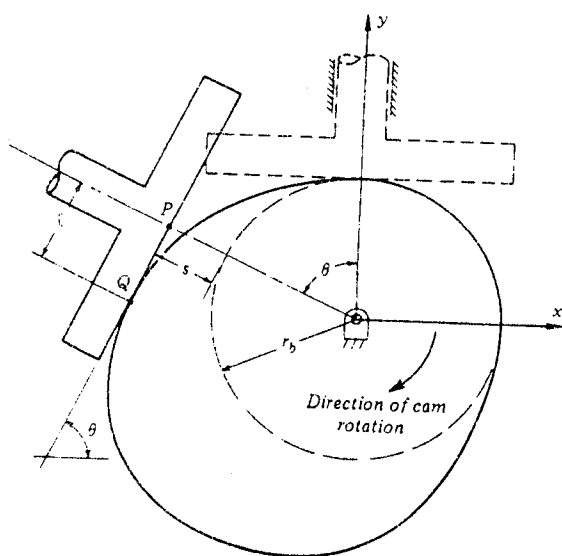


Figure 1

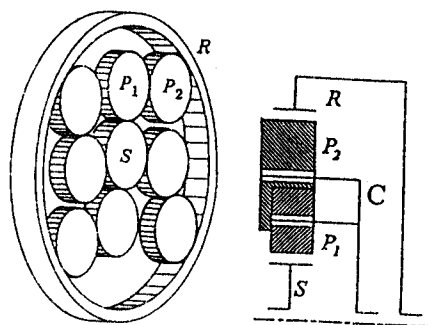


Figure 2

1. 正應變 (normal strain) 及剪應變 (shear strain) 的定義為何? (8%)
2. 一個物體受力時, 物體內任何一點的應力 (stress) 的定義為何? (6%) 應力是向量還是張量? (2%)
3. 材料力學中, 進行長圓柱扭轉 (torsion) 分析時所作的假設為何? (8%)
4. 材料力學中, 進行長樑彎曲 (bending) 分析時所作的假設為何? (8%)
5. 試說明挫屈 (buckling) 的現象。 (8%)
6. 試舉出五種方法, 以提升柱 (column) 結構之挫屈強度。 (10%)
7. 何謂平面應力 (plane stress) 及平面應變 (plane strain)? (4%) 採用應變規 (strain gauge) 量測計算應力時, 應使用平面應力或平面應變之虎克定律? 試說明理由。 (4%)
8. 試說明使用能量法時, 方程式 $\partial U^* / \partial \delta_i = \delta_i$ 中 U^* 代表的意義。 (4%) 另外卡氏定理 (Castigliano's Theorem) 的適用範圍為何? (4%)
9. 針對樑問題中之不同邊界形式: (a) 固定端 (fixed end) (b) 自由端 (free end) (c) 簡支端 (simply supported end), 說明其幾何邊界條件。 (6%)
10. 何謂降伏 (yielding)? (2%) 試舉四個預測降伏之理論, (4%) 並指出那一個理論最優? (2%)

11. 有一石頭質量 10 kg , 另有水桶裝 100 kg 的水, 起初石頭距離水面 10.2 m , 石頭和水有相同的溫度; 然後石頭落入水中, 在下列各種狀態中試求 ΔU , ΔKE , ΔPE , Q 和 W , 假設重力加速度為 9.8 m/s^2 .

- (a) 石頭即將進入水中 (5%)
- (b) 石頭剛靜止於水桶。 (5%)

12. 有一熱泵 (heat pump) 在冬天當暖房使用, 在夏天將此熱泵逆轉當冷房使用, 在冬天使房內溫度維持於 20°C , 而在夏天使房內溫度維持於 25°C , 假設熱經過房子的牆和天花板, 因內外溫差產生的熱傳為 $2400\text{ kJ/hr}\cdot\text{K}$

- (a) 若冬天外面氣溫是 0°C , 維持房內為 20°C 溫度, 運轉熱泵最小動力是多少? (5%)
- (b) 若 (a) 之運轉動力, 維持房內溫度為 25°C , 則房外夏天最高溫度可多少? (5%)

- (1) 針對應用於火力發電廠的 Rankine 循環而言, 利用 $T-s$ 圖及熱效率 (20%) 公式, 分析下列不同方法對熱效率及汽渦輪機內含濕氣 (moisture) 的影響。每一方法皆需繪出 $T-s$ 圖說明及比較。
- (a) 降低冷凝器 (condenser) 的壓力。
 - (b) 將鍋爐蒸氣加熱到過熱 (superheat) 蒸氣。
 - (c) 增加鍋爐 (boiler) 的壓力。
 - (d) 使用 reheat 循環。
- (2) 一部冷暖氣機可以在冬天時作為 heat pump, 但夏天作為 refrigerator, (10%) 請繪圖表示蒸發器 (evaporator)、冷凝器 (condenser)、膨脹閥 (expansion valve) 和壓縮機 (compressor) 的固定相關位置, 然後說明如何利用逆流閥 (reversing valve) 達到冬暖夏涼的目的, 並以 $T-s$ 圖輔助說明。
- (3) 考慮使用一般水壺燒開水的過程, 水壺壺蓋上有小孔與外界相通, 外 (10%) 界空氣總壓為 1 大氣壓, 加熱的水僅裝八分滿。利用 $T-u$ 圖說明壺中的液態水和氣態水的變化過程。為什麼開水沸騰時的溫度總是低於 99.6°C (1 大氣壓純物質的水, 沸點為 99.6°C)?
- (4) 試繪出氮和氧混合體系在 1 大氣壓時的液氣兩相平衡圖, 於圖中標示液相、 (10%) 氣相及兩相共存區域, 及其分界線; 並說明於兩相區中在給定溫度下, 如何由圖讀出氮和氧在液氣兩相的個別組成。(1 大氣壓下, 純物質的氮和氧之沸點分別為 77.3 K 和 90.2 K)。

(背面仍有題目, 請繼續作答)

(5) (25%)

- (a) A rigid insulated tank is initially evacuated. Atmospheric air at 0.1 Mpa and 20 C is allowed to leak into the tank until the pressure reaches 0.1 Mpa. What is the final temperature of the air within the tank, in degree Celsius?
- (b) Consider the data of part (a), except that the tank initially contains air at 0.04 Mpa and 20 C. Determine the final temperature, in degree Celsius?

(6) (25%)

Air at 200 Kpa and 950 K enters an adiabatic nozzle at low velocity and discharged at a pressure of 80 Kpa. If the adiabatic efficiency of the nozzle is 92%, First, sketch the process in a T(temperature)-S(entropy) diagram and then, determine

- (a) the maximum possible exit velocity (m/s)
- (b) the actual exit temperature in degree K
- (c) the actual exit velocity of the air in m/s

Note:

For Air: $C_p = 1.005 \text{ KJ/Kg K}$, $C_v = 0.718 \text{ KJ/Kg K}$