

Chapter 8 Applications of Plane Stress (Pressure vessels, Beams, and Combined Loadings)

8.1 Introduction

Plane stress conditions : buildings, machines, vehicles, and aircraft

In this chapter, the structures to be discussed are

pressure vessels

stress in beams : principle stresses, maximum shear stress

structures subjected to combined loadings

8.2 Spherical Pressure Vessels

pressure vessels are classified as shell structures, it is subjected to internal pressure p

to determine the stresses in the wall, let us cut through the sphere on a vertical diametral plane, the resultant pressure force is

$$P = p (\pi r^2)$$

where r is the inner radius of the sphere

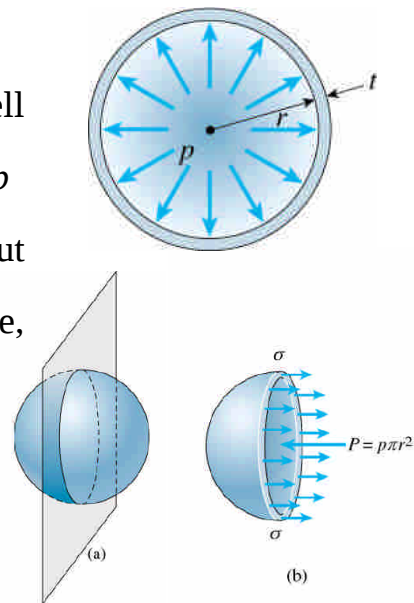
the resultant of the tensile stresses σ in the wall is

$$F = \sigma (2 \pi r_m t)$$

where t is the thickness of the wall and r_m is its mean radius

$$r_m = r + t / 2$$

equation of equilibrium in the horizontal direction

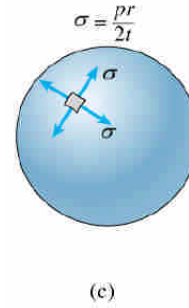


$$\Sigma F_{\text{horiz}} = 0$$

$$\sigma (2 \pi r_m t) - p (\pi r^2) = 0$$

the tensile stresses in the wall is

$$\sigma = \frac{p r^2}{2 r_m t}$$



for $r \gg t$ ($r > 10 t$), $r_m \simeq r$ then

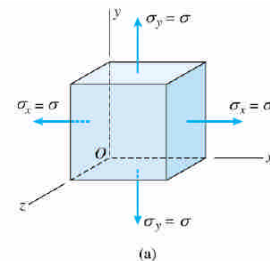
$$\sigma = \frac{p r}{2 t}$$

when we cut through the center of the sphere in any direction, we can conclude that the sphere is subjected to uniform tensile stress σ in all directions, they are known as membrane stresses

Stresses at the outer surface

$$\sigma_1 = \sigma_2 = \frac{p r}{2 t} \quad \sigma_3 = 0$$

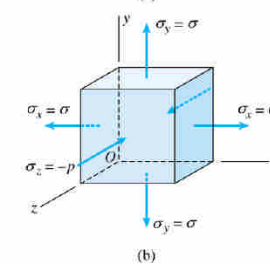
$$\text{then} \quad \tau_{\max} = \frac{\sigma}{2} = \frac{p r}{4 t}$$



Stresses at the inner surface

$$\sigma_1 = \sigma_2 = \frac{p r}{2 t} \quad \sigma_3 = -p$$

$$\text{then} \quad \tau_{\max} = \frac{\sigma + p}{2} = \frac{p r}{4 t} + \frac{p}{2} = \frac{p}{2} \left(\frac{r}{2t} + 1 \right)$$



Limitations of thin-shell theory

1. $r > 10 t$ or more
2. internal pressure must exceed external pressure

3. only pressure loading is considered for stress calculation
4. stress concentrations are not considered in the formula derived

Example 8-1

$$d = 18 \text{ in } t = 1/4 \text{ in}$$

$$\text{a. } \sigma_{\text{allow}} = 14,000 \text{ psi}, p_a = ?$$

$$\text{b. } \tau_{\text{allow}} = 6,000 \text{ psi } p_b = ?$$

$$\text{c. } \epsilon_{\text{allow}} = 0.0003 \quad E = 29 \times 10^6 \text{ psi}$$

$$\nu = 0.28 \quad p_c = ?$$

$$\text{d. } T_{\text{failure}} = 8.1 \text{ kip/in } n = 2.5 \quad p_d = ?$$

$$\text{e. } p_{\text{allow}} = ?$$

$$\text{a. } \sigma = pr/2t$$

$$p_a = \frac{2 t \sigma_{\text{allow}}}{r} = \frac{2 \times 0.25 \times 14,000}{9} = 777.8 \text{ psi}$$

$$\text{b. } \tau = pr/4t$$

$$p_b = \frac{4 t \tau_{\text{allow}}}{r} = \frac{4 \times 0.25 \times 6,000}{9} = 666.7 \text{ psi}$$

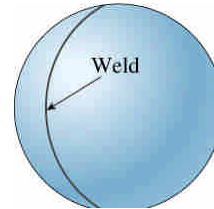
$$\text{c. } \epsilon_x = (\sigma_x - \nu \sigma_y) / E$$

$$\text{for } \sigma_x = \sigma_y = \sigma = pr / 2t$$

$$\text{then } \epsilon_x = \frac{\sigma}{E} (1 - \nu) = \frac{p r}{2t E} (1 - \nu)$$

$$\text{thus } p_c = \frac{2t E \epsilon_{\text{allow}}}{r (1 - \nu)} = \frac{2 \times 0.25 \times 29 \times 10^6 \times 0.0003}{9 (1 - 0.28)} = 671.3 \text{ psi}$$

$$\text{d. } T_{\text{allow}} = T_{\text{failure}} / n = 8.1 / 2.5 = 3.24 \text{ kip/in} = 3,240 \text{ lb/in}$$



$$\sigma_{\text{allow}} = T_{\text{allow}} / t = 3,240 / 0.25 = 12,960 \text{ psi}$$

$$p_d = \frac{2 t \sigma_{\text{allow}}}{r} = \frac{2 \times 0.25 \times 12,960}{9} = 720.0 \text{ psi}$$

$$e. \quad p_{\text{allow}} = \min[p_a, p_b, p_c \text{ and } p_d] = 666 \text{ psi}$$

for this p_{allow} , the tensile stresses in the shell are

$$\sigma = \frac{p r}{2 t} = \frac{666 \times 9}{2 \times 0.25} = 12,000 \text{ psi}$$

8.3 Cylindrical Pressure Vessels

consider a thin-walled circular tank

AB subjected to internal pressure

σ_1 : circumferential stress or hoop stress

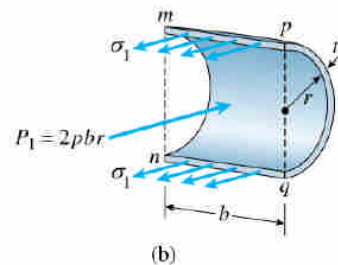
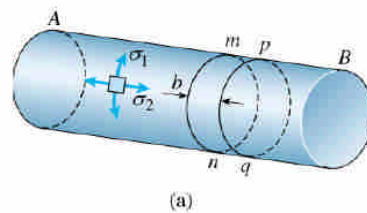
σ_2 : longitudinal stress or axial stress

from the free body $mpqn$, we

have the equation of equilibrium as

$$\sigma_1 (2 b t) - 2 p b r = 0$$

$$\text{then } \sigma_1 = \frac{p r}{t}$$

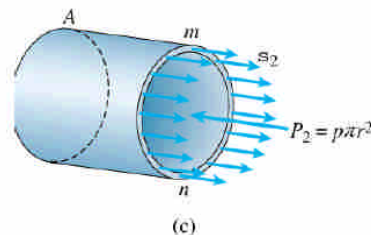


and the equation of equilibrium in longitudinal direction

$$\sigma_2 (2 \pi r t) - p \pi r^2 = 0$$

$$\text{then } \sigma_2 = \frac{p r}{2 t}$$

$$\text{now, we have } \sigma_1 = 2 \sigma_2$$



Stress at the outer surface

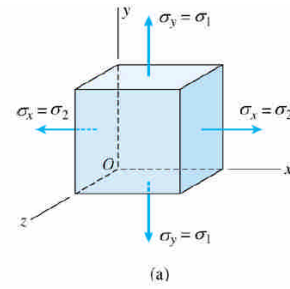
$$\sigma_1 = pr / t \quad \sigma_2 = pr / 2t \quad \sigma_3 = 0$$

$$(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t}$$

$$(\tau_{\max})_x = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

$$(\tau_{\max})_y = \frac{\sigma_2}{2} = \frac{pr}{4t}$$

$$\text{thus } \tau_{\max} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$



this stress occurs on a plane that has been rotated 45° about the x axis

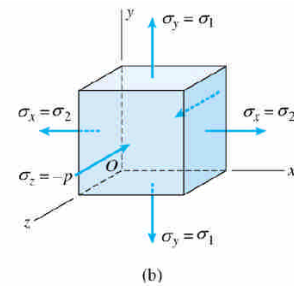
Stress at the inner surface

$$\sigma_1 = pr / t \quad \sigma_2 = pr / 2t \quad \sigma_3 = -p$$

$$(\tau_{\max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{2t} + \frac{p}{2}$$

$$(\tau_{\max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{pr}{4t} + \frac{p}{2}$$

$$(\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t}$$



Example 8-2

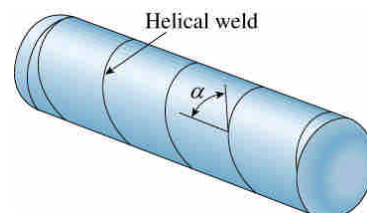
consider a cylindrical pressure vessel

$$r = 1.8 \text{ m} \quad t = 20 \text{ mm}$$

$$\text{weld angle } \alpha = 55^\circ$$

$$E = 200 \text{ GPa} \quad \nu = 0.30 \quad p = 800 \text{ kPa}$$

determine :



a. σ_1 and σ_2

b. the maximum in-plane and out-of-plane shear stress

c. ε_1 and ε_2

d. σ_w and τ_w in the welded seam

$$a. \quad \sigma_1 = p r / t = 800 \times 1800 / 20 = 72 \text{ MPa}$$

$$\sigma_2 = p r / 2 t = 800 \times 1800 / 2 \times 20 = 36 \text{ MPa}$$

$$b. \quad (\tau_{\max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{p r}{4 t} = 18 \text{ MPa} \quad (\text{in-plane shear})$$

$$(\tau_{\max})_x = \frac{\sigma_1}{2} = \frac{p r}{2 t} = 36 \text{ MPa} \quad (\text{out-of-plane shear})$$

$$c. \quad \varepsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) \quad \varepsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1)$$

$$\sigma_1 = p r / t \quad \sigma_2 = p r / 2 t$$

$$\text{then } \varepsilon_1 = \frac{\sigma_1}{2 E} (2 - \nu) = \frac{72 (2 - 0.3)}{2 \times 200 \times 10^3} = 306 \times 10^{-6}$$

$$\varepsilon_2 = \frac{\sigma_2}{E} (1 - 2 \nu) = \frac{36 (1 - 2 \times 0.3)}{200 \times 10^3} = 72 \times 10^{-6}$$

$$d. \quad \theta = 90 - \alpha = 35^\circ$$

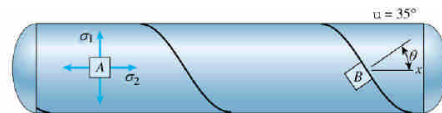
$$\sigma_x = \sigma_2 = p r / 2 t = 36 \text{ MPa}$$

$$\sigma_y = \sigma_1 = p r / t = 72 \text{ MPa}$$

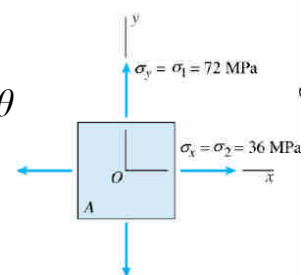
$$\tau_{xy} = 0$$

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x1y1} = - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$



(a)

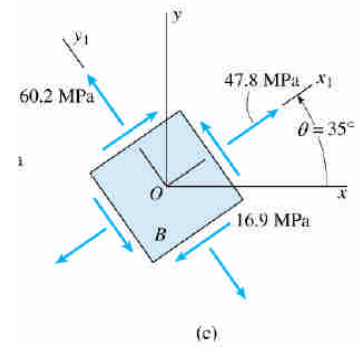


(b)

Then

$$\sigma_{x_1} = \frac{p r}{4 t} (3 - \cos 2\theta) = 47.8 \text{ MPa} = \sigma_w$$

$$\tau_{x_1 y_1} = \frac{p r}{4 t} \sin 2\theta = 16.9 \text{ MPa} = \tau_w$$



$$\sigma_{y_1} = \sigma_1 + \sigma_2 - \sigma_{x_1} = 60.2 \text{ MPa}$$

Mohr's circle

$$R = \frac{72 - 36}{2} = 18 \text{ MPa}$$

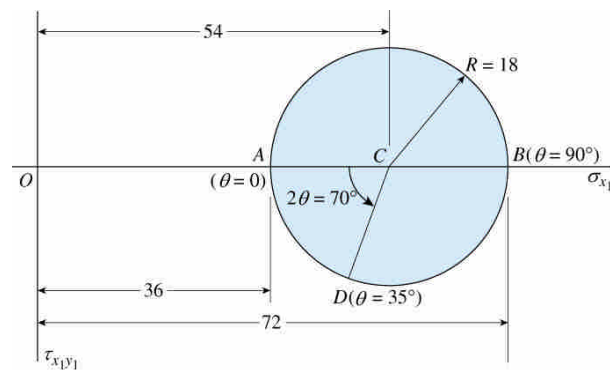
$$\frac{\sigma_1 + \sigma_2}{2} = \frac{27 + 36}{2} = 54 \text{ MPa}$$

$$\theta = 35^\circ \quad 2\theta = 70^\circ$$

$$\sigma_{x_1} = 54 - R \cos 70^\circ = 47.8 \text{ MPa}$$

$$\tau_{x_1 y_1} = R \sin 70^\circ = 16.9 \text{ MPa}$$

same results as earlier



8.4 Maximum Stresses in Beams

8.5 Combined Loadings