

Data Structures
Spring 2005, Final Exam, June 14
8:00-9:50

1. (20%) Consider the following undirected graph.
 - (a) Find a minimum-cost spanning tree by Prim's algorithm.
 - (b) Find a minimum-cost spanning tree by Kruskal's algorithm.
 - (c) Find a depth-first spanning tree starting at **a**.
 - (d) Find a breath-first spanning tree starting at **d**.

2. (10%) Let a hash table and a hashing function be defined as follows :
table : array [0..7] of hashnode;
The Hashing function is selected as:

$$h(x) = mid - squaring(x)$$

where x is calculated from x (an alphanumeric string) by summing the numbers which represent the alphanumeric characters. The A..Z alphabets will be represented as numbers 1..26 and 0..9 be as numbers 0..9. For example,

$$AZ6 = 1 + 26 + 6$$

Mid - squaring is calculated by

(1) $y = x^2$

(2) Take left 3-bit starting from the second right most bit of y .

The following operations are performed starting with an empty table.

insert 2B3

insert 11C

insert Y48

insert 183

insert DF3

insert 19Z

- (a) With linear probing as rehash technique, draw the closed hash table.
- (b) With chaining as overflow handling technique, draw the chained hash table.

3. (10%) Single source all destinations *ShortestPath* problem. Let us apply *Dijkstra* algorithm to the following directed graph shown below. If we wish to reconstruct the shortest path from the source to each vertex, then we can maintain another array P of vertices, such that $P[v]$ contains the vertex immediately before vertex v in the shortest path. Show the contents of P after running each iteration of *Dijkstra* algorithm. Suppose the source is vertex 1 and all elements in array P are initialized to 1.

4. (20%)
 - (a) (8%) Here are ten integers: 15, 24, 18, 9, 34, 26, 14, 36. Sort them using *quick sort*. Give the status of the list at the end of each phase.
 - (b) (3%) Dijkstra's algorithm is used to find single source all destinations shortest paths. To apply this algorithm, there is one constraint on the weight of edge. What is the constraint ?
 - (c) (3%) (internal sort) If we have only limited space in Main Memory and the number of elements to be sorted is very large, which sorting algorithm is not suited ? (insertion sort, quick sort, heap sort, merge sort)
 - (d) (6%) What is the lower bound on sorting problem ? Give the worst case time complexity of bin sort algorithm. Is your answer consistent with the result for lower bound analysis ? Explain why ?

5. (10%) (a) Describe a mathematical model for the following scheduling problem. Given tasks, $T_1, T_2, \dots, T_n$, which require times $t_1, t_2, \dots, t_n$ to complete, and a set of constraints, each of the form " T_i must be completed prior to the start of T_j ".
 (b) For the above model, show how to find the minimum time necessary to complete all tasks.

6. (10%) A *run* is a sorted sequence of elements. External sorting is to merge two runs into a single run by merging sort. Let input or output one block take t_{io} time. n runs of different sizes of block are to be merged into a single run by repeatedly merging pairs of runs until only one run remains. Now, given 10 runs of sizes: 3, 5, 6, 7, 9, 10, 14, 17, 18, 19. (a) Show how to merge the runs so that the total I/O time is minimum. (b) What is the minimum t_{io} required ?

7. (10%) Let $x_1, x_2, \dots, x_m$ be a sequence of 4-digit hexadecimal integers. Describe an algorithm to sort this sequence in time $O(m)$.

8. (10%) Let $G = (V, E, W)$ be a un-directed graph with $W \in \{1, 2, 3\}$ on all edges. Write an $O(|V| + |E|)$ C-like algorithm *ShortestPath*(i, V, E) which computes the value of shortest path from vertex i to all other vertices in G . (hint: graph traversal algorithm)