

Data Structures
Spring 2005, Final Exam, June 13
10:10-12:00

1. (13%) Consider the following undirected graph.
 - (a) (6%) Find a depth-first spanning tree starting at **1**. Let the graph be stored as shown in the following adjacency list.
 - (b) (7%) Find a minimum-cost spanning tree by Prim's algorithm. Let the starting node be **1**.

2. (10%) Construct Huffman Binary codes for the following 6 messages whose probabilities of appearance are $A = 0.25$, $B = 0.23$, $C = 0.21$, $D = 0.15$, $E = 0.09$, and $F = 0.07$. Suppose that E is encoded as 1111. What are codes for A and F ?

3. (10%) Let G be a un-directed graph with unit weight on all edges. Write an $O(|V| + |E|)$ C-like algorithm *ShortestPath*(i, V, E) which computes the value of shortest path from vertex i to all other vertices in G . (hint: graph traversal algorithm)

4. (a) (4%) Describe a graph model for a gate-level circuit where a gate g_i has delay d_i .
(b) (8%) Write an algorithm to find the longest delay of the circuit.

5. (10%) Use an example to explain why the single source all destinations *ShortestPath* algorithm (Dijkstra's algorithm) does not work for a graph with negative edge.

6. (10%) Write an $O(|V| + |E|)$ C-like procedure *IsConnected*(V, E) which returns the value *true* if the undirected graph with the vertex set V and edge set E is connected and return *false* otherwise.

7. (10%) Let a hash table and a hashing function be defined as follows :
 table : array [0..15] of hashnode;
 hashing function (mid squaring): $h(x) = \text{four bits starting from the 5th right most bit of } x^2$;

The following operations are performed starting with an empty table.

insert 23
 insert 11
 insert 8
 insert 18
 insert 3
 insert 19
 insert 34
 insert 15
 insert 27

- (a) With linear probing as rehash technique, draw the closed hash table.
 (b) With chaining as overflow handling technique, draw the chained hash table.
 (note: $23^2 = 1021_8$, $11^2 = 171_8$, $8^2 = 20_8$, $18^2 = 504_8$, $3^2 = 11_8$, $19^2 = 551_8$, $34^2 = 2204_8$, $15^2 = 341_8$, $27^2 = 1331_8$)
8. (a) (6%) Here are ten integers: 15, 24, 18, 9, 16, 48, 34, 26, 14, 99. Sort them using *quick sort*. Give the status of the list at the end of each phase.
 (b) (3%) What is the worst case time complexity for *quick sort* ? Describe an input sequence which has this time complexity.
 (c) (3%) If the number of elements to be sorted is small and the list of numbers is partially sorted, which sorting algorithm is best suited ?
 (d) (3%) Which sorting algorithm is best for external sort ?
9. (10%) Let $x_1, x_2, \dots, x_m$ be a sequence of integers in the range 1 to n^2 . Describe an algorithm to sort this sequence in time $O(n + m)$.