

一、Computer Organization and Design

1. **[Acronyms]** Example: DMA → Direct Memory Access. (a) VHDL (b) SoC (c) TLB (d) RAID (e) NUMA. (Within the context of computer architecture.) (10%)
2. **[Number System, IEEE 754]** Determine the base of the number system for the following operation to be correct: $23+44+13+32 = 223$. (5%) (b) Find the smallest normalized positive floating-point number in the IEEE-754 single-precision representation. (5%)
3. **[Cache]** Consider data transfers between two levels of hierarchical memory. If we logically organize all data stored in the lower level into blocks and store the most-frequently-used blocks in the upper level. Assume that the hit rate is H , the upper level latency is T_u , and the lower level latency is T_l , and the miss penalty is T_m .
 - (a) What is the average memory latency? (5%)
 - (b) What is the speedup by using the hierarchical memory system? (5%)
4. **[Pipelining]** (a) Assuming no hazards, a pipelined processor with s stages (each stage takes 1 clock cycle) can execute n instructions in _____ clock cycles. (2%) (b) Use the result in (a) to show that the ideal pipeline speedup equals the number of stages. (3%)
5. **[Approximation Circuits]**
 In a traditional adder design, the calculation must consider all input bits to obtain the final carry out. However, in real programs, inputs to the adder are not completely random and the *effective carry chain is much shorter* for most cases. Instead of designing for the worst-case scenario, it is possible to build a faster adder with a much shorter carry chain to *approximate* the result. Suppose we consider only the previous k inputs (look-ahead k -bits) instead of all previous input bits to estimate the i -th carry bit c_i , i.e.,

$$c_i = f(a_{i-1}, b_{i-1}, a_{i-2}, b_{i-2}, \dots, a_{i-k}, b_{i-k}) \text{ where } 0 < k < i+1 \text{ and } a_j, b_j = 0 \text{ if } j < 0.$$
 - (a) With random inputs, show that c_i will generate correct a result with a probability of $1 - \frac{1}{2^{k+2}}$. (5%)
 - (b) What is the probability of having a correct 'carry' result considering only k previous inputs for an N -bit addition? (5%)
 - (c) Design a logic circuit to detect when the approximate adder will generate an incorrect carry result for the i -th carry bit. (5%)

二、Operating Systems

1. Suppose that two processes, P_a and P_b , are running in a *uni-processor* system. P_a has *three* threads. P_b has *two* threads. All threads in both processes are CPU-bound and they never block for I/O. The operating system uses simple *round-robin scheduling*.

(a) (2%) Suppose that all of the threads are *user-level* threads, and that user-level threads are implemented using a single kernel thread per process. What percentage of the processor's time will be spent running P_a 's threads?

(b) (2%) Suppose instead that all of the threads are *kernel* threads. What percentage of the processor's time will be spent running P_a 's threads?

2. Answer the following questions about deadlock.

(a) (3%) A system has three processes sharing seven units of resource

R. Each process may request up to three units of R. Is there a danger of deadlock?

(b) Consider a system in which the total available memory is 48K and in which memory once allocated to a process *cannot* be preempted from that process.

Three processes A, B, and C have declared in advance that the *maximum* amount of memory that they will require is 25K, 15K, and 41K words respectively. When the three processes are all in execution and using 3K, 9K, and 24K words of memory respectively, consider the following additional requests *individually*:

i) (1%) A requests 9K words more. Can this request be granted with a guarantee that deadlock will not occur? (yes or no)

ii) (1%) B requests 6K words more. Can this request be granted with a guarantee that deadlock will not occur? (yes or no)

iii) (1%) C requests 7K words more. Can this request be granted with a guarantee that deadlock will not occur? (yes or no)

(c) (2%) Explain why an OS designer needs to worry about deadlock involving physical memory but not the CPU.

(d) (2%) Does a deadlock prevention algorithm necessarily prevent starvation as well?

3. Given the following variables and their initial values as follows:

- x : integer, initial value is 0
- y : integer, initial value is 23
- m : *general semaphore*, initial value is 1
- m1 : *general semaphore*, initial value is 0

For each of the following program, list values *possible* for x when the program terminates. Note that “**cobegin** P1; || P2; **coend**” means P1 and P2 are executed concurrently, and “:=” is the assignment operator.

- (a) (2%) **cobegin** $x := x + 1;$ || $x := y + 1;$ **coend**
- (b) (2%) **cobegin** $P(m); x := x + 1; V(m);$ || $P(m); x := y + 1; V(m);$ **coend**
- (c) (2%) **cobegin** $P(m1); x := x + 1; V(m);$ || $P(m); x := y + 1; V(m1);$ **coend**
- (d) (2%) **cobegin** $V(m1); x := x + 1; P(m);$ || $V(m1); x := y + 1; P(m1);$ **coend**

4. (a) (3%) Suppose that you wish to design a virtual memory system with the following characteristics:

- i) The size of a page table entry is 4 bytes.
- ii) Each page table must fit into a single physical frame.
- iii) The system must be able to support virtual address spaces as large as 2^{38} bytes (256 GB).

Suppose that you decide to use a *multi-level paging scheme*. What are the respective *minimum* page sizes that your system must have if you determine to use *two levels* or *three levels* of page tables? Your answer includes two parts: one for two-level and one for three-level.

(b) (3%) For a certain page trace starting with no page in the memory, a demand-paged memory system operated under the least-recently used (*LRU*) *replacement policy* results in 9 and 11 *page faults* when the primary memory is of 6 and 4 pages, respectively. When the same page trace is operated under the *optimal policy*, what are the numbers of page faults may be for the same primary memory of 6 and 4 pages?

(c) (3%) When we switch from *statically* linked libraries to *dynamically* linked libraries, would there be any effects on the working set size of a process and on the working set size of all the running processes?

5. One operating system performs process scheduling by assigning *base priorities* (a high of -20 to a low of +20 with a median of 0), and making *priority adjustments*. Priority adjustments are computed in response to changing system conditions. These are added to base priorities to compute *current priorities*; the process with the *highest* current priority is dispatched first.

The priority adjustment is strongly biased in favor of process that have recently used relatively little CPU time. The scheduling algorithm "forgets" CPU usage quickly to give the benefit of the doubt to processes with changing behavior. The algorithm "forgets" 90% of recent CPU usage in $5 * n$ seconds; n is the average number of runnable processes in the last minute.

Based on the above description of the process scheduling algorithm, answer each of the following questions briefly.

- (a) (2%) Are CPU-bound processes favored more when the system is heavily loaded or when it is lightly loaded? (heavily or lightly)
 - (b) (2%) Will an I/O-bound process receive favored or unfavored response immediately after an I/O completion? (favored or unfavored)
 - (c) (2%) Can CPU-bound processes suffer indefinite postponement? (yes or no)
 - (d) (2%) When a process's behavior changes from CPU-bound to I/O-bound, it will immediately receive favored response. True or false?
 - (e) (2%) When a process's behavior changes from I/O-bound to CPU-bound, it will immediately receive favored response. True or false?
6. (a) (3%) Among the first-come-first-serve (FCFS), shortest seek time next (SSTF), LOOK, SCAN, C-LOOK, C-SCAN, and N-Step SCAN disk-head scheduling policies, which are subject to *starvation* at high loads (i.e., very large number of requests arrive in a short time)?
- (b) (2%) Response times are more predictable in preemptive systems than in non-preemptive systems. True or false.
 - (c) (2%) Describe a program that works fine with round-robin scheduling, but makes other program starving under first-come-first-serve (FCFS) scheduling.
 - (d) (2%) The LRU replacement scheme is *inappropriate* for managing disk cache of sequential files? True or false.

(一)

1. (12%) When we encode members of Z_2^m into Z_2^n ; such a code is called an (m, n) code. The first m components of a code word are the information digits, and the last r ($r = n - m$) components are the check digits. Please answer (1) which of the following are perfect codes, and (2) which are single-error correcting codes?

(a) $(12, 7)$ (b) $(15, 11)$ (c) $(5, 3)$
2. (12%) How many nonnegative integer solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$.
3. (12%) Please solve the recurrence relation
$$\begin{cases} a_{n+2} = 4a_{n+1} - 4a_n, n \geq 0 \\ a_0 = 1, a_1 = 3 \end{cases}$$
4. (12%) Let $f : S \rightarrow T$, and $g : T \rightarrow U$. Then the composition function, $g \circ f$, is a function from S to U defined by $(g \circ f)(s) = g(f(s))$. Consider the following statements, which are false statement(s)?

(a) If f is one-to-one then $f \circ g$ is one-to-one.
(b) If f and g are onto then $f \circ g$ is onto.
(c) If f and g are one-to-one and onto then $f \circ g$ is one-to-one and onto.
(d) If $f \circ g$ is one-to-one then f is one-to-one.
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(f) If $f \circ g$ is onto then f is onto.
(g) If $f \circ g$ is onto then g is onto.
5. (12%) Please (1) design a minimum rail network connecting the seven cities, shown in the mileage chart below, (2) show sum of the network.

	City-1	City-2	City-3	City-4	City-5	City-6	City-7
City-1	0	500	400	600	100	550	300
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City-3	400	620	0	525	520	900	420
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(二)
(機率部分的題目每一題都需要寫出計算過程)

1 (12%) 令 $f(x)=cx(1-x)$ 是 random variable X 的 probability density function，其中 c 為一個常數。若 X 的分佈區間是 $[0,1]$ ，回答下列問題。

1.1 (4%) 求 c 的值

1.2 (4%) 令 $Y=3X+2$ ，求 $E[Y]$ 的值

1.3 (4%) 求 $VAR[X]$ 的值

2 (13%) 已知變數 X 和 Y 之間的關係是 $X=aY^b$ ，假定我們已經獲知在 $Y=y_i$ 時， $X=x_i$ ， $i=1,2,3,4$ 四組數據，利用這四組抽樣數據求出和表示 a 和 b 的值。

3 (15%) 假定莊家有甲和乙兩個骰子，甲乙兩骰子擲出數字的機率如下表。在任一賭局中莊家只會使用甲乙兩骰子的其中一個，並且會先試擲其所使用的骰子三次。在某一次賭局中，莊家在試擲時擲出 1，4 和 6。若我們假設每一次試擲均為獨立事件，並且假設莊家使用甲乙兩個骰子的機率各為 0.6 和 0.4，在該次賭局中，莊家使用甲乙兩骰子的機率各為多少？

數字	1	2	3	4	5	6
甲骰子	0.1	0.2	0.2	0.1	0.3	0.1
乙骰子	0.2	0.1	0.1	0.3	0.1	0.2

(一) 1. Let $V_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid b > 0 \right\}$, $V_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a - b + c = 0 \right\}$.

Are V_1 and V_2 subspaces of $\mathbb{R}^3$? Explain. [10%]

2. Let A be a 4×5 matrix.

(a) If the rank of A is 3, what is the dimension of its column space? Explain. [6%]

(b) If the rank of A is 3, what is the dimension of the solution space of the homogeneous system $Ax = 0$? Explain. [6%]

3. Let $L: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be defined by

$$L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_2 \\ a_3 + a_4 \\ a_1 + a_3 \end{bmatrix}.$$

(a) Is L a linear transformation? Justify your answer. [6%]

(b) Find a basis for $\text{Ker } L$. [6%]

(c) Find a basis for $\text{range } L$. [6%]

(二)

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(二)

「計算機數學與網路」科之「計算機網路概論」部份

- [10%] 1. (a) What is the main difference between TCP and UDP?
(b) Choose the following applications that are suitable to use UDP as the transport protocol rather than TCP?
ftp, email, Voice Over IP, Video On Demand, http.
- [10%] 2. If we use ftp to download files via wireless LAN (WLAN) access to Internet, sometimes we suffer large delay.
Can you explain at least 5 reasons for the large delay?
- [4%] 3. Many companies have a policy of having two (or more) routers connecting the company to the Internet to provide some redundancy in case one of them goes down. Is this policy still possible with NAT (Network Address Translation)? Explain your answer.
- [6%] 4. The wireless LANs used protocols such as CSMA/CA (or MACA) instead of using CSMA/CD. Under what conditions, if any, would it be possible to use CSMA/CD instead?
- [10%] 5. With the growth of multimedia applications, quality of service (QoS) is needed. Can you list at least 3 QoS requirements for these applications? Explain briefly the techniques for achieving good QoS in each requirement.

1. (10%)

Consider a disk on PC has the following characteristics

Rotation speed = 7200 revolution/minute

Arm movement time = 1 msec fixed startup time + 0.1 msec for each track crossed (the 1 msec is a constant no matter how far the arm moves)

Number of surfaces = 2 (a single read/write arm holds both read/write heads)

Number of tracks per surface = 100

Number of sectors per track = 20

Number of characters per sector = 512

- (1) How many characters can be stored on a single disk?
- (2) The access time to any individual sector of the disk is made up of three components. Please list these three components.
- (3) What is the best case disk access time for this disk?
- (4) What is the worst case disk access time for this disk?

2. (10%)

Two major internal data structures in the assembler are the Operation Code Table (OPTAB) and the Symbol Table (SYMTAB).

- (1) Please list the functional requirements of OPTAB.
- (2) Which of the following data structures are beneficial for the OPTAB: (a) sorted array (b) binary search tree (c) AVL tree (d) minimum spanning tree (e) static hashing, (f) dynamic hashing (g) minimum perfect hashing (h) heap (i) graph (j) inverted file? Why?
- (3) Please list the functional requirements of SYMTAB.
- (4) Which of the following data structures are beneficial for the SYMTAB: (a) sorted array (b) binary search tree (c) AVL tree (d) minimum spanning tree (e) static hashing (f) dynamic hashing (g) minimum perfect hashing (h) heap (i) graph (j) inverted file? Why?

3. (8%)

- (1) There are several basic methods of passing parameters to subprograms in programming languages. Please list the basic methods of passing parameters in C Languages?
- (2) In C language, a typical statement to output the value of an integer variable xvar is

`printf("%d", xvar);`

while that to input a value to an integer variable xvar is

`scanf("%d", &xvar);`

Please give the reasons why there exists the difference between the two notations of the variables for I/O statements.

4. (4%)

Given a two-dimension array $x[1024, 1024]$. The size of each row of this array is design to be the size of a page in main memory. Assume that there is only one frame which is just only one page size in main memory. How many page faults will occur for the following program segment?

```
for (i=0; i<=1023; i++)
    for (j=0; j<=1023; j++)
        x[i, j]=0;
```

5. (10%)

In the relational data model, the *referential integrity constraint* states that the value of the *foreign key* in one relation S must refer to an existing value of the referred *primary key* in the other relation R.

STUDENT	
StudID	Name
001	Grace
003	James
006	Mary

SCORE		
CourseNo	StudentNo	Score
003	001	90
005	001	85
001	003	90
002	006	80

For example, given the above two tables STUDENT and SCORE, the primary key of table Student is the attribute StudID while that of table SCORE is the composition of attributes CourseNo and StudentNo. Moreover, StudentNo is a foreign key of table SCORE which refers to the primary key StudID of table STUDENT. Under referential integrity constraint, each instance of the foreign key StudentNo in table SCORE must match the instance of the primary key StudID of some tuple in table STUDENT. Note that in the relational data model, each table must have a primary key. However, some tables may have no foreign keys.

- (1) Given a relational database schema with three tables R, S, T and associated primary keys A, B, C respectively. Table R has a foreign key X which refers to the primary key C of table T while table T has a foreign key Z which refers to the primary key B of table S. Assume that after the construction of the database schema, there is no tuples in these tables. To import tuples from the raw text data to these tables, it is important to consider the importing order of tables for the referential integrity constraint. Please list the order for importing these tables.
- (2) Given a relational database schema with n relational tables, the associated primary keys and the foreign keys, please design the data structure and algorithm, under the *referential integrity constraint*, to find a correct order for importing the data to these tables based on a well-known graph algorithm.

6. (8%) 選擇題 (每小題 2 分, 答錯倒扣 1 分。此大題最低 0 分)

- (1) () Which one of the following phases checks the sequence of tokens to see whether it is

syntactically correct according to the rules of the programming language.

- (a) lexical analysis
 - (b) semantic analysis and code generation
 - (c) parsing
 - (d) code optimization
- (2) () Which one of the following technologies is the most similar to the operation of proxy servers ?
- (a) virtual memory
 - (b) time-sharing
 - (c) cache memory
 - (d) pipeline
- (3) () Which one of the following protocols support the fast connectionless service.
- (a) UDP
 - (b) TCP
 - (c) SMTP
 - (d) ARP
- (4) () Given the following program segment, what is the value of the variable svar after execution of this statement?

```
svar = x(4)
int x(n)
{   if (n <= 1)
    return 1;
    else
    return(x(n-1)+x(n-2)); }
```

- (a) 3
- (b) 4
- (c) 5
- (d) 6

說明：1. 請書寫必要之解題過程。過程正確但答案錯誤，可能有部分分數。如題目之解答非顯而易見者，僅書寫答案而缺乏必要之過程，亦無法獲得該題之滿分。

2. 可使用中文或英文作答。

7. (10%, 2% for each problem) 選擇題(select a number).

- (1) Which of the following items is not true for a binary search tree (BST)?
 - (a) It takes constant time, $O(1)$, in finding the minimum or maximum in a BST.
 - (b) For any node x in a BST tree, let y denote any element in the left subtree of x and let z denote any element in the right subtree of x , we have $y < x < z$. (Assume that all the elements in the tree are distinct.)
 - (c) The insert and delete operations takes $O(\log n)$ time for a BST with n nodes.
 - (d) The worst case dynamic set operations on a BST take $O(n)$ time.
- (2) The worst case performance of quicksort is
 - (a) $O(n)$
 - (b) $O(n \log n)$
 - (c) $O(n^2)$
 - (d) $O(2^n)$
- (3) The worst case of insertion operation on a BST takes
 - (a) $O(\log n)$
 - (b) $O(n)$
 - (c) $O(n \log n)$
 - (d) $O(n^2)$
- (4) In depth-first-search algorithm, the order that the node is processed is similar to that of
 - (a) BFS
 - (b) inorder traversal
 - (c) preorder traversal
 - (d) postorder traversal
- (5) The elements in a queue is processed in FIFO fashion, which is similar to
 - (a) BFS
 - (b) DFS
 - (c) preorder traversal
 - (d) postorder traversal

8. (14%, 2% for each problem) 是非題(True or False):

- (1) Adjacency matrix is good for representing sparse graph.
- (2) The minimum spanning tree of a n -node graph has $O(\log n)$ edges.
- (3) $2^{2^n} = O(2^n)$.
- (4) Membership test in a linked list requires $O(\log n)$ time, in the worst case, for input size n .
- (5) Both insert and delete operations of a queue take $O(1)$ time.
- (6) A binary tree can be used to represent a tree with more than 2 child nodes of a (parent) node.
- (7) A 2-3 tree with height h has at least $2^h - 1$ leaves and at most $3^h - 1$ leaves.

9. (10%)

There are two algorithms, the Kruskal's algorithm and the Prim's algorithm, for finding the Minimum Spanning Tree(MST) of a graph $G=(V,E)$, where V denotes the set of vertices and E denotes the set of edges. These algorithms are given below:

Algorithm Kruskal(G)

```
//Kruskal's algorithm for constructing a MST
//Input: A weighted connected graph  $G=(V,E)$ 
//Output:  $E_T$ , the set of edges composing a MST of  $G$ 
Sort  $E$  in non-decreasing order of the edge weights
      so that  $w(e_{i_1}) \leq w(e_{i_2}) \leq \dots \leq w(e_{i_{|E|}})$ 
 $E_T \leftarrow \emptyset$ ;  $ecounter \leftarrow 0$  //Initialize the set of
      tree edges and its size
 $k \leftarrow 0$  //Initialize the number of
      processed edges

while  $ecounter < |V| - 1$ 
   $k \leftarrow k + 1$ 
  if  $E_T \cup \{e_{i_k}\}$  is acyclic
     $E_T \leftarrow E_T \cup \{e_{i_k}\}$ ;  $ecounter \leftarrow ecounter + 1$ 
return  $E_T$ 
```

Algorithm Prim(G)

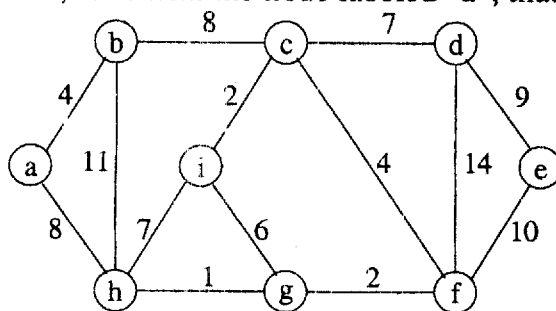
```
//Prim's algorithm for constructing a MST
//Input: A weighted connected graph  $G=(V,E)$ 
//Output:  $E_T$ , the set of edges composing a MST of  $G$ 
 $V_T \leftarrow \{v_0\}$  //Initialize the set of tree vertices to  $v_0$ 
 $E_T \leftarrow \emptyset$ 
for  $i \leftarrow 1$  to  $|V| - 1$  do
  find a minimum - weight edge  $e^* = (v^*, u^*)$ 
    among all the edges  $(v, u)$ 
    such that  $v^*$  is in  $V_T$  and  $u^*$  is in  $V - V_T$ 
   $V_T \leftarrow V_T \cup \{u^*\}$ 
   $E_T \leftarrow E_T \cup \{e^*\}$ 
return  $E_T$ 
```

Answer the following questions according to these two algorithms and the diagram below:

(1) (5%) Find the 6th edge added to the MST of the following graph using Kruskal's algorithm.

(2) (5%) Find the 6th edge added to the MST of the following graph using Prim's algorithm.

(If there are two edges with the same edge weight, select the one with "smaller" alphabetical order first. For the Prim's algorithm, start with the node labeled "a", that is, v_0 is set to the "a" node.)



10. (6%)

The recurrence equation, as described below, can be used to analyze the complexity of divide and conquer method.

$$T(n) = a T\left(\frac{n}{b}\right) + f(n)$$

Answer the following:

- (1) (2%) the role of the function $f(n)$,
- (2) (2%) the restriction on b , and,
- (3) (2%) why we need such a restriction.

11. (10%)

- (1) (3%) What is the lower bound of the comparison based sorting algorithms. (Just state the answer.)
- (2) (2%) The time complexity of the radix sort can be proved to be $O(d(n+k))$, for n d -digit numbers in which each digit ranges from 0 to k . Is this result conflict with the result in (1)?
- (3) (5%) Justify your answer in (2).