

考試科目	計算機數學(二)	所別	資訊科學	考試時間	4月21日 上午 9:00 至 12:00
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I 選擇與填充 (35 %; 不倒扣)

- (4%) Let $f(n)$ be an increasing function satisfying the recurrence relation: $f(n) = 7f(n/2) + 15n^2/4$. Then which of the following statements are correct? (多選)
 - $f(n) = O(n^2)$
 - $f(n) = o(n^2)$
 - $f(n) = O(n^3)$
 - $f(n) = o(n^3)$
 - $f(n) = \Theta(n^3)$
- (4%) Which of the following statement is *not* valid in Boolean algebras.
 - $x(y+z) = xy + xz$
 - $x(x+y) = x$
 - $(x+y)(x+z) = x + yz$
 - $x(\sim y) + (\sim x)y = x + y$
- (4%) What is the proposition a proof of which *does not* imply the validity of the implication $p \rightarrow q$?
 - $p \rightarrow \text{false}$
 - $\sim p \rightarrow \sim q$
 - $\sim p \text{ or } q$
 - $\sim q \rightarrow \text{false}$
 - $\sim(p \text{ and } \sim q)$
- (4%) What is the number of non-negative integer solutions to the equation $x + y + z < 18$ with $x \geq 1$, $y \geq 2$ and $z \geq 3$.
- (4%) What is the probability of winning the 3 out-of-6 prize in playing Taiwan lottery for correctly choosing 3 (but not 4, 5 or 6) numbers out of six integers chosen between 1 and 42, inclusively, by a fair random process.
- (5%) What is the least positive integer x satisfying the system of congruences: $x \equiv 2 \pmod{5}$, $x \equiv 5 \pmod{11}$ and $x \equiv 11 \pmod{17}$.
- (10%) Which of the following statements are correct? (多選; 每答對一小題給 2%)
 - All spanning trees of a graph have the same number of edges.
 - If $G = (V, E)$ is a multigraph containing no isolated or pending vertices, then the number of edges $|E|$ of G is always less than or equal to the number of vertices $|V|$ of G .
 - The chromatic number of every graph is less than or equal to 4.
 - There exists a Euler circuit for every complete simple graph K_5 with 5 vertices.
 - There exists one and only one path between every two vertices of a tree.

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II 計算與證明 (25 %)

8. (10 %) Let G be a directed graph with n vertices, x and y two distinct vertices of G . Show that if there is a path from x to y then there must exist a path of length less than n from x to y .
9. (15%) Let $\leq$ be a preorder on a nonempty set S (i.e., $\leq$ is a reflexive and transitive relation on S). Define a new relation $\equiv$ on S as follows : for all $x, y \in S$, $x \equiv y$ iff $x \leq y$ and $y \leq x$.
- (a) Show that $\equiv$ is an equivalence relation on S . [6 %]
- Let $\ll$ be a relation on the set $S/\equiv$ of all equivalence classes of S such that for all A and B belonging to $S/\equiv$, $A \ll B$ iff there exist $x \in A$ and $y \in B$ with $x \leq y$.
- (b) Show that the relation $\ll$ is a partial order on $S/\equiv$. [9 %]

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IV.

設 $A = \begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 2 & 2 & -1 & 0 & 1 \\ 1 & 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

(八分) (1) 將 A 化為簡約列梯形 (reduced row-echelon form). (必須列出過程)

(八分) (2) 求齊次系統 (homogeneous system) $Ax = 0$ 的解空間 (solution space) 之基底 (basis), 其中 $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, 0 是零向量.

(八分) (3) A 的秩是多少? (必須說明理由)

IV
(八分) (i) R^3 中的三個向量 $v_1 = (3, 0, -4)$, $v_2 = (5, -1, 2)$, $v_3 = (1, 1, 3)$ 是否線性獨立 (linearly independent)? 並將 $v = (16, 1, 9)$ 表示成 v_1, v_2, v_3 的線性組合.

(八分) (2) $W = \left\{ \begin{bmatrix} a & c \\ c & b \end{bmatrix} \mid a, b, c \text{ 是實數} \right\}$ 是不是

$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \text{ 是實數} \right\}$ 的子空間?

(必須說明理由). 如果是, 也求出 W 的基底與維數 (dimension).

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回答問題請列計算過程。

1. (10分) Assume that floating numbers are represented in a 16-bit format as follows:

- Bit 0 is the sign of the mantissa (0=positive, 1=negative),
- Bits 1-6 are the exponent. The implied base of the exponent is 8. The 6-bit exponent is stored in binary using excess-32,
- The 9-bit mantissa is normalized so that the binary point is to the left of the entire mantissa.

- (a) What range of numbers can be represented in this format?
(b) Represent the decimal number 21 in the format.

Hint: use base-8 number for the answers.

2. (10分) Consider the recursive paradigm in computational algorithms.

- (a) What is the typical structure of recursive functions? Explain it by implementing the factorial function ($n!$) in recursive manner.
(b) What are the potential problems in using recursive functions? How the problem can be reduced?

3. (10分) Analyze the computational complexity of selection sort and merge sort algorithms with pseudocodes. Express the complexity in big-O notation. Compare them and draw a conclusion.

4. (10分) A finite-state machine is defined by a set of possible *inputs* I , a set of possible *outputs* O , and a set of finite *states* S , together with two functions: $output = f(current_state)$ and $next_state = g(current_state, input)$. Suppose $I = \{0, 1\}$, $O = \{\text{上, 榜, 了}\}$, $S = \{x, y, z\}$, $f(x) = \text{榜}$, $f(y) = \text{上}$, $f(z) = \text{了}$, $g(x, 0) = z$, $g(x, 1) = y$, $g(y, 0) = z$, $g(y, 1) = x$, $g(z, 0) = x$, $g(z, 1) = z$.

- (a) Draw a state graph for the machine.
(b) Find the output if the initial state is x and the input string is 101.
(c) Under what situation will the output be “上榜了”?

5. (10分) 是非題每題二分，答錯倒扣一分。

- (1) When an object image is reduced in size and enlarged later, the detail of the image can be recovered exactly.
(2) Correcting errors is as easy as detecting them.
(3) Two entire structures $p1$ and $p2$ can be tested for equality by (if $p1 == p2$) as long as they are the same data type.
(4) Thrashing is caused by the creation of multiple threads in a process.

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6. (8%) Given the following algorithm, please analyze

- (1) the best time complexity of the comparison operation.
- (2) the best time complexity of the exchange operation.
- (3) the worst time complexity of the comparison operation.
- (4) the worst time complexity of the exchange operation.

```
void X-Algorithm(int A[], int N)
{
    int j, p, m;
    for (p=0; p < N-2; p++)
    {
        m=p;
        for (j=p+1; j < N-1; j++)
            if (A[j] < A[m])
                m=j;
        exchange (A[p], A[m]);
    }
}
```

7. (6%) Given the modified Merge sort algorithm below, please analyze

- (1) the worst time complexity of this modified algorithm.
- (2) the best time complexity of this modified algorithm.

```
void Msort(int A[], int Tmp[], int Left, int Right)
{
    int M, Center;
    if (Left < Right)
    {
        Center=(Left+Right)/2;
        M is the index such that A[M] is the mediam of (A[Left], A[Center], A[Right])
        Msort(A, Tmp, Left, M);
        Msort(A, Tmp, M+1, Right);
        Merge(A, Tmp, Left, M+1, Right);
    }
}
```

8. (6%) Given a binary search tree T, please give the linear time algorithms to list all nodes of this tree

- (1) in ascending order of values.
- (2) in descending order of values.

9. (8%) Given the following activity graph of a hypothetical project, represented as an adjacency matrix,

- (1) please list the earliest completion time for each vertex.
- (2) please list the latest completion time for each vertex.
- (3) please list the critical path.
- (4) please list the slake time for each edge.

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	V0	V1	V2	V3	V4	V5	V6	V7	V8	V9
V ₀	0	3	∞	∞	∞	∞	∞	∞	∞	∞
V ₁	∞	0	1	4	∞	∞	∞	∞	∞	∞
V ₂	∞	∞	0	∞	0	∞	∞	∞	∞	∞
V ₃	∞	∞	∞	0	0	∞	∞	∞	∞	∞
V ₄	∞	∞	∞	∞	0	1	∞	∞	∞	∞
V ₅	∞	∞	∞	∞	∞	0	9	5	∞	∞
V ₆	∞	∞	∞	∞	∞	∞	0	∞	0	∞
V ₇	∞	∞	∞	∞	∞	∞	∞	0	0	∞
V ₈	∞	∞	∞	∞	∞	∞	∞	∞	0	2
V ₉	∞	∞	∞	∞	∞	∞	∞	∞	∞	0

10. (7.5%) 是非題，每題 1.5 分，答錯倒扣 1 分

- (1) () When the input keys are strings, hashing techniques does not work.
- (2) () Both Prime's algorithm and Kruskal's algorithm for minimum cost spanning tree are based on the greedy strategy.
- (3) () The next (failure) function of KMP algorithm, which searches for the occurrence of a pattern B in a string A, is independent of the string A.
- (4) () If an NP-complete problem X is polynomial reducible to a problem Y, then Y is an NP-complete problem.
- (5) () In depth-first searching of a directed graph, starting from any vertex will traverse the whole graph..

11. (7.5%) 選擇題，每題 1.5 分，答錯到扣 1 分

- (1) () Which the following sorting algorithm takes the least number of comparisons for sorting of the following sequence of data (9, 16, 66, 80, 96, 137, 170, 229) ?
 (a) insertion sort
 (b) quick sort
 (c) heap sort
 (d) merge sort
- (2) () Which the following sequence of data will produce the binary search tree with the least number of depth ?
 (a) 1, 2, 3, 4, 5, 6
 (b) 6, 5, 4, 3, 2, 1
 (c) 4, 1, 5, 1, 2, 6
 (d) 3, 2, 5, 4, 1, 6

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- (3) () A logical expression contains three types of operators, namely, **in order of precedence**, ~ (not), $\vee$ (or), $\wedge$ (and). Which is the postfix expression of the logical expression $G \wedge (A \vee B) \vee (C \vee (F \vee \sim D \wedge E))$
- (a) $GAB\vee\wedge CFD\sim E\vee\vee\vee\wedge$
 (b) $GAB\vee CFD\sim\vee E\wedge\vee\vee\wedge$
 (c) $GAB\vee\wedge CFD\sim E\wedge\vee\vee\vee$
 (d) $GAB\vee CF\sim D\vee E\wedge\vee\vee\wedge$
- (4) () Which of the following data structure is **unlikely** to be useful in the Dijkstra's shortest path algorithm?
- (a) queue
 (b) stack
 (c) heap
 (d) none of the above is useful.
- (5) () Given the following algorithm, which is likely to be the result if the input variable N is -5?

```

Boolean Magic(int N)
{
    if (N==1)
        Return True
    else if (N==0)
        Return False
    else { N= N-2;
          Return Magic(N);
        }
}

```

- (a) True
 (b) False
 (c) Stack overflow
 (d) Arithmetic overflow
12. (7%) A point quad-tree is a 4 way tree used to index two-dimensional data. Each interior node corresponds to a square region in two dimensions and contains a pair of coordinates and pointers to four children which represent four quadrants, NW, NE, SW, and SE. If the number of points in a square is no larger than what will fit in a block, then we can think of this square as a leaf of the tree, and it is represented by the block that holds its points. If there are too many points to fit in one block, then we treat the square as an interior node, with children corresponding to its four quadrants. For example, the first figure below shows the data points organized into regions that correspond to nodes of a quad tree shown in the second figure below. It is assumed that only two

考試科目

資料結構與演算法

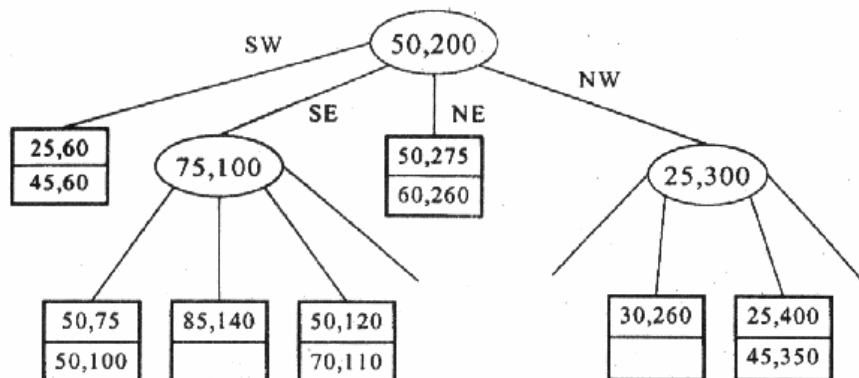
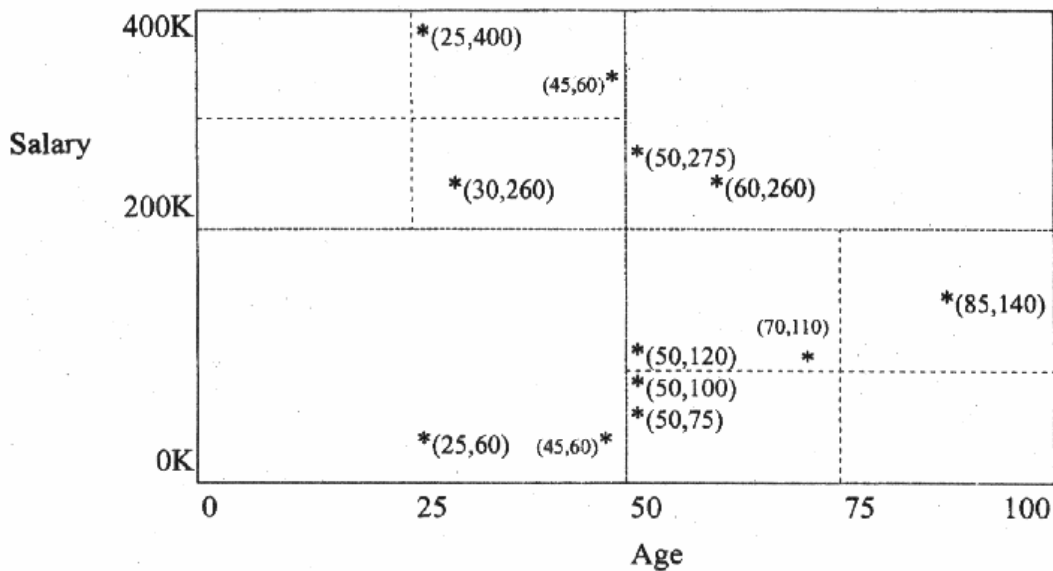
所別

資訊科學

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星期二 下午第一節

records can fit in a block. Note that if the data is located in the vertical border, it is assigned to the eastern region. If the data is located in the horizontal border, it is assigned to the southern region.



- (1) Given the following 12 records describing characteristics of some PC, please construct the quad-tree with dimensions Speed and RAM. Assume that each block can store two records, the range for Speed is 100 to 500, and for RAM it is 0 to 256.
- (2) How many interior nodes of the constructed quad-tree are accessed to process the range query that selects the model where $(250 < \text{Speed} < 370)$ and $(32 < \text{RAM} < 96)$?

model	A	B	C	D	E	F	G	H	I	J	K	L
Speed	300	333	400	350	450	400	450	233	266	300	350	400
RAM	32	64	64	32	96	128	128	32	64	64	64	128

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一、Computer Organization and Design

1. **[Overflow Detection]** (a) Assume that A and B are 32-bit integers. It is trivial to show that overflow can not occur when adding operands with different signs. Therefore, we only need to consider the following four conditions. Fill in the blanks to indicate when overflow actually occurs. (2%)

Operation	Operand A	Operand B	Result	Overflow? (Yes or No)
A+B	≥ 0	≥ 0	≥ 0	
A+B	≥ 0	≥ 0	< 0	
A+B	< 0	< 0	≥ 0	
A+B	< 0	< 0	< 0	

(b) Repeat part (a) for subtraction. What conditions need to be analyzed for subtraction? Why? (4%)

(c) A simple check for overflow during addition is to see if the CarryIn to the most significant bit is not the same as the CarryOut of the most significant bit. Prove that this check is the same as the results shown in (a) and (b) by building a table that shows all the possible combinations of Sign (of A, B, and Result) and CarryIn to the sign bit position and derive the CarryOut, Overflow and related information. (4%)

2. **[Computer Arithmetic]** An elegant approach to multiplying signed numbers is called *Booth's algorithm*.

(a) Demonstrate *Booth's algorithm* using the example: 3×-2 represented by 4-bit numbers. List all the steps involved and the intermediate values of the product. (6%)

(b) To compute axb where a is the multiplier and b is the multiplicand (both are 32-bits), one can recast Booth's algorithm in terms of the bit value of the multiplier (a_i refers to the i -th bit of a):

a_i	a_{i+1}	Operation
0	0	Do nothing
0	1	Add b
1	0	Subtract b
1	1	Do nothing

With the above table and the fact that shifting the multiplicand left respect to the Product register is equivalent to multiplying by a power of 2, show that Booth's algorithm does in fact perform 2's complement multiplication of a and b . (4%)

3. **[Cache]** Suppose a computer's address size is k bits (using byte addressing), the cache size is S bytes, the block size is B bytes, and the cache is A -way set associative. Assume

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that B is a power of two, so $B=2^b$. Figure out what the following quantities are in terms of S, B, A, b and k :

- (a) the number of sets in the cache (3%)
- (b) the number of index bits in the address (3%)
- (c) the number of bits needed to implement the cache. (4%)

4. **[Integrated Circuit Cost]** The cost of an integrated circuit can be expressed through three simple equations:

Cost per die = Cost per wafer / (Dies per wafer $\times$ yield)

Dies per wafer = Wafer area / Die area

Yield = $1 / (1 + \text{Defects per area} \times \text{Die area} / \alpha)^{\alpha}$

- (a) What is the yield assuming $0.8/\text{cm}^2$ defect density and a die area of 90 mm^2 for $\alpha=2$? (4%)
- (b) What is the approximate relationship between cost and die area? (Hint: Cost = $f(\text{Die area})^x$) for some x . Determine x in terms of α (4%)
- (c) What implication does the above relationship have for designers (assume that $\alpha \geq 2$)? (2%)

5. **[Disk Array I/O]** Assume that we have the following two magnetic disk configurations: a single disk and an array of 4 disks.

- i. Each disk has 64 sectors per track, each sector holds 1000 bytes, and the disk revolves at 10000 rpm.
- ii. Seek time is 6ms.
- iii. The delay of the disk controller is 1 ms per transaction, either for a single disk or for the array.

Assume that the performance of the I/O system is limited only by the disks and the controller. Remember that the consecutive sectors on a single disk will be spread on one sector per disk in the array.

- (a) How much time does each I/O operation take for the single disk if the workload consists of 4 KB reads from sequential sectors? (2%)
 - (b) How much time does each I/O operation take for the disk array if the workload consists of 4 KB reads from sequential sectors? (2%)
 - (c) Compare the I/O performance per second of these two disk organizations, assuming that the requests are random reads, half of which are 4 KB and half of which are 16 KB of data from sequential sectors. (6%)
- (For simplicity, assume that the sectors may be read in any order. The rotational latency is one-half the revolution time for the single disk read of 16 sectors and the disk array read of 4 sectors.)

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二. There are five problems in this computer systems: operating systems examination. Please do all of them. The weights for each problem is indicated after the problem number.

1. (a) (3 points) What resources are used when a thread is created? How do they differ from those used when a process is created?
- (b) (3 points) What are two differences between user-level threads and kernel-level threads?
- (c) (4 points) Describe the actions taken by a kernel to context switch between kernel-level threads?
2. The *dining-philosophers* problem is considered to be a classic synchronization problem. It is a simple representation of the need to allocate several resources among several processes in a deadlock- and starvation-free manner. One simple solution is to represent each chopstick by a semaphore. A philosopher tries to grab the chopstick by executing **wait** operation on that semaphore; she (or he) releases her chopsticks by executing the **signal** operation on the appropriate semaphores. Thus, the shared data are

```
semaphore chopstick[5];
```

where all the elements of **chopstick** are initialized to 1. The structure of philosopher *i* is shown as the following:

```
do{
    wait(chopstick[i]);
    wait(chopstick[(i+1) % 5]);
    .....
    eat
    .....
    signal(chopstick[i]);
    signal(chopstick[(i+1) % 5]);
    .....
    think
    .....
} while(1);
```

- (a) (3 points) Why we reject the above program codes to solve the *dining-philosophers* problem?
- (b) (7 points) Please show a correct program structure to solve the *dining-philosophers* problem using high-level synchronization construct **monitor**?

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3. A page-replacement algorithm should minimize the number of page faults. We can do this minimization by distributing heavily used pages evenly over all of memory, rather than having them compete for a small number of page frames. We can associate with each page frame a counter of the number of pages that are associated with that frame. Then, to replace a page, we search for the page frame with the smallest counter. Define a page-replacement algorithm using this basic idea. Specifically address the problems of:

- what the initial value of the counters is,
- when counters are increased,
- when counters are decreased,
- how the page to be replaced is selected.

(a) (5 points) How many page faults occur for your algorithm for the following reference string, for five page frames?

1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2

(b) (5 points) What is the minimum number of page faults for an optimal page-replacement strategy for the reference string in part (a) with five page frames? Is it the page faults number the same as (a)'s, please comments on (a) and (b)'s results?

4. (a) (5 points) Consider a system that supports the strategies of contiguous, linked, and indexed allocation. What criteria should be used in deciding which strategy is best utilized for a particular file?

(b) (5 points) A UNIX i-node has 10 disk addresses for data blocks, as well as the addresses of single, double, and triple indirect blocks. If each of these holds 512 disk addresses, what is the size of the largest file that can be handled, assuming that a disk block is 2K?

5. Consider the operating systems that support three domains ($user_1$, ..., $user_3$) and three objects: file F_1 , file F_2 , file F_3 . $user_1$ is the owner of file F_1 , $user_2$ is the owner of file F_2 , $user_3$ is the owner of file F_3 . Furthermore, the operating systems allow $user_1$ to do the domain switch to $user_2$ and $user_2$ to do the domain switch to $user_3$. The control right in $access(user_2, user_3)$ was also included in the access matrix.

(a) (5 points) How would you denote this protection scheme in access matrix?

(b) (5 points) How would you implement of (a)'s access matrix in access control list(ACL)?

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第一單元：離散數學部分 (60 %)

I 選擇與填充 (35 %; 不倒扣)

- (4%) Let $f(n)$ be an increasing function satisfying the recurrence relation: $f(n) = 7f(n/2) + 15n^2/4$. Then which of the following statements are correct? (多選)
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 - $f(n) = o(n^2)$
 - $f(n) = O(n^3)$
 - $f(n) = o(n^3)$
 - $f(n) = \Theta(n^3)$
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 - $\sim p \rightarrow \sim q$
 - $\sim p \text{ or } q$
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 - $\sim(p \text{ and } \sim q)$
- (4%) What is the number of non-negative integer solutions to the equation $x + y + z < 18$ with $x \geq 1$, $y \geq 2$ and $z \geq 3$.
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 - If $G = (V, E)$ is a multigraph containing no isolated or pending vertices, then the number of edges $|E|$ of G is always less than or equal to the number of vertices $|V|$ of G .
 - The chromatic number of every graph is less than or equal to 4.
 - There exists a Euler circuit for every complete simple graph K_5 with 5 vertices.
 - There exists one and only one path between every two vertices of a tree.

考試科目	計算機教學(-)	所別	資訊科學	考試時間	4月21日上午第 節 星期日 下
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II 計算與證明 (25 %)

8. (10 %) Let G be a directed graph with n vertices, x and y two distinct vertices of G . Show that if there is a path from x to y then there must exist a path of length less than n from x to y .
9. (15%) Let $\leq$ be a preorder on a nonempty set S (i.e., $\leq$ is a reflexive and transitive relation on S). Define a new relation $\equiv$ on S as follows : for all $x, y \in S$, $x \equiv y$ iff $x \leq y$ and $y \leq x$.
- (a) Show that $\equiv$ is an equivalence relation on S . [6 %]
- Let $\ll$ be a relation on the set $S/\equiv$ of all equivalence classes of S such that for all A and B belonging to $S/\equiv$, $A \ll B$ iff there exist $x \in A$ and $y \in B$ with $x \leq y$.
- (b) Show that the relation $\ll$ is a partial order on $S/\equiv$. [9 %]

考試科目	計算機數學(-) (機率)	所別	資科系	考試時間	4月21日 星期日	下午第 節
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第二單元：機率 (本單元共計 40 分)

[14%] 1. Suppose X and Y are independent random variables uniformly distributed over $\{1, 2, \dots, n\}$.

(a) Find $P(X \geq Y)$

(b) Find $P(Z=z)$, where $Z = \max\{X, Y\}$.

[12%] 2. Consider a directed graph G with n nodes. Let X_{ij} be a variable defined so that

$$X_{ij} = \begin{cases} 0 & \text{if there is no edge between node } i \text{ and node } j \\ 1 & \text{otherwise} \end{cases}$$

Assume that the $\{X_{ij}\}$ are mutually independent Bernoulli random variables with parameter p . The corresponding graph is called a **p-random-graph**. Find the pmf (probability mass function), the expected value, and the variance of the total number of edges X in the graph.

[14%] 3. Jobs arriving to a computing server have been found to require CPU time that can be modeled by an exponential distribution with parameter $1/140 \text{ ms}^{-1}$. The CPU scheduling discipline is quantum-oriented so that a job not completing within a quantum of 100 ms will be routed back to the tail of the queue of waiting jobs.

(a) Find the probability that an arriving job is forced to wait for a second quantum.

(b) Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum?