

考試科目	計算機數學	所別	資訊科學	考試時間	月 星期	日 上 下	午 第	節
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6. How many equivalence classes are there induced by the R relation ?
(a) 11 (b) 13 (c) 17 (d) 25
7. Let $S_0, S_1, S_2, \dots$ be a sequence defined as follows: $S_0 = 8, S_1 = 10$, and $S_k = 5S_{k-1} - 6S_{k-2}$ for all $k \geq 2$. Let the solution of S_n be of the form: $S_n = a 2^n + b 3^n$ for all $n \geq 0$. Then what is the value of $a + b$?
(a) 6 (b) 8 (c) -6 (d) -4
8. What is the maximal number of leaves for a rooted 3-ary tree of height $n \geq 0$. Note the height of a tree with only a single node is 0.
9. What is the sum of all coefficients of all terms in the expansion of the $(3x - y)^{20}$?
10. Find a positive integer $n < 231$ such that $n \equiv 2 \pmod{3}$, $n \equiv 5 \pmod{7}$ and $n \equiv 2 \pmod{11}$.

II 計算與證明 (共 30 分)

11. (10 pts) For all integer $n > 0$, define $H(n) = 1 + 1/2 + 1/3 + \dots + 1/n$.
(a) Show that for all integer $n \geq 0$, $(n+2)/2 \leq H(2^n) \leq n+2$.
(b) Using the above result to show that $H(n) = \Theta(\log n)$.
12. (20pts) A tournament is a directed graph without self-loop such that if $u \neq v$ are two vertices of the graph, then exactly one of (u,v) and (v,u) is an edge of the graph.
(a) How many different tournaments are there in a set of $n > 0$ vertices ? (5 pts)
(b) What is the sum of the in-degrees and out-degrees of all vertices in a tournament with $n > 0$ vertices? (5pts)
(c) Show that every tournament has at least a Hamilton path ? (Note: A Hamilton path is a path passing through all vertices exactly once. This theorem can be proved by induction) (10 pts)

考試科目	計算機數學(二) 線性代數	所別	資科系	考試時間	月 日 上午 第 節 星期 下
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IV. (1) True or false: a brief proof or counterexample is needed. (24%)

- (a) Every invertible matrix can be written as a product of elementary matrices.
- (b) Similar matrices have the same eigenvalues and eigenvectors.
- (c) If W is a subspace of a vector space of V , then every basis of W can be expanded into a basis for V . Conversely, every basis of V can be reduced to a basis of W .

(2) Find all solutions to the following linear system by reducing the associated matrix to row echelon form.

$$2x - y - 3z + w = 2$$

$$x - 2z + w = 1 \quad (8\%)$$

$$-3x + y + z + 2w = 3$$

(3) Let V be the space of all 2×2 matrices with real entries. Let $T: V \rightarrow V$ be defined by $T(A) = A'$ where A' is the transpose matrix of A . Show that T is a linear transformation. Is T diagonalizable? Explain your answer. (8%)

考試科目

計算機概論

所別

資訊科學系

考試時間

4月22日(日) 下午第1節
星期日

[7%] 1. (a) (2%) In an introductory course of computer science, we usually introduce base-2 numbering system instead of base-10. Can you explain the reason briefly?

(b) (5%) Using 2-input AND, OR, and NOT gates, design a circuit to implement the following truth table. Can you use as minimum number of gates as possible?

a	b	output
0	0	1
0	1	1
1	0	0
1	1	1

[8%] 2. (a) (4%) Consider the following structure of the instruction register.

What is the maximum number of distinct operation codes that can be recognized and executed by the processor on this machine? What is the maximum memory size on this machine?

op code	address-1	address-2
6 bits	18 bits	18 bits

(b) (4%) Assume that our MAR (Memory Address Register) contains 20 bits, enabling us to access up to 2^{20} memory cells, which is 1MB. However, our computer has 4MB of memory. Explain how it might be possible to address all 4MB memory cells using a MAR that contains only 20bits.

[6%] 3. Assume that any individual program spends about 50% of its time waiting for I/O operations to be completed. What percentage of time is the processor doing useful work (called processor utilization) if there are three programs loaded into memory? How many programs should we keep in memory if we want processor utilization to be at least 95%?

[8%] 4. (a) (4%) Describe the language defined by the following grammar:

```

<goal>      ::= <letter> | <letter><next>
<next>      ::= ,<letter>
<letter>    ::= A

```

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(b) (4%) Create a BNF grammar that describes simple Boolean expressions of the form

var AND var var OR var

where var is one of the symbols w, x, y, and z.

[8%] 5. Write an iterative C/C++ function and a recursive C/C++ function to compute the n -th element of Fibonacci sequence. The sequence is defined:

$$f_0 = 0, \quad f_1 = 1, \quad f_{i+1} = f_i + f_{i-1} \quad \text{for } i = 1, 2, \dots$$

[8%] 6. (a) (4%) What are differences between packet switching and circuit switching networks? You should compare their advantages and disadvantages.

(b) (2%) For delay-sensitive and loss-sensitive applications, which one (packet switching or circuit switching) is suitable, respectively? Why?

(c) (2%) In packet switching networks, considering some applications which need quality of service (QoS), what technologies can you use to achieve QoS?

[5%] 7. One definition of Computer Science is the study of algorithms, including (i) their formal and mathematical properties (ii) their hardware realizations (iii) their linguistic realizations (iv) their applications.

Recently, Internet is very popular and can be the focus for any fields of computer science. The famous domains are like ISP (Internet Service Provider), ICP (Internet Content Provider), ASP (Application Service Provider), IDC (Internet Data Center), etc. Can you pick up one of the above and write a short essay to explain how you can play the right role in it according to the above definition of computer science. (Note: The more concretely and clearly you describe, the higher score you will get.)

考試科目

計算機概論

所別

資科所

考試時間

月 日 上 午 第 節
星期 下

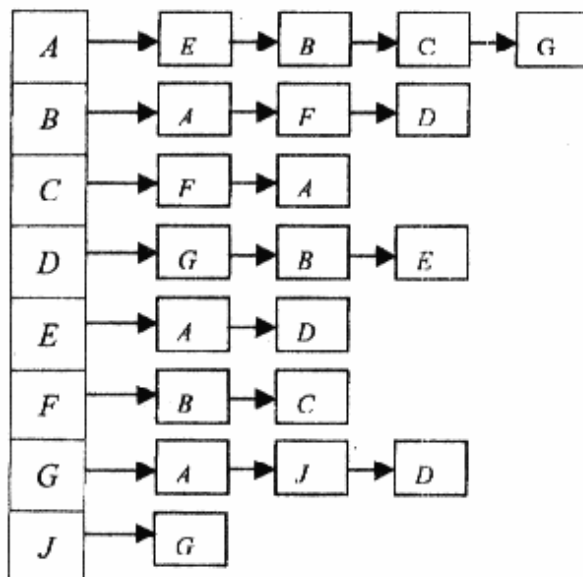
8. (10%) A logical expression contains three types of operators, namely, in order of precedence, $\sim$ (not), $\vee$ (or), $\wedge$ (and). Given the following logical expression

$$A \wedge \sim (B \vee (\sim D \wedge (C \wedge E \vee F))) \wedge G,$$

(1) Please give the postfix expression.

(2) Assume that A , B , D and F are false, C , E and G are true. Please evaluate this logical expression based on the postfix form. Please elaborate the evaluation process step by step.

9. (8%) Given the following graph represented as the adjacency list representation, please list the traversal order of vertices using Depth First Traversal and Breadth First Traversal, respectively, of the graph starting from the vertex A .



10. (10%) Given two strings A and B , please design an efficient algorithm to change A string to B string character by character with minimum editing cost. Three types of edit steps are allowed:

(1) insert: insert a character, each insertion is assigned a cost of 2

(2) delete: delete a character, each deletion is assigned a cost 3

(3) replace: replace one character with another character, each replacement is assigned a cost which equals the ASCII code difference between two characters.

The algorithm should take $O(m \times n)$ time in the worst case, where m , n is the number of characters of the string A and B respectively. Please describe your algorithm by the example where $A = \text{"bacd"}$, $B = \text{"adc"}$.

11. (10%) Please design an efficient data structure and corresponding algorithms to support the following operations:

Insert(x): insert x into the data structure only if it is not already there.

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試題隨卷繳交

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Delete(x): delete x from the data structure if it is there.

Find(k): find the k -th smallest element in the data structure.

All these operations should take $O(\log n)$ time in the worst case, where n is the number of elements in the data structure.

12. (12%) 是非題，每題 1.5 分，答錯倒扣 1.5 分

- (1) () The minimum spanning tree is a binary tree.
- (2) () The third element in a minimum heap must be one of the children of the root.
- (3) () Every problem which can be solved in polynomial time by deterministic algorithm must be a nondeterministic polynomial problem.
- (4) () Up to now, none of the NP-Complete problems can be solved by any polynomial time algorithm.
- (5) () The Dijkstra's algorithm for shortest path problem is based on the greedy strategy.
- (6) () Merge sort is stable whereas insertion sort is not.
- (7) () Binary searching of a sorted n element linked list takes $O(\log n)$ time in the worst case.
- (8) () It is impossible to generate the topological ordering for the cyclic graph.

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Computer Organization and Design

1. **Performance** You are trying to decide which system to purchase and are considering two different computers M1 and M2:

Instruction Class	CPI for M1	CPI for M2
A	4	2
B	6	4
C	8	3

The machine M1 has a clock rate of 1 GHz and M2 has a clock rate of 500 MHz.

- (a) [3 points] Compare the two processors' peak performance in terms of MIPS.
(b) [3 points] Consider the three compilers C1, C2, and C3 that produce the frequency results shown below:

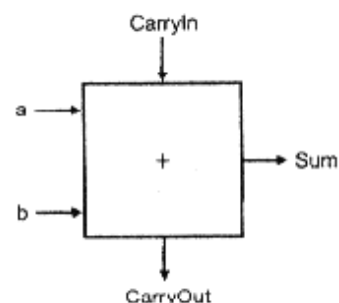
Instruction Class	C1	C2	C3
A	30%	30%	50%
B	50%	20%	30%
C	20%	50%	20%

For compiler C1, compare M1 and M2. Which processor is faster and by how much?

- (c) [4 points] Assuming that any of the three compilers can be used on M1 or M2, which of these two machines and which compiler should be purchased?

2. **Bit Pattern** [10 points] Given the bit pattern: 1000 1111 1110 1111 1100 0000 0000 0000, what does it represent, assuming that it is
- a. a two's complement integer? b. an unsigned integer?
- c. a single precision floating-point number?
- cf: You are not required to calculate the final results for parts (a) and (b), simply express them in terms of the summation of 2's power, such as $2^{16} + 2^{10} + \dots + 2 + 1$.

3. **1-bit full adder** [10 points] Express the CarryOut and Sum bits as logical equations of the inputs a, b and CarryIn, then implement the Sum bit by logic gates.



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4. Pipelining

- (a) [4 points] Explain the three types of hazards encountered in pipelining. Which one of the hazards can be handled by forwarding?
- (b) [6 points] Identify all of the data dependencies in the following code. Which dependencies are data hazards that will be resolved via forwarding?

```
add $2, $5, $4
```

```
add $4, $2, $5
```

```
sw $5, 100($2)
```

```
add $3, $2, $4
```

5. Performance of Multilevel Caches Suppose we have a processor with a base CPI of 1.0, assuming all references hit in the primary cache, and a clock rate of 1GHz. Assume a main memory access time of 150ns, including all the miss handling. Suppose the miss rate per instruction at the primary cache is 6%.

- (a) [4 points] What is the effective CPI for this machine?
- (b) [6 points] We would like to double the performance of the CPU by adding a second level cache. Suppose the amount of the 2nd level cache is large enough to reduce the miss rate in memory to 2%, calculate the maximum allowable access time of this 2nd level cache.

考試科目	計算機系統 (作業系統)	所別	資訊科學系	考試時間	月 日 上午 第 節 星期 下
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There are five problems in this computer systems: operating systems examination. Please do all of them. The weights for each problem is indicated after the problem number.

6. (10%) The first known correct software solution to the critical-section problem for two processes was developed by Dekker. The two processes, P_0 and P_1 , share the following variables:

```
var flag: array[0..1] of boolean; (*initially false*)
    turn: 0..1;
```

The structure of process P_i ($i = 0$ or 1), with P_j ($j = 1$ or 0) being the other process, is shown in the following figure:

repeat

```
flag[i] := true;
while flag[j]
do if turn = j
then begin
    flag[i] := false;
    while turn = j do no-op;
    flag[i] := true;
end;
```

critical section

```
turn := j;
flag[i] := false;
```

remainder section

until false;

Figure 1: The structure of process P_i in Dekker's algorithm.

Prove that the algorithm satisfies all three requirements for the critical-section problem.

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7. (10%)(a) Multithreading is a commonly used programming technique. Describe three different ways that threads could be implemented.
(b) Explain how these ways compare to the Linux clone mechanism. When might each alternative mechanism be better or worse than using clones?

8. (10%)(a) When do page faults occur? Describe the actions taken by the operating system when a page fault occurs.
(b) An operating system supports a paged virtual memory, using a central processor with a cycle time of 2 microsecond. It costs an additional 2 microsecond to access a page other than the current one. Pages have 2000 words, and the paging device is a drum that rotates at 3600 revolutions per minute, and transfers 1 million words per second. The following statistical measurements were obtained from the system:

- 2 percent of all instructions executed accessed a page other than the current page.
- Of the instructions that accessed another page, 85 percent accessed a page already in memory.
- When a new page was required, the replaced page was modified 40 percent of the time.

Calculate the effective instruction time on this system, assuming that the system is running one process only, and that the processor is idle during drum transfers.

9. (10%)(a) SUN NFS aims to support heterogeneous distributed systems by the provision of an operating system-independent file service. What are the key decisions that the implementer of an NFS server for an operating system other than UNIX would have to take?
(b) What constraints should an underlying filing system obey to be suitable for the implementation of NFS servers?

10. (10%)(a) PGP (Pretty Good Privacy) is widely used to secure email communication. Describe the steps that a pair of users using PGP must take before they can exchange email messages with privacy and authenticity guarantees. (The PGP negotiation is an instance of the hybrid scheme.)
(b) How would email be sent to a large list of recipients using PGP or a similar scheme? Suggest a scheme that is simpler and faster when the list is used frequently.

備 考 試 題 隨 卷 繳 交

考試科目	計算機數學(+) 所別	資訊科學	考試時間	月	日	上	午	第	節
				星期		下			

計算機數學(離散數學部份) (此部份共計 60 分)

I. 選擇或填充 (不倒扣, 每題三分)

- Let $\wedge, \vee, \rightarrow$ and $\sim$ denote the logical AND, OR, implication and NOT operations, respectively. Then which of the following sentences is *not* a tautology?
 - $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
 - $p \vee (q \wedge r) \rightarrow (p \vee q) \wedge r$
 - $\sim(p \rightarrow q) \rightarrow \sim q$
 - $p \wedge (q \vee r) \rightarrow (p \wedge q) \vee r$
- Let A and B be two regular languages over $\{0,1\}$. Then which of the following languages is not a regular language?
 - $\{x \in \{0,1\}^* \mid \text{the last 3rd bit of } x \text{ is "1"}\}$
 - $\{x \in \{0,1\}^* \mid \text{There is a bit string } y \text{ with } |y| = |x| \text{ and } xy \in A\}$
 - $\{xx \mid x \in \{0,1\}^*\}$
 - $\{xy \mid x \in A \text{ and } y \notin B\}$
- Which of the following statements about tree is FALSE?
 - There exists a tree with degrees 3,3,2,2,1,1,1,1.
 - Every tree is bipartite.
 - Every tree is planar.
 - If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.
- Which of the following statements is TRUE?
 - There exists a connected planar simple graph with 7 vertices, 9 edges and 5 regions.
 - For all connected multi-graphs G, if the vertices of G have degrees 2,2,2,3,4,4, respectively, then it has an Euler circuit.
 - The chromatic numbers of all planar graphs are less than 4.
 - There exists an irreflexive and transitive relation which is not antiymmetric.

Let $A = \{(x,y) \mid x, y \in \{1,2,3,4,6\}\}$ and define a relation R on A with $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 y_2 = x_2 y_1$. It is easy to show that R is an equivalence relation on A. Now answer the following two questions:

- What is the size of the equivalence class $[(2,4)]$?
 - 2
 - 3
 - 4
 - 5

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(a) How many different tournaments are there in a set of $n > 0$ vertices ? (5 pts)
(b) What is the sum of the in-degrees and out-degrees of all vertices in a tournament with $n > 0$ vertices? (5pts)
(c) Show that every tournament has at least a Hamilton path ? (Note: A Hamilton path is a path passing through all vertices exactly once. This theorem can be proved by induction) (10 pts)

考試科目	計算機數學(一) 機率	所別	政研系	考試時間	月 日 上 午 第 節 星期 下
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四、(1) Three fair dice are rolled. You are allowed to bet 1 dollar on the occurrence of one of the integers 1, 2, 3, 4, 5, 6. Suppose you bet on the occurrence of a 5. Then if one 5 occurs (on the 3 dice) you win 1 dollar, if two 5's occur you win 2 dollars, and if three 5's occur you win 3 dollars. If no 5's occur you lose 1 dollar. Let V be the net amount you win in one play of this game. What is the probability density (or mass) function of V ? What are its mean and variance? Is this a fair game? (10%)

(2) Suppose that we are going to drive a car to an athletic event. From past experience in making this trip we are willing to assume that our driving time is equally likely to be anywhere from 20 to 30 minutes. If we let X be the number of minutes it will take us to get there, what is the probability density function of X ? What is the cumulative distribution function of X ? What are the mean and variance of X ? (10%)

(3) At a certain manufacturing plant, accidents have been occurring at the rate of 1 every 2 months. Assuming that the accidents occur independently, what is the expected number of accidents per year? What is the standard deviation of the number of accidents per year? What is the probability of there being no accidents in a given month? (10%)

(4) Students at a large university are required to take an entrance exam when entering the school (scored on a basis of 100 points possible). Let X be the score made by a particular incoming student (who does go on to graduate) and let Y be his graduating quality point ratio (4 points = A). The joint density function of X and Y

$$\text{is observed to be } f_{X,Y}(x,y) = \begin{cases} \frac{1}{50}, & \text{for } 2 < y < 4, 25(y-1) < x < 25y \\ 0, & \text{otherwise} \end{cases}$$

Find μ_X , μ_Y , and ρ_{XY} . Explain the significance of ρ_{XY} . (10%)

考試科目	計算機數學	所別	資訊科學	考試時間	4月22日 上午第4節
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計算機數學(離散數學部份) (此部份共計 60 分)

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