

第一單元：離散數學 (本單元共計 60 分)

- (10 %) Let A_n ($n \geq 2$) be the number of strictly increasing sequence of positive integers that have 1 as the first term and n as the last term. That is, sequences $x_1, x_2, \dots, x_k$ where $x_1 = 1$ and $x_k = n$ and $x_j < x_{j+1}$ for $1 \leq j < k$.
 - What are A_2 , A_3 and A_4 ? [3%]
 - Find a recurrence relation for A_n ($n > 1$). [5 %]
 - Solve the correct recurrence relation you obtained at (b) with A_2 as the initial condition for A_n ($n \geq 2$). [2%]
- (15%) The rooted Fibonacci trees T_n are defined recursively as follows:
 (Basis:) T_1 and T_2 are both rooted trees consisting of a single vertex.
 (Recursion:) T_n ($n > 2$) is constructed from a root with T_{n-1} as its left subtree and T_{n-2} as its right subtree.
 - What is the number of leaves of T_n for $n > 1$? [5%]
 - What is the number of internal vertices of T_n for $n > 1$? [5%]
 - What is the height of T_n for $n > 1$? [5%]
- (10 %) Let A be a set with 10 elements. Then
 - How many ternary relations are there on A ? [3%]
 - How many asymmetric relation are there on A ? [3%]
 - What is the number of all simple graphs which have A as the vertex set? [4 %]
- (10 %) Show that in a connected simple graph with $n > 0$ vertices, there must exist a path of length $< n$ between every two distinct vertices.
- (15 %) Two sets A and B are said to be equipotent (, written $A \sim B$,) if there is a 1-1 and onto mapping between them. Let N be the set of positive integers and $K = \{x \mid x \text{ is a real number and } 0 \leq x \leq 1\}$. Show that
 - $\sim$ is an equivalence relation on the class of all sets, [5%]
 - $K \sim K^2$ where $K^2 =_{\text{def}} \{(x, y) \mid x, y \in K\}$. [5%]
 - K and N are not equipotent. [5%]

Hint for (b) and (c) : Each number in K can be represented *uniquely* as an infinite sequence of digits : $0. x_1 x_2 x_3 \dots$. For example, $1/3 = 0.33333333\dots$ and $1/2 = 0.500000\dots$. This means K and the set $\{f \mid f \text{ is a function from } N \text{ to } \{0, \dots, 9\}\}$ are equipotent.

考試科目 Course	計算機數學 (機率)	學級 資訊科學	日期 Date, Period	4月23日	試題編號 Course No.
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一. 設隨機變數 X 和 Y 的結合密度函數為

$$f(x, y) = kxy, \text{ 其中 } k \text{ 是常數, } 0 \leq x, y \leq 1$$

(6分) (1) 求 k 值.

(6分) (2) 求 $P(Y \geq X | Y \geq \frac{1}{2})$.

二. 設隨機變數 X 和 Y 的結合密度函數為

$$f(x, y) = 3x, \quad 0 < y < x < 1.$$

(6分) (1) 求 Y 的邊緣密度函數 (marginal density function).

(6分) (2) 給定 $Y = \frac{1}{2}$, 求 X 的條件密度函數 (conditional density function).

(6分) (3) 給定 $Y = \frac{1}{2}$, 求 X 的條件變異數 (conditional variance).

三. 設隨機變數 X 在區間 $(0, 1)$ 上具有均勻分配

(10分) (uniform distribution). 求 $Y = -\frac{1}{2} \ln(1-X)$ 的密度函數 (density function).

- [10%] 1. Determine which of the following statements are true? If the answer is false, justify it briefly.
- (a) (2%) All problems have algorithmic solutions, but some of the solutions are impractical.
 - (b) (2%) The stored program concept is about representing data as binary values and storing them in memory.
 - (c) (2%) In storage management, fixed-size records incur internal fragmentation while variable-size records incur external fragmentation.
 - (d) (2%) Suppose that A is an assertion, and "If A then B" is a given rule, then B is an assertion.
 - (e) (2%) In COM (Component Object Model), source code is reused through interface inheritance.

- [6%] 2.(a) (3%) What is the principle of locality?
 (b) (3%) Please give an example where this principle is used to improve the performance of a computer system.

[5%] 3. You are given the following two tables (Student and Record) in a relational database. Please use the two tables and the common operators (SELECT, PROJECT, and JOINT) for relational database to write statements that generate a new table called *FailSP*. This table should contain only the names of the students who failed the exam (Score < 60) and the names of their parents. Any intermediate tables being generated should be kept as small as possible.

Student		
StudID	StudName	Parent
002	John	Joe
004	Mary	Maria
007	Tom	Tim

Record		
CourseID	StudID	Score
001	002	46
001	002	89
002	007	55
003	004	90

- [6%] 4. (a) (3%) Explain why Ethernet loses efficiency as traffic increases in LAN.
 (b) (3%) What are the corresponding positions of the following protocols in the OSI networking model: TCP, IP, Ethernet (CSMA/CD)?

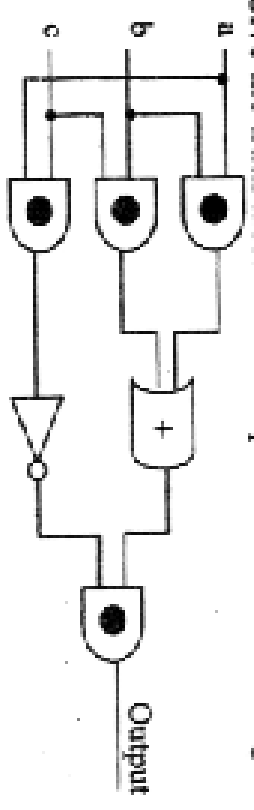
[4%] 5. Use the following BNF grammar to draw the parse tree for the following assignment statement: $x := x + x - y$

$\langle \text{assignment} \rangle ::= \langle \text{var} \rangle := \langle \text{expr} \rangle$
 $\langle \text{expr} \rangle ::= \langle \text{var} \rangle \mid \langle \text{expr} \rangle \langle \text{oper} \rangle \langle \text{var} \rangle$
 $\langle \text{var} \rangle ::= X \mid Y$
 $\langle \text{oper} \rangle ::= + \mid -$

4-2

- [4%] 6. A rule-based system attempts to (list the letters that apply)
- (a) mimic the human ability to reference pertinent facts and string them together in a logical fashion
 - (b) simulate the connectionist architecture of the human brain
 - (c) carefully distinguish between objects and their attributes
 - (d) contain both a set of facts about the subject matter and a mechanism for selecting the relevant facts and reasoning from them in a logical way.

- [6%] 7. (a) (3%) Give the truth table that corresponds to the following circuit.



- (b) (3%) For the truth table obtained in part (a), give an equivalent Boolean expression that contains the minimum number of "transistors" when implemented in circuit? What is this minimum number?

- [9%] 8. (a) (6%) Write a recursive C function

```
tree_node *BtreeFindNode(tree_node *root, int target);
```

to search a binary tree for the value target in a depth-first manner. This function should return the found node or null if no such node exists. The structure for a node in the tree is given as follows.

```
typedef struct _tree_node {
    int value;
    struct _tree_node *left;
    struct _tree_node *right;
} tree_node;
```

(b) (3%) What are the advantages and disadvantages of using recursive routines?

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4-3

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9. (6%) Please answer and explain the following questions?

- (a) the worst-case time complexity of searching in a binary search tree of n elements?
- (b) the worst-case time complexity of searching in an AVL tree of n elements?
- (c) the average time complexity of searching in an unsorted array of n elements?
- (d) the average time complexity of searching in a sorted linked list of n elements?

10. (6%) Please determine the time complexity of merge sort for

- (a) sorted input
- (b) reverse-ordered input
- (c) random input.

11. (8%) Two binary trees are similar if they are both empty or both nonempty and have similar left and right subtrees. Given the following declaration in C, please write a function to decide whether two binary trees are similar. What is the running time of your program.

```
typedef struct Treenode *PtrToNode;
typedef struct PtrToNode Tree;
struct Treenode
{
    int Element;
    Tree Left;
    Tree Right;
}
```

12. (10%) Suppose that there exists a city divided into 6 districts. Each district has a switching center. The distance between each pair of switching centers is shown in Table 1. Two switching centers are connected if there is a wire between them. The

switching center of caller automatically directs the phone call to another switching

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4-4

13. (10%) Please design the data structure and $O(\log n)$ algorithms to support the following two operations in dynamic environment: *Insert* and *Delete-Runner-Lp*. The *Insert* operation inserts an element while the *Delete-Runner-Lp* operation deletes the second minimum element, instead of the minimum one. You should describe the data structure and write down the two algorithms.

14. (10%) Suppose that there exists an algorithm X . With the input of a set of n integers and an integer k , algorithm X will answer "yes" or "no," indicating whether there is a subset of the n numbers whose sum is k . Please design an algorithm Y to find the subset of the n numbers whose sum is k by using the algorithm X $O(n)$ times. You should describe the idea of your algorithm.

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- (5 points) Both data and instructions are stored in main memory. Can CPU determine whether a main memory location contains an instruction or an operand only by inspecting its contents? If yes, how? If no, what does CPU do?
- (5 points) Registers R1 and R2 of a computer contain the *decimal* values 3000 and 4400, respectively. What is the effective address of the *memory operand* in each of the following instructions? (a) Add $-(R1), R5$ (*autodecrement*) (b) Move 50(R2), R5 (*index*) (c) Load 500, R5 (*absolute*) (d) Subtract (R2)+, R5 (*autoincrement*) (e) Store R5, 80(R1, R2) (*index*)
- (10 points) What is the job of the control unit in CPU? Explain the *micro-programmed* approach to implement the control unit. What are its advantages and disadvantages?
- (10 points) Consider the following decomposition of the instruction processing: (1) Fetch Instruction, (2) Decode Instruction, (3) Calculate Operand Address, (4) Fetch Operands, (5) Execute Instruction, (6) Write Operand.
 - Consider the scope of executing seven instructions only. Show the timing diagram for instruction pipeline operation. What is the performance gain of *pipelining over sequential execution*?
 - Suppose instruction 3 is a conditional branch to instruction 9. After instruction 3 is completed (a branch instruction does not need the WO stage), instruction 9 is executed next. Show the timing diagram for instruction pipeline operation over nine fetched instructions. Explain what is *branch penalty* and show its quantity in term of number of cycle.
- (10 points) A six-stage pipelined processor has TWO branch delay slots. Assume that 20 percent of the instructions executed on a computer are branch instructions. An optimizing compiler can fill one of these slots 90 percent of the time and can fill the second slot only 25 percent of the time.
 - What is the percentage of improvement in performance achieved by this optimization?
 - Suppose an instruction is executed in time T. What is the throughput of the processor in term of the number of instructions executed per T?
- (10 points) A byte-addressable computer has a small data cache capable of holding four 16-bit words. Each cache block consists of two 16-bit words. When a given program is executed, the processor reads data from the following sequence of *hexadecimal* addresses: 310, 386, 312, 384, 310, 320, 386, 322, 310, 312. This pattern is repeated TWO times. Show the process of how the contents of the cache are changed if a *direct mapped* cache is used. Compute the hit rate for this example. Assume that the cache is initially empty.

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7. (10%) Consider a computer system consisting of four resources of the type A and four resources of type B that are shared by three processes. Type A and type B resources are bundled in the sense that a process always gets an A if it gets a B, and vice versa. Each of these processes requires no more than two resources of type A and two resources of type B. **Is this system deadlock-free? Show a proof for your answer.**

8. (10%) Consider a single-CPU computer system that adopts the well-known FCFS (first-come, first-serve) algorithm for CPU scheduling. Assume that the CPU is idle before three independent processes simultaneously arrive at the system at time 5, and that the execution times of these processes have a common normal distribution with mean 3 and standard deviation 3. **When is the system expected to complete all three processes?** Now assume that the system adopts the round-robin scheduling algorithm, that the time quantum is 1 time unit, and that it takes 1 time unit for the system to carry out context switch. **When is the system expected to complete all three processes?**

Note that you must show and explain how you get your answer in your answer sheet. Answers without explanation will receive no credits.

9. (10%) Consider the following programs to be executed in a computer system that employs demand paging for memory management.

```
main()
{
  int i, j, fun[200][200];
  for (i=0; i<200; i++)
    for (j=0; j<200; j++)
      fun[i][j] = 0;
  return 0;
}
```

```
main()
{
  int i, j, fun[200][200];
  for (j=0; j<200; j++)
    for (i=0; i<200; i++)
      fun[i][j] = 0;
  return 0;
}
```

Assume that, in this language, the array is stored column major, and that the operating system allocates 128 pages of 256 words to an individual program. **Which program, left one or right one, would run faster? Why? Answer these questions again, but assume that the operating system allocates 256 pages of 512 words to programs?**

10. (10%) Answer the following questions about computer security.

10.1 (5%) A company modernized its information system by introducing Intranet and Internet services. The general manager hired a top computer specialist to install a waterproof firewall to protect the computers of the company from unauthorized accesses. The system run pretty well for a while until one day the main host computer of the company was broken in, and important documents were apparently damaged. **Do you think this could happen in reality? Provide a good technical reason for your answer.**

10.2 (5%) UNIX systems that employ encryption function to encode users' passwords typically do not allow people to use as passwords words that can found in dictionaries. **Provide a technical reason for the precautionary check.**

11. (10%) Briefly and precisely answer the following questions.

11.1 (3%) Explain the main similarity and difference between job scheduling and CPU scheduling.

11.2 (4%) Explain the main similarity and difference among the following techniques: segmentation, paging and caching.

11.3 (3%) Explain why a process may be "swapped".