

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 1 頁, 共 2 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

回答問題時, 請將“題號”標示清楚, 例如 1A, 2D,

1. Given two polynomials $f(x, y) = \sum_{i=0}^n \sum_{j=0}^n a_{ij} x^i y^j$ and $g(x, y) = \sum_{i=0}^n \sum_{j=0}^n b_{ij} x^i y^j$. Let

k be the maximum of number of nonzero a_{ij} 's and number of nonzero b_{ij} 's.

Please answer each of the following questions (all of A, B, and C) by no more than one page. A solution spends more than one page will get zero point.

A. 10 points.

Please design a space complexity efficient algorithm to construct the $f(x, y)$ where $k = \Theta(n)$ and specify the space complexity and time complexity of your design. (You can reference any known algorithm in your design without details, for instance, just say “use quicksort to sort” instead of describing what quicksort doing)

B. 10 points.

Please design a space complexity efficient algorithm to construct the $f(x, y)$ where $k = \Theta(n^2)$ and specify the space complexity and time complexity of your design. (You can reference any known algorithm in your design without details, for instance, just say “use quicksort to sort” instead of describing what quicksort doing)

C. 5 points.

Please given an efficient algorithm for computing $f(x, y) + g(x, y)$ by using your solution in either a or b and specify the time complexity of your algorithm. (You can reference any known algorithm in your design without details, for instance, just say “use quicksort to sort” instead of describing what quicksort doing)

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 2 頁, 共 2 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

2. 是非題, 下列五題, A-E, 請判斷並回答每一句的敘述是 True(T)

或 False (F), 每題兩分, 不倒扣。

- A. Heap sort and merge sort all take $O(n \log n)$ time to sort n numbers. Quick sort is the fastest sorting algorithm. It has $O(n \log n)$ average performance to sort n numbers. Since they all use comparison operation to sort, the lower bound for any comparison sort algorithm is $\Omega(n \log n)$.
- B. The correctness of the radix sort is based on the fact that counting sort is stable.
- C. Given a graph $G=(V,E)$, $|V|$ and $|E|$ are respectively the number of vertices and edges in G . If the graph is stored in an adjacency matrix, depth first search of the graph takes $O(|V|+|E|)$ time.
- D. Breadth first search a graph $G=(V,E)$ results a breadth first search tree, T . From T and the back edges (those edges in G but not in T), we can compute the articulation points and then determine the biconnected components.
- E. Given a undirected graph $G=(V,E)$, where $V=\{A, B, C, D, E, F, G, H, I\}$, and $E=\{(A,B), (A,C), (B,C), (B,D), (C,D), (D,E), (D,F), (E,F), (C,G), (G,H), (G,I), (H,I)\}$. The subgraph $G'=(V',E')$ in G , $V'=\{A, B, C\}$, $E'=\{(A,B), (A,C), (B,C)\}$ is a biconnected component in G .

3. 下列四題(A-D)為選擇題, 四選一, 每題四分, 不倒扣。

- A. $\Theta(n)$ and $O(n)$ actually are set of functions. But we often say $\Theta(n)+O(n)$. Which one is the best to present the result of the addition?
 - 甲、 $\Omega(n)$
 - 乙、 $O(n)$
 - 丙、 $\Theta(n)$
 - 丁、 $o(n)$

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 3 頁, 共 7 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

- B. Minimum spanning tree of a weighted graph can be solved by using the Kruskal's algorithm. Given a weighted graph $G=(V,E)$, there are $|V|$ vertices and $|E|$ edges. 選出錯誤的敘述
- 甲、Kruskal's algorithm is a greedy approach, i.e., we iteratively select the least weight edge and test if the edge can be a tree edge.
 - 乙、To select the least weight edge can be implemented by using a heap or presorting the edges by their weights. Using a heap could be more efficient than presorting the edges.
 - 丙、To test whether a selected edge is a tree edge, we can use the Union-Find algorithm. The testing takes $O(|V| \alpha(m,n))$ time, where $\alpha(m,n)$ is the inverse of the Ackermann's function.
 - 丁、There could be many spanning trees that are all minimum spanning tree in G .
- C. Considering quick sort n numbers, 選出錯誤的敘述
- 甲、Quick sort has $O(n \log n)$ average performance.
 - 乙、Quick sort has worst case $O(n^2)$ performance.
 - 丙、We can use a random number generator to improve the performance of Quick sort, we then have the worst case performance $O(n \log n)$.
 - 丁、The best case performance of quick sort is $\Theta(n \log n)$.
- D. Divide and conquer, 選出錯誤的敘述
- 甲、We can solve a problem by 1. dividing the problem into two sub-problems of equal size, 2. solving the two sub-problems recursively, 3. merging the solutions to the two sub-problems to get the solution to the original problem.
 - 乙、Let the total time to solve the problem be $T(n)$. $T(n) = 2T(n/2) + f(n)$ where $f(n)$ is the time required for dividing the problem and merging the solutions.
 - 丙、If $f(n) = O(n^2)$, then $T(n) = O(n^2 \log n)$.
 - 丁、When we divide the problem into two sub-problems, if we cannot divide it into equal size, but we can make sure the size of the sub-problem is always between $1/3$ and $2/3$ of the original size, the total time ($T(n)$) required is the same as equally dividing the problem.

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 4 頁, 共 7 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

4. 下列五題(A-E)為選擇題, 單選題, 每題四分, 不倒扣。

A. Given a complete binary tree in array representation:

3 1 4 6 5 2

which binary tree below is not one of the intermediate trees present during "heapification" (i.e. when turning the original tree into a max heap)?

甲、6 3 4 1 5 2

乙、6 5 4 1 3 2

丙、4 1 3 6 5 2

丁、3 6 4 1 5 2

B. Following the previous question, which one below is not among the intermediate trees during heap sort:

甲、5 2 4 1 3 6

乙、2 3 4 1 5 6

丙、1 3 2 4 5 6

丁、2 1 3 4 5 6

戊、4 5 2 1 3 6

C. Consider the min-max heap in array representation:

1 12 9 7 2 3 5 10 11 8 6 4

which node exchange below results in a non-min-max heap?

甲、7 - 8

乙、2 - 4

丙、8 - 6

丁、5 - 4

戊、8 - 11

D. Consider the deap below in array representation:

_ 1 11 4 2 8 10 5 6 3 9 7

which one below is not among the intermediate trees present during min deletion?

甲、_ x 11 4 2 8 10 5 6 3 9 7

乙、_ 2 11 4 x 8 10 5 6 3 9 7

丙、_ 2 11 4 3 8 10 5 6 x 9 7

丁、_ 2 11 4 3 8 10 5 6 7 9

戊、_ 2 11 4 3 8 7 5 6 10 9

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 5 頁, 共 7 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

- E. Given two min-binomial heaps (B-heaps) each represented as a sequence of heaps represented in array representation:

1	5 6 8
4 7	2 3

Which one below is not among the intermediate B-heaps during a heap combine operation followed by a sequence of min deletion operations?

- 甲、 1
4 7
5 6 8
2 3
- 乙、 4 7
5 6 8
2 3
- 丙、 4 5 7 6 8
2 3
- 丁、 5 6 8
2 4 3 7
- 戊、 3
4 5 7 6 8

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 6 頁, 共 7 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

5. 下列五題(A-E)為選擇題, 請自甲乙丙丁中選出最適當的答案, 不倒扣, 每提 4 分。

A. Which of the following statements for binary tree are correct?

- (a) The max. number of nodes on level h of a binary tree are $2^h - 1$, $h \geq 1$.
- (b) In worst case a skewed tree of depth k requires $2^k - 1$ spaces.
- (c) For complete binary trees, array representation wastes no space.
- (d) For skewed tree less than half the array is utilized.

The answers are (甲) a, c (乙) b, d (丙) a, b, c (丁) a, b, c, d.

B. Which of the following statements are correct for selection trees?

- (a) The tree has to be restructured each time a record is merged into the output file.
- (b) The time to restructure the selection tree for k ordered sequences is $O(\log k)$.
- (c) The time required to set up the selection tree the first time is $O(k)$
- (d) The total time needed to merge the k runs of n records is $O(n \log k)$ for selection trees.

The answers are (甲) a, c (乙) b, d (丙) a, b, c (丁) a, b, c, d.

C. Which of the following statements are correct for static hashing?

- (a) In worst case performance for hashing an insertion in a hash table with n identifiers may take $O(n)$ times.
- (b) For static hashing, if no overflows occur, the time required to enter, delete, or search for identifiers is $O(1)$.
- (c) For static hashing overflow handling with linear open addressing may create clusters of identifiers.
- (d) Let $\alpha = n/b$ be the loading density of a hash table using a uniform hash function f . then the expected number of identifier comparisons will be approximate to be α for rehashing when a search is made for identifier not in the hash table.

The answers are (甲) a, c (乙) b, d (丙) a, b, c (丁) a, b, c, d.

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：資料結構(101) 考試日期：93 年 4 月 17 日 第 1 節

系所班別：資訊科學系 組別：甲組 第 1 頁, 共 1 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

- D. Which of the following statements are correct for directory dynamic hashing?
- (a) It may double the directory size when a page overflows
 - (b) It guarantees only one disk access for retrieving any page.
 - (c) Using a directory to represent a trie allows the table of identifiers to grow and shrink dynamically.
 - (d) The space utilization should be at most 50%.

The answers are (甲) a, c (乙) b, d (丙) a, b, c (丁) a, b, c, d.

- E. Which of the following statements are correct?
- (a) There are four links to be reconnected by using an LR rotation to construct an AVL tree.
 - (b) In the case of n -node AVL trees the worst case insertion time is $O(\log n)$.
 - (c) for 2-3 tree an individual rotation or combine operation takes $O(1)$ time.
 - (d) Both deletion from or insertion into a 2-3 tree with n elements take $O(\log n)$ time.

The answers are (甲) a, c (乙) b, d (丙) a, b, c (丁) a, b, c, d.

6. (9%) Show your hash table, containing 7 buckets and each bucket containing at most 2 records, after storing 6, 8, 27, 25, 112, 13, 10, 12, 9, 26, if the hashing function is based on division (show your hashing function) and the overflow handling is linear open addressing.