

科目 微積分 類組別 A-2,A-3,A-4,A-5,B-5,B-6 共 2 頁第 2 頁 *請在試卷答案卷(卡)內作答

二. 計算證明題. 共 40 分 (需寫出計算及證明過程, 否則不予計分)

1. (10 分)

Find

$$\lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{n}{k^2 + n^2} \right).$$

2. (10 分)

Let $0.a_1a_2a_3a_4\cdots$ be the decimal expansion of the rational number $\frac{5}{7}$. Let $b_k = a_{2k}, k = 1, 2, \dots$. The decimal $0.b_1b_2b_3b_4\cdots$ also represents a rational number $\frac{a}{b}$. Find $\frac{a}{b}$.

3. (10 分)

Find the shortest distance from the point $(1, 2, 0)$ to the elliptic cone $z = \sqrt{x^2 + 2y^2}$.

4. (10 分)

Evaluate the surface integral $\int \int_S (x^4 + y^4 + z^4) d\sigma$, where $d\sigma$ is the surface element and $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$.