

科目名稱：離散數學與機率(104) 考試日期：93 年 4 月 17 日 第 2 節

系所班別：資訊科學系 組別：甲組 第 1 頁, 共 4 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

PART A. Discrete Mathematics

1. (5 points) Let N be the set of all natural numbers. Let M be the set of all finite subsets of N . Is there a one-to-one, onto mapping from N to M ? Why or why not?

2. (5 points) A natural number n may be written as $p_1^{a_1} p_2^{a_2} \dots p_k^{a_k}$. Calculate the product of all the divisors of a number n .

3. (5 points) Define $\pi(i)$ as the sum of all digits of the natural number i . For instance, $\pi(257) = 2 + 5 + 7 = 14$. Define

$$\theta(i) = \begin{cases} i & \text{if } i < 10 \\ \theta(\pi(i)) & \text{otherwise} \end{cases}$$

For instance, $\theta(257) = \theta(\pi(257)) = \theta(14) = \theta(\pi(14)) = \theta(5) = 5$. Compute $\theta[\sum_{i=0}^{999} \theta(i)]$.

4. (5 points) Let $a_1, a_2, \dots, a_n$ denote a sequence L of n integers. Show that there must be a consecutive subsequence of L whose sum is divisible by n . Note that a consecutive subsequence has the form $a_j, a_{j+1}, \dots, a_{j+k}$.

5. (5 points) Is the following claim true? Why or why not?

There is a Hamiltonian path in an n -dimensional cube, for $n \in N$.

For problems 6~11, give your answers directly. No computation is necessary.

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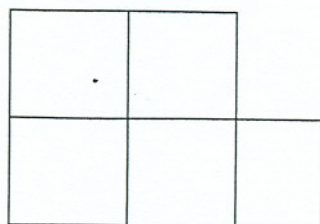
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6. For each of these relations on the set $\{1, 2, 3, 4\}$ decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive?
 - a) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$ (1%)
 - b) $\{((1,1), (1,2), (2,1), (2,2), (3,3), (4,4))\}$ (1%)
 - c) $\{(2,4), (4,2)\}$ (1%)
 - d) $\{(1,2), (2,3), (3,4)\}$ (1%)
 - e) $\{(1,1), (2,2), (3,3), (4,4)\}$ (1%)
7. An employee joined a company in 1999 with a starting salary of \$50,000. Every year this employee receives a raise of \$1000 plus 5% of the salary of the previous year.
 - a) Set up a recurrence relation for the salary of this employee n year after 1999. (2%)
 - b) What will be the salary of this employee in 2007? (1%)
 - c) Find an explicit formula for the salary of this employee n year after 1999. (2%)
8. Let G be a simple graph with the smallest number of vertices such that 1) the total degrees of G are exactly 240 and 2) all vertices of G have the same degree. How many vertices G has? (3%)
9. How many divisions are required to find $\gcd(210, 111)$ using the Euclidean algorithm? (3%)
10. Consider the graph G whose adjacency matrix is

$$\begin{matrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{matrix}$$
 - a) How many spanning trees has in the graph G ? (3%)
 - b) Draw one of its spanning trees. (1%)
11. Let H be the graph shown below where the edges represent the streets in a city. A police officer wants to make a trip to patrol each street exactly once. Can such a trip be designed and give your reason? (The police officer may start at any vertex in the graph). (5%)



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PART B.

Problems for Probability

Problem 1 (5%)

In a state lottery, a player must choose 8 of the numbers from 1 to 40. The lottery commission then performs an experiment that selects 8 of these 40 numbers. Assuming that the choice of the lottery commission is equally likely to be any of the $\binom{40}{8}$ combinations, what is the probability that a player has at least 6 of the number selected?

Problem 2 (5%)

Suppose we have 10 coins such that if the i th coin is flipped, heads will appear with probability $\frac{i}{10}$, $i = 1, 2, \dots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

Problem 3 (5%)

If the distribution function of a random variable X is given by

$$F(t) = \begin{cases} 0 & t < 0 \\ 1/2 & 0 \leq t < 1 \\ 3/5 & 1 \leq t < 2 \\ 4/5 & 2 \leq t < 3 \\ 9/10 & 3 \leq t < 4 \\ 1 & t \geq 4 \end{cases}$$

calculate the probability mass function of X .

Problem 4 (10%)

A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.

Problem 5 (6%)

Let X be a continuous random variable with range $[0, 1]$ and mean $E(X) = \mu$. Show the inequality $P(X \geq \varepsilon\mu) \geq (1 - \varepsilon)\mu$ for any $0 \leq \varepsilon \leq 1$.

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Problem 6 (6%)

A student measures the speed of a running train 20 times and take the average of the measurements as the result. If each measurement has an independent random error with standard the standard normal distribution $N(0, 1)$ (in kilometers per hour). What is the probability that the result differs from the actual speed by less than 0.1 kilometers per hour?

Problem 7 (13%; 7% for part a and 6% for part b)

Let X have a uniform distribution over $(-5, 5)$ and Y have a uniform distribution over $(0, 20)$. Assume that X and Y are independent.

- Compute the distribution function of $Z = \max(3X, 2Y)$.
- Compute $E(Z)$ and $\text{Var}(Z)$.