

國立交通大學九十二學年度碩士班入學考試試題

134, 144

科目名稱：離散數學與機率(104) 114, 124 考試日期：92年4月20日 第2節

系所班別：資訊科學系 組別：甲組 ~ 戊組 第1頁, 共3頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

- 1 Use strong induction to prove that every integer $n > 1$ is the product of finitely many primes. (7%)
- 2 A computer network consists of eight computers. Each computer is directly connected to at least one of the other computers. Use pigeonhole principle to show that there are at least two computers in the network that are directly connected to the same number of other computers. (5%)
- 3 Prove that big- O estimates for the function $(x+1)^n$ is $O(x^n)$. (5%)
- 4 The Fibonacci numbers, $f_0, f_1, f_2, \dots$, are defined by equations $f_0 = 0, f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n = 2, 3, 4, \dots$
 - a) Give a recursive algorithm using pseudocode for computing Fibonacci number. (2%)
 - b) Give an iterative algorithm using pseudocode for computing Fibonacci number. (2%)
 - c) How many additions have been used to find f_n with the recursive approach? (2%)
 - d) How many additions have been used to find f_n with the iterative approach? (2%)

For problems 5~12, give your answers directly.

No explanation or computation is necessary.

- 5 How many nonisomorphic partially ordered sets with three elements are there? (3%)
- 6 Consider the poset $(P(S), \subseteq)$, where $P(S)$ is the set of all subsets of the set $S = \{1, 2, 3, 4\}$. At least how many disjoint chains can the set $P(S)$ be expressed as their union? (3%)
- 7 For any positive integer n , let $\equiv_n$ be a binary relation on the set of integers such that $a \equiv_n b$ if $a \equiv b \pmod{n}$. Clearly, $\equiv_n$ is an equivalence relation. Given positive integers m and n , find k that satisfies $\equiv_m \cap \equiv_n = \equiv_k$. (3%)
- 8 The inversion table $b_1, b_2, \dots, b_n$ of a permutation on $1, 2, \dots, n$ is defined by letting b_k be the number of elements to the left of k that are greater than k .
Let

2 1 0 2 1 0

be the inversion table of a given permutation. What would be the inversion table of the permutation obtained by applying one bubble-sort pass to the given permutation? (3%)

(A bubble-sort pass interchanges $a_i \leftrightarrow a_{i+1}$ iff $a_i > a_{i+1}$ for $i = 1, 2, \dots, n-1$)

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- 9 How many permutations on $1, 2, \dots, n$ ($n \geq 2$) are sorted by at most two bubble-sort passes? (3%)
- 10 Let $P_G(x)$ be the number of ways of properly coloring (i.e. no two adjacent vertices have the same color) the vertices of the graph G with x or fewer colors. $P_G(x)$ is called the *chromatic polynomial* of G (in terms of x). Determine the chromatic polynomial of a tree with n vertices. (3%)
- 11 Which is a legal chromatic polynomial? (3%)
- a) $x^4 - 4x^3 + 6x^2 - 4x + 1$
 - b) $x^4 - 4x^3 + 6x^2 - 4x$
 - c) $x^4 - 4x^3 - 3x^2 + 6x$
 - d) $x^4 - 4x^3 + 6x^2 - 3x$
 - e) $2x^4 - 4x^3 + 5x^2 - 3x$
- 12 A sequence $a_1, a_2, \dots, a_n, \dots$ is linear recursive of degree k if there are constants $c_1, c_2, \dots, c_k$ such that $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$ for $n > k$. Find the recursion that generates the linear recursive sequence: $1, 4, 9, 16, 25, \dots, n^2, \dots$ (4%)

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Problem 1 (15%)

A box contains N balls, labeled by numbers $1, 2, \dots, N$, respectively. We draw n balls from the box randomly and without replacement, where $n > 1$. Let Y and Z be the random variables for the minimum and maximum numbers labeled in the drawn balls, respectively.

- What is the probability mass function of Z ?
- What is the joint probability mass function of Y and Z ?
- Let $W = Y + Z$. What is the probability mass function of W ?
- For $N = 8$ and $n = 6$, what is $E(W)$?

Problem 2 (10%)

Let X_1 and X_2 be mutually independent exponential random variables with parameter λ . Find the cumulative distribution function for $Y = X_1 - X_2$.

Problem 3 (5%)

You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability 0.8; with water it will die with probability 0.15. You are 90 percent certain that your neighbor will remember to water the plant.

If the plant is alive, what is the probability your neighbor forgot to water it?

Problem 4 (5%)

The random variable X has probability density function

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[X] = 0.6$, find a and b .

Problem 5 (5%)

Suppose that the cumulative distribution function of the random variable X is given by

$$F(x) = 1 - e^{-x^2} \quad x > 0$$

Use the results $E[X^n] = \int_0^\infty nx^{n-1}P\{X > x\} dx$ and $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ to find $\text{Var}(X)$.

Problem 6 (5%)

If n balls are randomly selected from an urn containing N balls of which m are white, find the expected number of white balls selected.

Problem 7 (5%)

It is known that the moment generating function of a normal random variable N with mean μ and variance σ^2 is given by $M_N(t) = \exp\left\{\frac{\sigma^2 t^2}{2} + \mu t\right\}$. Use this information to show that if X and Y are two independent normal random variables with means μ_X and μ_Y and variances σ_X^2 and σ_Y^2 , respectively, then $X + Y$ is also a normal random variable with mean $\mu_X + \mu_Y$ and variance $\sigma_X^2 + \sigma_Y^2$.