

國立交通大學九十一學年度碩士班入學考試試題

科目名稱：離散數學與機率(094)

考試日期：91 年 4 月 20 日 第 2 節

系所班別：資訊科學系

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*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

Problems 1~3: 3 points each

- 1 For each statement below, indicate whether it is true or not.
 - a) $O(f(n)) - O(f(n)) = O(f(n))$
 - b) $f(n) + O(g(n)) = O(f(n) + g(n))$
 - c) $\log f(n) = O(\log g(n))$ implies that $f(n) = O(g(n))$
- 2 Let R_1 and R_2 be two binary relations on a set. For each statement below, indicate whether it is true or not.
 - a) If R_1 and R_2 are symmetric, so are $R_1 \cup R_2$ and $R_1 \cap R_2$.
 - b) If R_1 and R_2 are asymmetric, so are $R_1 \cup R_2$ and $R_1 \cap R_2$.
 - c) If R_1 and R_2 are antisymmetric, so are $R_1 \cup R_2$ and $R_1 \cap R_2$.
- 3 For each statement below, indicate whether it is true or not.
 - a) There is an infinite sequence of infinities.
 - b) Any k -dimensional space, $k \geq 1$, has the same cardinality as the set $[0, 0.999\dots)$ of real numbers.
 - c) The set of computable numbers is countably infinite.
(N.B. A real number is computable, if, for any positive integer n , there is an algorithm which gives the n th digit of the decimal expansion of the real in finitely many steps.)

Problems 4~7: 4 points each

- 4 Let D_n be the set of all divisors of a positive integer n .
 - a) When is the poset $(D_n, |)$ a totally ordered set?
 - b) When is the lattice $(D_n, |)$ a complemented lattice?
- 5 a) The following algorithm decides whether an integer $n \geq 2$ is composite.

for $d = 2$ to $\lfloor \sqrt{n} \rfloor$ do

if $n \bmod d = 0$ then Answer "yes"

Answer "no"

Is this a polynomial-time algorithm?

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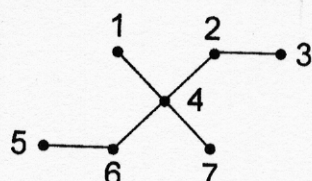
- b) The following algorithm decides whether $k = x_i$ for some i , $1 \leq i \leq n$
- ```
for $i = 1$ to n do
 if $k = x_i$ then Answer "yes"
Answer "no"
```

Is this a polynomial-time algorithm?

- 6 Given a labeled tree with  $n \geq 3$  vertices, the Prüfer code of the tree is an  $n-2$  tuple of integers  $1, 2, \dots, n$  defined as follows:

- (i) Choose a vertex of degree one of the tree with the smallest label
- (ii) Make the unique vertex adjacent to the vertex chosen in (i) the first item in the tuple
- (iii) Remove the vertex chosen in (i) and its adjacent edge from the tree
- (iv) Repeat these steps until there are only two vertices left in the tree

- a) Determine the Prüfer code for the labeled tree



- b) Determine the labeled tree whose Prüfer code is  $(3, 3, 4, 4)$ .

- 7 a) A natural number may be represented as a set, as described below.

$$0 = \emptyset$$

$$1 = \{\emptyset\}$$

$$2 = \{\emptyset, \{\emptyset\}\}$$

$$3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \text{ and so on}$$

With this representation, define the successor function  $\text{succ}(n)$ , where  $n$  is a natural number.

- b) A natural number may also be represented as a function, as below.

$$0 = \lambda f. \lambda x. x$$

$$1 = \lambda f. \lambda x. f x$$

$$2 = \lambda f. \lambda x. f (f x)$$

$$3 = \lambda f. \lambda x. f (f (f x)), \text{ and so on}$$

Define the successor function within this representation.



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8. (7 points) Mathematical induction is a very common method for proving mathematical theorems. Please explain why theorems proved with the mathematical induction is always correct.

9. (3 points) Please show that the set of real numbers between 0 and 1 (excluding 0 and 1) is an infinite set.

10. (3 points) Please show that the set of real numbers between 0 and 1 (excluding 0 and 1) is an uncountable set.

11. (6 points) Let  $(A, *)$  be an algebraic system. Let  $e \in A$ . Assume that  $e$  is a left identity. Show that either  $e$  is also a right identity or there is no right identity at all.

12. (6 points) Show that the transitive closure of a symmetric relation is also symmetric.

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- ( 5%) 1. An urn contains  $b$  black balls and  $r$  red balls. One of the balls is drawn at random, but when it is put back in the urn,  $c$  additional balls of the same color are put in with it. Now, suppose that we draw another ball. Find the probability that the first ball was black, given that the second ball drawn was red.
- ( 5%) 2. Let the random variable  $X$  have probability density  $f_X$ . Find the probability density function of the random variable  $aX + b$ , where  $a$  and  $b$  are constants,  $a \neq 0$ .
- (15%) 3. A group of  $N$  people throw their hats into the center of a room. The hats are mixed up, and each person randomly selects one. Let  $X$  be the number of people that select their own hats. Find the expectation and variance of  $X$ .
- ( 5%) 4. List the three axioms of probability.
- (10%) 5. A discrete random variable  $X$  is called *geometric* if

$$P\{X = n\} = (1 - p)^{n-1}p \quad n = 1, 2, \dots$$

Find the variance of a geometric random variable.

- (10%) 6. Suppose that the joint density of  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find  $P\{X > 1 | Y = y\}$ .