

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

離散數學

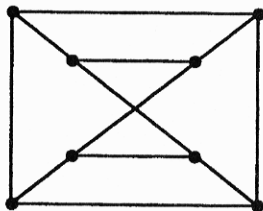
1. Find the smallest positive integer x such that $13x = 1 \pmod{2436}$. (5%)
2. Prove that at a party where there are at least two people, there are two people who know the same number of other people there. (Assume that the acquaintance relation is a symmetric but non-reflexive relation.) (5%)
3. Suppose that a valid code is an n -digit number in decimal notation containing even number of 0's. Let a_n denote the number of valid code words of length n . Find an explicit formula for a_n (not recurrence relation). (5%)
4. Suppose that a connected planar simple graph with 300 edges. If a planar representation of this graph divides the plane into 200 regions, how many vertices does this graph have? (5%)
5. How many nonisomorphic unrooted trees are there with 5 vertices? (5%)

6 填充題

1~2 格, 每格 2 分; 3~9 格, 每格 3 分

請直接寫答案, 計算過程不要寫在答案卷上

Consider the following graph

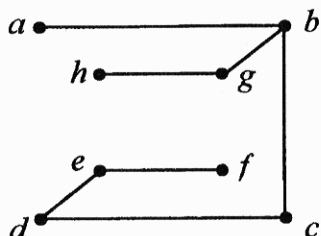


Is it a planar graph? 1

Does it contain a Hamiltonian circuit? 2

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Below is a spanning tree of the above graph.



Determine the fundamental cut-set corresponding to the edge (a,b) . 3

Determine the fundamental circuit corresponding to the edge (h,e) . 4

Let $A = \{0,1\}$ and B be the set of all functions from A^2 to A .

Define a binary relation $\triangleleft$ on B such that $f \triangleleft g$ if $f(x,y) \leq g(x,y) \forall x,y$

Then, $(B, \triangleleft)$ is a lattice.

Then, $\text{lub}(f,g) = \underline{5}$ and the universal lower bound is 6.

Mergesort takes 7 comparisons to sort 5 distinct elements in the worst case.

Any comparison-based sorting algorithm takes at least 8 comparisons to sort 5 distinct elements in the worst case.

Let $A = \{1,2,\dots,2048\}$ and R be a binary relation on A such that aRb if a divides b .

Then, the set A can be partitioned into at least 9 antichains.

國立交通大學九十學年度碩士班入學考試試題

科目名稱：離散數學與機率(074)

考試日期：90年4月22日 第2節

系所班別：資訊科學系

第3頁, 共3頁

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機率

1. What are the axioms of probability? (5%)
2. Let X_1 and X_2 be jointly continuous random variables with probability density function f_{X_1, X_2} . Let $Y_1 = X_1 + X_2$, $Y_2 = X_1 - X_2$. Find the joint density function of Y_1 and Y_2 in terms of f_{X_1, X_2} . (10%)
3. Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 75 and variance 25. What can be said about the probability that this week's production will be between 60 and 90? (10%)
(Hint: Use Chebyshev's inequality.)

4. Suppose that X takes on one of the values 0, 1, 2. If for some constant c ,

$$P(X = i) = cP(X = i - 1), \quad i = 1, 2,$$

find $\text{Var}(X)$. (5%)

5. Let X , Y , and Z be independent Poisson random variables with parameters λ_1 , λ_2 and λ_3 , respectively. For nonnegative integers x , y , and z , find

$$P(X = x, Y = y, Z = z | X + Y + Z = x + y + z).$$

(10%)

6. A standard Cauchy random variable has density function

$$f(x) = \frac{1}{\pi(1 + x^2)}, \quad -\infty < x < \infty$$

If X is a standard Cauchy random variable, find the probability density function of $1/X$. (10%)