

國立交通大學八十九學年度碩士班入學考試試題

科目名稱：離散數學與機率(074)

考試日期：89 年 4 月 22 日 第 2 節

系所班別：資訊科學學系

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*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

一、離散數學

1. (8%) Let $(A, \star)$ and $(B, *)$ be two algebraic systems.

The Cartesian product of $(A, \star)$ and $(B, *)$ is defined as the algebraic system $(A \times B, \triangle)$, where

$$(a_1, b_1) \triangle (a_2, b_2) = (a_1 \star a_2, b_1 * b_2)$$

Show that the Cartesian product of two groups is a group.

2 (8%) Solve the recurrence relation:

$$a_r + 3a_{r-1} + 2a_{r-2} = f(r),$$

where $f(2) = 1$, $f(r) = 0$ if $r \neq 2$, with the boundary condition

$$a_0 = a_1 = 0.$$

3. (9%) A chessboard contains intervening white squares and black squares. Cut off one white square and one black square from a standard 8×8 chessboard and we will obtain a defective chessboard. Note that the two squares that are cut off need not be adjacent. Show that the defective chessboard can always be covered with exactly thirty-one 2×1 dominoes.

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4 Find the smallest positive integer x such that

$$\begin{cases} x \equiv 37 \pmod{99} \\ x \equiv 44 \pmod{101} \\ x \equiv 170 \pmod{199} \end{cases} \quad (10\%)$$

5 (a) State the halting problem.

(b) Prove that there is no procedure that solves the halting problem. (10%)

6 Let G be a simple graph with the smallest number of vertices such that (1) G has exactly 120 edges and (2) all vertices of G are of the same degree. How many vertices G has. (5%)

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二. 機率

以下 1 至 6 題是填充題 只需給答案 不必寫任何計算式

- 3% 1. Let X and Y be independent random variables having normal densities $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$. Find the density function of $X+Y$.
(You must give the exact formula of the density function!)
- 4% 2. Let X be a random variable having the normal density $n(0, \sigma^2)$. Find the density function of X^2 .
- 4% 3. Let X and Y be independent random variables each having the normal density $n(0, \sigma^2)$. Find the density function of X^2+Y^2 .
- 6% 4. Let X be a uniformly distributed random variable over $(0, 1)$, and let Y be a uniformly distributed random variable over $(0, X)$.
(i) Find the joint density of X and Y .
(ii) Find the marginal density of Y .
- 4% 5. Let $X_1, X_2, \dots, X_n$ be independent random variables.
(i) If X_i has the binomial distribution with parameters n_i and p . Find the distribution of $X_1+X_2+\dots+X_n$.
(ii) If X_i has the Poisson distribution with parameter λ_i . Find the distribution of $X_1+X_2+\dots+X_n$.
- 4% 6. (i) State the Chebyshev's inequality.
(ii) State the Central Limit Theorem.

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7. An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn, c additional balls of the same color are put in with it. Now, suppose we draw another ball. What is the probability that the first ball was black, given that the second ball drawn was red? (5%)

8. For some constant c , the random variable X has probability density function

$$f(x) = \begin{cases} cx^4 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E[X]$. (4%)

9. A standard Cauchy random variable has density function

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$$

If X is a standard Cauchy random variable, show that $1/X$ is also a standard Cauchy random variable. (6%)

10. The random variables X and Y have a joint density function given by

$$f(x) = \begin{cases} \frac{2e^{-2x}}{x} & 0 \leq x < \infty, 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

Compute $\text{Cov}(X, Y)$. (10%)