

科目名稱：線性代數與機率(103) 考試日期：93 年 4 月 17 日 第 2 節

系所班別：資訊科學系 組別：甲組 第 1 頁, 共 5 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

A. LINEAR ALGEBRA

1 True or False (10 points)

- a) The set of all vectors in a plane not parallel to a fixed vector under the usual operations of vector addition and scalar multiplication is a vector space.
- b) If two matrices have the same column space, row space, and null space then one is a multiple of the other.
- c) $\begin{vmatrix} A & B \\ B & A \end{vmatrix} = |A+B| \cdot |A-B|$ where A and B are two matrices of order n .
- d) The vectors
 $A_1 = (1, -2, -2, 1)$, $A_2 = (0, 1, 1, 0)$, $A_3 = (1, -1, -1, 1)$
 and
 $B_1 = (0, -1, -1, 0)$, $B_2 = (1, 0, 0, 1)$
 span the same linear subspace of $\mathbf{R}^4$.
- e) Let V be a 3-dimensional subspace of $\mathbf{R}^4$, then every basis of V can be extended to a basis of $\mathbf{R}^4$ by adding one more vector.

2 For each problem below, give your answer directly. (15 points)

No explanation or computation is required.

- a) Find the largest possible number of independent vectors from the following vector set.

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \quad v_5 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_6 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

- b) Under what condition on the numbers
- p
- ,
- q
- , and
- r
- will the following system of linear equations be consistent?

$$-2x + y + z = p$$

$$x - 2y + z = q$$

$$x + y - 2z = r$$

- c) Let
- $A = LU$
- be the LU-factorization of
- $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$
- .

Find L^{-1} .

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- d) Find the complete solution in the form $x_p + x_h$ to the nonhomogeneous system

$$x + y + z = 5$$

(x_p is a particular solution of the nonhomogeneous system and x_h is a solution to the corresponding homogeneous system.)

- e) Let

$$A_1 = (1, 0, 0, 0), A_2 = (0, 1, 0, 0), A_3 = (0, 0, 1, 0), A_4 = (0, 0, 0, 1)$$

and

$$B_1 = (2, 1, -1, 1), B_2 = (0, 3, 1, 0), B_3 = (5, 3, 2, 1), B_4 = (6, 6, 1, 3)$$

be two bases for $\mathbf{R}^4$.

Find a nonzero vector which has the same coordinates with respect to the above two bases.

以下各計算題, 請清楚標出你最後的答案. (加底線或加框)

3. (1) In $\mathbf{R}^3$, consider the subspace S , where 4%
 $S = \text{span} \{(1, 1, 0), (1, 1, 1)\}.$

Find the orthogonal projection of $(1, 0, 0)$ onto the subspace S .

- (2) In $\mathbf{R}^3$, find the rotation matrix that rotates an angle θ about y -axis 2%
 counterclockwise.

- (3) In $\mathbf{R}^3$, $B_1 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$, $B_2 = \{(2, 2, 0), (0, 1, 1), (1, 0, 1)\}$
 What is the transition matrix from B_1 to B_2 ? 2%

You do not need to simplify the answer or to find the inverse matrix.

- (4) Let T be a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ given by 5%
 $T(x, y) = (x+3y, 2x).$

Find $T^n(x, y)$, where $T^n(x, y) = T(T(T(\dots T(T(x, y))\dots))$, n times.

4. For each of the following two cases, are the following two matrices similar?
 You must explain your reason. 4%

(1) $A = \begin{bmatrix} -2 & 7 \\ -3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

(2) $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

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5. Let A and B be two matrices. Of the following five statements, which ones are correct and which ones are incorrect. (No partial credit for partial answer)

- (1) The row space of AB is a subspace of the row space of A .
- (2) The column space of AB is a subspace of the column space of A .
- (3) The row space of A is a subspace of the row space of AB .

(4) $\text{rank}(AB) \geq \text{rank}(A)$.

(5) $\text{rank}(AB) \leq \text{rank}(B)$.

3%

6. In $\mathbb{R}^3$, consider the inner product $\langle u, v \rangle = au_1 v_1 + bu_2 v_2 + cu_3 v_3$ where $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$. 5%

(1) State all possible values for which a , b , and c can be, such that $\langle u, v \rangle$ is an inner product.

(2) Write the Cauchy-Schwarz inequality for this particular inner product.

(3) In the subspace $S = \text{span} \{(1, 1, 1)\}$, find the vector v which is closest to $(1, 2, 3)$.

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B. Problems for Probability

Problem 1 (5%)

In a state lottery, a player must choose 8 of the numbers from 1 to 40. The lottery commission then performs an experiment that selects 8 of these 40 numbers. Assuming that the choice of the lottery commission is equally likely to be any of the $\binom{40}{8}$ combinations, what is the probability that a player has at least 6 of the number selected?

Problem 2 (5%)

Suppose we have 10 coins such that if the i th coin is flipped, heads will appear with probability $\frac{i}{10}$, $i = 1, 2, \dots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

Problem 3 (5%)

If the distribution function of a random variable X is given by

$$F(t) = \begin{cases} 0 & t < 0 \\ 1/2 & 0 \leq t < 1 \\ 3/5 & 1 \leq t < 2 \\ 4/5 & 2 \leq t < 3 \\ 9/10 & 3 \leq t < 4 \\ 1 & t \geq 4 \end{cases}$$

calculate the probability mass function of X .

Problem 4 (10%)

A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.

Problem 5 (6%)

Let X be a continuous random variable with range $[0, 1]$ and mean $E(X) = \mu$. Show the inequality $P(X \geq \varepsilon\mu) \geq (1 - \varepsilon)\mu$ for any $0 \leq \varepsilon \leq 1$.

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Problem 6 (6%)

A student measures the speed of a running train 20 times and take the average of the measurements as the result. If each measurement has an independent random error with standard the standard normal distribution $N(0, 1)$ (in kilometers per hour). What is the probability that the result differs from the actual speed by less than 0.1 kilometers per hour?

Problem 7 (13%; 7% for part a and 6% for part b)

Let X have a uniform distribution over $(-5, 5)$ and Y have a uniform distribution over $(0, 20)$. Assume that X and Y are independent.

- a. Compute the distribution function of $Z = \max(3X, 2Y)$.
- b. Compute $E(Z)$ and $\text{Var}(Z)$.