

國立交通大學九十一學年度碩士班入學考試試題

科目名稱：線性代數與機率(093)

考試日期：91 年 4 月 20 日 第 2 節

系所班別：資訊科學系

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*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

Problem 1: 9 points

- A1 For each statement below, indicate whether it is true or not.
- a) The determinant of an idempotent matrix is either 0 or 1.
 - b) The only vector space that contains a finite number of vectors is the zero vector space $Z = \{0\}$.
 - c) Square matrices A and B are invertible if and only if AB is invertible.
 - d) A square matrix is a nilpotent matrix of index k if and only if its transpose is a nilpotent matrix of index k .
 - e) Let E and F be subsets of a vector space, then $\text{span}(E) \subseteq \text{span}(F)$ if and only if $E \subseteq F$.
 - f) The product of two orthogonal matrices is orthogonal, so is the sum of two orthogonal matrices.
 - g) $\det: M_{n,n} \rightarrow R$ is a linear transformation.
 - h) The column space of a 2 by 2 matrix has the same dimension as its row space.
 - i) The only possible eigenvalues of a projection matrix are 0 and 1.

Problems 2~5: 4 points each

- A2 Consider the linear system

$$ax + y + z = 1$$

$$x + ay + z = 0$$

$$x + y + az = 0$$

For what values of a does the linear system have a unique solution?
Also, use Cramer's rule to find the solution.

- A3 Find the orthogonal complement of the following subspaces in R^3 .

- a) $\{(x, y, z) \mid x + 2y + 3z = 0\}$
- b) $\{(x, y, z) \mid x + y + z = 0 \text{ and } x - y + z = 0\}$

- A4 Compute $A^{100}v$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- A5 Let $T: R^3 \rightarrow R^3$ be the linear transformation given by the formula

$$T(x, y, z) = (y + z, x + z, y + x)$$

Compute the matrices A , B and P that satisfy $A = P \cdot B \cdot P^{-1}$, where A is the matrix of T relative to the standard basis of R^3 , and B is the matrix of T relative to the basis $\{(1, 1, 1), (1, -1, 0), (1, 1, -2)\}$

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- (10%) 6. Let V be the real vector space of polynomials of degree strictly less than 3 and let W be the real vector space of polynomials of degree strictly less than 2. Define the transformation T from V to W by

$$T(v) = \frac{v(t) - v(0)}{t}$$

Let the ordered basis for V be $B = \{1 + t, t + t^2, 1 + t^2\}$ and let the ordered basis for W be $C = \{1 + t, 1 - t\}$. Find the matrix representation of T .

- (10%) 7. Consider the 5×5 matrix

$$A = \begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

It is easy to see that $\lambda_1 = 7$ and $\lambda_2 = 4$ are the eigenvalues of A with algebraic multiplicities $m_1 = 2$ and $m_2 = 3$, respectively. Find the geometric multiplicities of λ_1 and λ_2 .

- (5%) 8. Prove that $\lambda = 0$ is an eigenvalue of A if and only if A is singular.

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- (5%)B1. An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn, c additional balls of the same color are put in with it. Now, suppose that we draw another ball. Find the probability that the first ball was black, given that the second ball drawn was red.
- (5%)B2. Let the random variable X have probability density f_X . Find the probability density function of the random variable $aX + b$, where a and b are constants, $a \neq 0$.
- (15%)B3. A group of N people throw their hats into the center of a room. The hats are mixed up, and each person randomly selects one. Let X be the number of people that select their own hats. Find the expectation and variance of X .
- (5%)B4. List the three axioms of probability.
- (10%)B5. A discrete random variable X is called *geometric* if

$$P\{X = n\} = (1 - p)^{n-1}p \quad n = 1, 2, \dots$$

Find the variance of a geometric random variable.

- (10%)B6. Suppose that the joint density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find $P\{X > 1 | Y = y\}$.