

科目名稱：線性代數與機率(073)

考試日期：90年4月22日 第2節

系所班別：資訊科學系

第1頁, 共3頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

線性代數

1. Let $B_1 = \left\{ \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}, \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} r_1 \\ s_1 \\ t_1 \end{bmatrix}, \begin{bmatrix} r_2 \\ s_2 \\ t_2 \end{bmatrix}, \begin{bmatrix} r_3 \\ s_3 \\ t_3 \end{bmatrix} \right\}$.

be two distinct ordered bases of $\mathbb{R}^3$.(A) Write down the change of basis matrix (from B_1 to B_2).(B) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Write down the matrix representation of T with respect to the bases B_1 and B_2 7%

2. Consider the vector space $C[-1,1]$ with the inner product $\langle f, g \rangle = \int_{-1}^1 f(x)g(x)dx$

(A) Write down the Cauchy-Schwarz inequality (with respect to this particular inner product).

(B) Consider the subspace $W = \text{span}\{1, x, x^2\}$. Find an orthonormal basis for W .(C) Find the orthogonal projection of x^3 onto the subspace W . 7%3. Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined as follows.

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \text{rotating } \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ counter clockwise by an angle } \theta$$

(A) Write down the matrix representation A of T with respect to the standard basis.(B) Find the eigenvalues of A and the corresponding eigenvectors. 7%4. Find the values of k so that the vectors $[3-k \ -1 \ 0]^T$, $[-1 \ 2-k \ -1]^T$, and $[0 \ -1 \ 3-k]^T$, span a two-dimensional space. (7%)

5. Find an orthonormal basis for the solution space of the following homogeneous system. (7%)

$$x + y - z + w = 0$$

$$2x - y + z + 2w = 0$$

6. Find the matrix P such that $P^{-1}AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \quad (7\%)$$

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7. Determine whether the given set, together with the standard operations, is a vector space. (8%) (是非題, 答錯一題倒扣2分, 扣完為止)

- (a) The set of all 2×2 singular matrices.
- (b) The set of all 2×2 nonsingular matrices.
- (c) The set of all 2×2 diagonal matrices.
- (d) The set of all 2×2 matrices of the form

$$\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$$

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機率

1. What are the axioms of probability? (5%)
2. Let X_1 and X_2 be jointly continuous random variables with probability density function f_{X_1, X_2} . Let $Y_1 = X_1 + X_2$, $Y_2 = X_1 - X_2$. Find the joint density function of Y_1 and Y_2 in terms of f_{X_1, X_2} . (10%)
3. Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 75 and variance 25. What can be said about the probability that this week's production will be between 60 and 90? (10%)
(Hint: Use Chebyshev's inequality.)

4. Suppose that X takes on one of the values 0, 1, 2. If for some constant c ,

$$P(X = i) = cP(X = i - 1), \quad i = 1, 2,$$

find $\text{Var}(X)$. (5%)

5. Let X , Y , and Z be independent Poisson random variables with parameters λ_1 , λ_2 and λ_3 , respectively. For nonnegative integers x , y , and z , find

$$P(X = x, Y = y, Z = z | X + Y + Z = x + y + z).$$

(10%)

6. A standard Cauchy random variable has density function

$$f(x) = \frac{1}{\pi(1 + x^2)}, \quad -\infty < x < \infty$$

If X is a standard Cauchy random variable, find the probability density function of $1/X$. (10%)