

國立交通大學八十九學年度碩士班入學考試試題

科目名稱：線性代數與機率(073)

考試日期：89年4月22日 第2節

系所班別：資訊科學學系

第 / 頁, 共 4 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

一. 線性代數

以下 1 至 6 題是填充題, 只需給答案 不必寫出任何計算式子

$$8\% \quad 1. \quad \text{matrix } A = \begin{bmatrix} -2 & -1 & -2 & 1 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 6 & 9 \\ -2 & -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 4 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Find ① $\det A$, ② $\text{rank}(A)$,

- ③ a basis of the row space of A
 ④ a basis of the column space of A
 ⑤ a basis of the image space of the linear transform A
 ⑥ a basis of the null space of the linear transform A

$$5\% \quad 2. \quad \text{Matrix } A = \begin{bmatrix} -4 & -12 & 11 & -1 \\ 2 & 6 & -4 & 2 \\ 1 & 4 & -3 & 4 \\ -4 & -12 & 11 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ -4 & 0 & 3 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find ① $\det A$ ② $\det(A^T)$ ③ $\det(A^{-1})$

④ $\det(\text{adj } A)$ ⑤ $(\text{adj } A)^{-1}$

$$3\% \quad 3. \quad \text{Matrix } A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 3 & 4 \\ 0 & 2 & 1 & 1 \\ 0 & 4 & 2 & 5 \\ 0 & 6 & 3 & 1 \end{bmatrix}$$

Find ① $\det(A^T)$ ② $\text{rank}(A)$

3% 4. Let A and B be two matrices. Compare the following ranks and fill in the blank with $=$, $>$, $<$, $\geq$, or $\leq$

- ① $\text{rank}(AB)$ _____ $\text{rank}(A)$.
 ② $\text{rank}(AB)$ _____ $\text{rank}(B)$.
 ③ A is nonsingular $\text{rank}(AB)$ _____ $\text{rank}(A)$.
 ④ A is nonsingular $\text{rank}(AB)$ _____ $\text{rank}(B)$.
 ⑤ $\text{rank}(A)$ _____ $\text{rank}(A^T)$.

國立交通大學八十九學年度碩士班入學考試試題

科目名稱：線性代數與機率(073)

考試日期：89年4月22日 第2節

系所班別：資訊科學學系

第2頁，共4頁

*作答前，請先核對試題、答案卷（試卷）與准考證上之所組別與考試科目是否相符!!

3% 5. Find all possible matrices X for which $AX=0$, where A is

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 5 & -6 \\ 2 & -3 & 6 \end{bmatrix}$$

3% 6. Find all right-hand sides b for which $Ax=b$ has solutions, and find all solutions, where

$$A = \begin{bmatrix} 4 & -1 & 2 & 6 \\ -1 & 5 & -1 & -3 \\ 3 & 4 & 1 & 3 \end{bmatrix}$$

7. Suppose that

$$B = \{v_1; v_2; \dots; v_p\} \quad \text{and} \quad B' = \{v_p; v_{p-1}; \dots; v_1\}$$

are two ordered bases for $\mathbb{R}^p$. Find the matrix M that translates between coordinates with respect to these two bases. (5%)

8. Suppose that $\mathcal{T}$ is a linear transformation from $\mathcal{V}$ to $\mathcal{W}$, both p -dimensional. Prove that the null space of $\mathcal{T}$ equals $\{0\}$ if and only if the image space of $\mathcal{T}$ equals $\mathcal{W}$. (10%)

9. Find the eigenvalues of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

together with their algebraic and geometric multiplicities, and find as many eigenvectors (in a linearly independent set) as possible. (10%)

國立交通大學八十九學年度碩士班入學考試試題

科目名稱：線性代數與機率(073)

考試日期：89年4月22日 第2節

系所班別：資訊科學學系

第3頁，共4頁

*作答前，請先核對試題、答案卷（試卷）與准考證上之所組別與考試科目是否相符!!

二. 機率

以下 1 至 6 題是填充題 只需給答案 不必寫任何計算式

- 3% 1. Let X and Y be independent random variables having normal densities $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$. Find the density function of $X+Y$.
(You must give the exact formula of the density function!)
- 4% 2. Let X be a random variable having the normal density $n(0, \sigma^2)$. Find the density function of X^2 .
- 4% 3. Let X and Y be independent random variables each having the normal density $n(0, \sigma^2)$. Find the density function of X^2+Y^2 .
- 6% 4. Let X be a uniformly distributed random variable over $(0, 1)$, and let Y be a uniformly distributed random variable over $(0, X)$.
(i) Find the joint density of X and Y .
(ii) Find the marginal density of Y .
- 4% 5. Let $X_1, X_2, \dots, X_n$ be independent random variables.
(i) If X_i has the binomial distribution with parameters n_i and p . Find the distribution of $X_1+X_2+\dots+X_n$.
(ii) If X_i has the Poisson distribution with parameter λ_i . Find the distribution of $X_1+X_2+\dots+X_n$.
- 4% 6. (i) State the Chebyshev's inequality.
(ii) State the Central Limit Theorem.

國立交通大學八十九學年度碩士班入學考試試題

科目名稱：線性代數與機率(073)

考試日期：89年4月22日 第2節

系所班別：資訊科學學系

第 4 頁, 共 4 頁

*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

7. An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn, c additional balls of the same color are put in with it. Now, suppose we draw another ball. What is the probability that the first ball was black, given that the second ball drawn was red? (5%)

8. For some constant c , the random variable X has probability density function

$$f(x) = \begin{cases} cx^4 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E[X]$. (4%)

9. A standard Cauchy random variable has density function

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$$

If X is a standard Cauchy random variable, show that $1/X$ is also a standard Cauchy random variable. (6%)

10. The random variables X and Y have a joint density function given by

$$f(x) = \begin{cases} \frac{2e^{-2x}}{x} & 0 \leq x < \infty, 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

Compute $\text{Cov}(X, Y)$. (10%)