

考試科目	微積分	所別	應用數學系 8/11, 8/16	考試時間	3月6日(六)第一節
------	-----	----	---------------------	------	------------

1. (16%) Prove or disprove (by a counter example): Let f be a real-valued differential odd function defined on $\mathbb{R}$. Let $g(x) = |f(x)|$. Suppose that $f(x_0) = 0$ for some $x_0 \in \mathbb{R}$. Then g is not differentiable at the point $x = x_0$.

2. (16%) Let $f(x, y, z) = \max\{x, y, z\}$ for all $(x, y, z) \in \mathbb{R}^3$. Is f a continuous function? Justify your answer.

3. Let f be a continuous real-valued function defined on $[a, b] \subset \mathbb{R}$. Suppose that f is differentiable on (a, b) . Let

$$G(x) = \int_a^x f(t) dt.$$

(a) (6%) Show that G is well-defined.

(b) (10%) Show that $G'(x) = f(x)$.

4. (a) (5%) State the Stoke's theorem.

(b) (15%) Let $\mathbf{F}(x, y, z) = (3y, -xz, yz^2)$ be a vector field on $\mathbb{R}^3$. Let S be portion of the surface $z = (x^2 + y^2)/2$ below the plane $z = 2$, with the curve C as the boundary. Verify Stoke's theorem.

5. (16%) Let f be a real-valued function on $\mathbb{R}$. Suppose that

$$f''(x) = (x-1)^{71}(x-2)^{52}(x-3)^{111}$$

and $f'(0) < f'(1) < f'(5) < f'(3)$. Show that $5f'(0) + f(0) \leq f(x) \leq 5f'(3) + f(0)$, for all $x \in (0, 5)$.

6. (16%) Find all possible real differentiable function on $\mathbb{R}$ such that the derivative of the function is itself. Justify your answer.

考試科目

線性代數

所別

8111, 8116
應用數學

考試時間

3月6日(六)第二節

* Show all your work.

1. (15 %) Let V be a finite-dimensional vector space over a field F , and let $\beta = \{x_1, x_2, \dots, x_n\}$ be an ordered basis for V . Let Q be an $n \times n$ invertible matrix with entries from F . Define

$$x'_j = \sum_{i=1}^n Q_{ij} x_i, \text{ for } 1 \leq j \leq n,$$

and set $\beta' = \{x'_1, x'_2, \dots, x'_n\}$. Prove that β' is a basis for V and hence that Q is the change of coordinate matrix changing β' -coordinates into β -coordinates.

2. (15 %) Let A be an $m \times n$ matrix with rank m and B be an $n \times p$ matrix with rank n . Determine the rank of AB . Justify your answer.
3. (15 %) Let $A, B \in M_{m \times n}(F)$ be such that $AB = -BA$. Prove that if n is odd and F is not a field of characteristic two, then A or B is not invertible.
4. (20 %) Two linear operators T and U on a finite-dimensional vector space V are called *simultaneously diagonalizable* if there exists an ordered basis β for V such that $[T]_\beta$ and $[U]_\beta$ are diagonal matrices. Prove that if T and U are diagonalizable linear operators on a finite-dimensional vector space V such that $UT = TU$, then T and U are simultaneously diagonalizable.
5. (15 %) Let A be an $n \times n$ matrix with complex entries. Prove that $AA^* = I$ if and only if the rows of A form an orthonormal basis for $\mathbb{C}^n$.
6. (20 %) For the matrix $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -1 & 3 \end{pmatrix}$, find a basis for each generalized eigenspace of L_A consisting of a union of disjoint cycles of generalized eigenvectors. Then find a Jordan canonical form J of A .

備

註

試題隨卷繳交