

考試科目	微積分	所別	應用物理系 (8161) (8166)	考試時間	3月14日 星期六	第3節
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1. Find the following limits if exists. (20%)

(a) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k(k+1)}$.

(b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{k}}{k+1}$.

(c) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4n}{n^2+k^2}$.

(d) $\lim_{x \rightarrow \infty} \frac{\sin x}{\sqrt{x}}$.

2. Evaluate the following integrals. (20%)

(a) $(\ln x)^2 dx$.

(b) $\int t^2 \sqrt{t-1} dt$.

(c) $\int \sin^2 2x dx$.

(d) $\int e^x \cos x dx$.

3. Given that $f(2) = 8$, $g(2) = 2$, $f'(2) = 1$, and $g'(2) = -2$. Find

(a) $(f^2 + g^2)'(2)$.

(b) $(\sqrt{f/g})'(2)$. (10%)

4. Using differential to approximate the values $e^{-0.01}$ and $\sqrt{0.99^2 + 0.03^2}$. (10%)

5. Find the maximum and minimum values of the function $f(x, y, z) = x + 2y - z$ on the sphere $x^2 + y^2 + z^2 = 24$. (10%)

6. Let $f(x) = x^3 - 4x$ and $g(x) = 5x$. Find the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$. (10%)

備 考 試 題 隨 卷 繳 交

命 題 委 員 : (簽章)

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7. Suppose that a child is inflating a spherical balloon and we know that the balloon is being inflated at a rate of $100 \text{ cm}^3/\text{sec}$. Using this information to determine the rate at which the radius is increasing when the radius is 9 cm. (10%)

8. A company determines that the profit resulting from the sale of x of its products per week is $P(x) = 160 - 96x + 18x^2 - x^3$ dollars. Find the maximum and minimum values for this function on the interval $[0,10]$ and the sales level x at which each occurs. (10%)



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考 試 科 目	普通物理(-) General Physics I	所 別	應用物理 (8162) (8163)	考 試 時 間	3 月 14 日 星期六	第 三 節
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NOTE: Please answer the following questions one by one, and the detailed calculations should be given.

1. Please show that the acceleration vector in uniform circular motion is $-\frac{v^2}{r}\hat{r}$, where r is the radius of the circle and v is the speed of the particle. (10 points)
2. Consider a long thin rod with rotation axis through its center, Prove its moment of inertia is $\frac{1}{12}ML^2$, where M and L are corresponding to mass and length of the long thin rod, respectively. (10 points)
3. Give a brief description about the operating principle of antilock braking systems (ABS). (10 points)
4. A sinusoidal wave on a string is described by the following equation $y = (0.15m)\sin(0.8x - 50t)$ where x and y are in meters and t is in seconds. If the mass per unit length of this string is 12 g/m, determine the power transmitted to the wave. (10 points)
5. A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane. Find the period of oscillation if the amplitude of the motion is small. (10 points)
6. Consider a disk of radius R and mass M rolling without slipping on an inclined plane of angle θ . Find the static friction between disk and inclined plane. (10 points)
7. How to measure the flow speed of an incompressible fluid? Give an example to address. (10 points)
8. Give an example to address that why the total entropy of an isolated system always increases when the process is irreversible. (10 points)
9. Consider a heat engine operated under a Carnot cycle, in which an ideal gas is contained. Prove the efficiency of Carnot engine as $e_c = 1 - \frac{T_c}{T_h}$, where T_h is an operation temperature during isothermal expansions and T_c is operated during isothermal compressions. (20 points)

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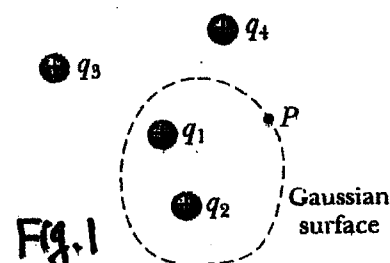
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考試科目	普通物理(二)	所別	應用物理所 ⁸¹⁶² ₈₁₆₃	考試時間	3月14日 星期六	第四節
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[Answers in both Chinese and English are acceptable.]

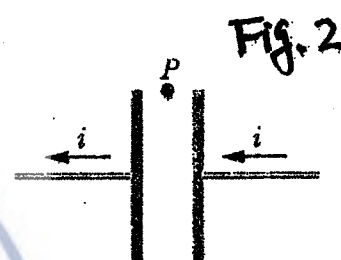
1 (32%) (simple answers are sufficient).

(1) In Fig. 1, a full Gaussian surface encloses two of the four positively charged particles. (a) Which of the particles contribute to the electric field at point P on the surface? (b) Which net flux of electric field through the surface is greater (if either): that due to q_1 and q_2 or that due to all four charges?

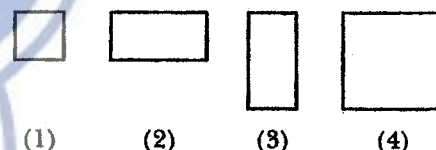


(2) Three wires, of the same diameter, are connected in turn between two points maintained at a constant potential difference. Their resistivities and lengths are ρ and L (wire A), 1.2ρ and $1.2L$ (wire B), and 0.9ρ and L (wire C). Rank the wires according to the rate at which energy is transferred to thermal energy within them, greatest first.

(3) An electron in an external magnetic field $\vec{B}_{ext}$ has its spin angular momentum S_z antiparallel to $\vec{B}_{ext}$. If the electron undergoes a *spin flip* so that S_z is then parallel with $\vec{B}_{ext}$, must energy be supplied to or lost by the electron?



(4) Figure 2 shows a parallel-plate capacitor and the current in the connecting wires that is discharging the capacitor. Are the directions of (a) electric field $\vec{E}$ and (b) displacement current i_d leftward or rightward between the plates? (c) Is the magnetic field at point P into the page or out of the page?



(5) Figure 3 shows four choices for the rectangular opening of a source of light waves. The sides have lengths of either L or $2L$, with L being 3.0 times the wavelength of the waves. Rank the openings according to the extent of (a) left-right spreading and (b) up-down spreading of the waves due to diffraction, greatest first.

Fig. 3

(6) According to the theory of relativity, which of the following statements is true?

(a) moving clocks run fast; (b) energy is not conserved in high speed collisions; (c) the speed of light must be measured relative to the ether; (d) momentum is not conserved in high speed collisions; (e) none of the (a), (b), (c) and (d) are true.

(7) While standing outdoors one evening, you are exposed to: (a) yellow light from a sodium street lamp, (b) radio waves from an AM radio station, (c) radio waves from an FM radio station, and (d) microwaves from an antenna of a communications system. Rank these four types of waves in terms of increasing photon energy, lowest first.

(8) Consider the element chlorine ($Z = 17$). In its part of the periodic table, the subshells of the electronic states are filled in the sequence $1s\ 2s\ 2p\ 3s\ 3p\ 3d\ 4s\ 4p\ \dots$. Identify the highest occupied subshell and state how many electrons are in it.

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2. (10%) In Fig. 4, a sphere, of radius a and charge $+q$ uniformly distributed throughout its volume, is concentric with a spherical conducting shell of inner radius b and outer radius c . This shell has a net charge of $-q$. Find expressions for the electric field, as a function of the radius r , (a) within the sphere, (b) between the sphere and the shell, (c) inside the shell. (e) What are the charges on the inner and outer surfaces of the shell?

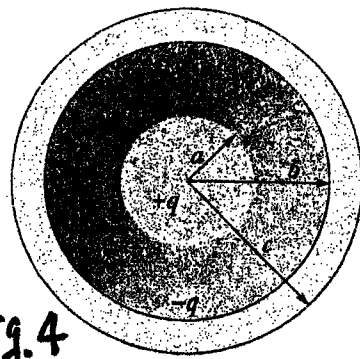


Fig. 4

3. (9 %) A parallel-plate capacitor with capacitance C is charged by a battery to a potential difference V between its plates. The charging battery is now disconnected and a porcelain slab with dielectric constant κ is slipped between its plates. What is the potential energy of the capacitor-slab device, both before and after the slab is put into place?

4. (9 %) A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$), and an α particle (charge $+2e$, mass $4m_p$) are accelerated through a common potential difference ΔV . Each of the particles enters a uniform magnetic field $\vec{B}$, with its velocity in a direction perpendicular to $\vec{B}$. The proton moves in a circular path of radius r_p . Determine the radii of the circular orbits for the deuteron, r_d , and the alpha particle, r_α , in terms of r_p .

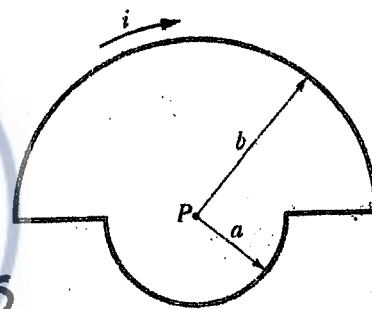


Fig. 5

5. (9 %) A length of wire is formed into a closed circuit with radii a and b , as shown in Fig. 5, and carries a current i . (a) What are the magnitude and direction of $\vec{B}$ at point P ? (b) Find the magnetic dipole moment of the circuit.

6. (10 %) Figure 6 shows a lens with radius of curvature R lying on a flat glass plate and illuminated from above by light with wavelength λ . Find the radii r of the interference maxima (called Newton's rings) assuming $r/R \ll 1$.

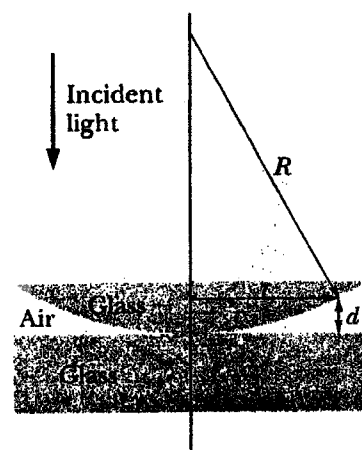


Fig. 6

7. (9 %) A particle of charge q and mass m has been accelerated from rest to a nonrelativistic speed through a potential difference ΔV . Find an expression for its de Broglie wavelength.

8. (12 %) Seven electrons are confined to a square corral that is a two-dimensional infinite potential well with widths $L_x = L_y = L$. Assume that the electrons do not electrically interact with one another.

- (a) What is the electron configuration for the ground state of the system of seven electrons?
 (b) What is the total energy of the seven-electron system in its ground state, as a multiple of $h^2/8mL^2$?
 (c) How much energy must be transferred to the system for it to jump to its first excited state, and what is the energy of that state?

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