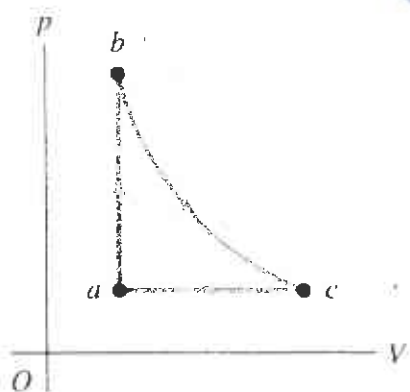


考 試 科 目	普通物理學	所 別	應用物理研究所	考 試 時 間	2 月 26 日 (六) 第 三 節
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1. A uniform rectangular wooden board of mass M is pivoted horizontally along its top edge. The vertical edge of the board has length L . (a) Calculate the moment of inertia of the board. A bullet of mass $M/4$ and horizontal velocity v strikes the board at the position $2L/3$ from the top edge and is embedded in the board. (b) Calculate the angular velocity of the board right after the impact and express it in terms of v and L . (c) What is the ratio of the kinetic energies before and right after the impact? (15%)
2. The potential energy of a KCl molecule is $U(r) = A [(R_0^7/8r^8) - 1/r]$, where both A and R_0 are constants and r is the distance between the two atoms. (a) Find the radial force $F(r)$ on each atom. (b) What is the minimum potential energy? What is the meaning of R_0 ? (c) If the force is considered elastic, find the force constant k and express k in terms of A and R_0 , using the small amplitude approximation $r = R_0 + x$ with x being much smaller than R_0 . (d) What are the vibration frequency and the vibration energy of the molecule? Assume the masses of K and Cl are m_1 and m_2 respectively. (20%)
3. A heat engine operates using the cycle shown in the figure. Process $b \rightarrow c$ is adiabatic. If $p_b/p_a = 3.00$ and $\gamma = C_p/C_v = 1.40$, find (a) V_c/V_a (5%) and (b) the efficiency of the engine (must be a number) (10%).



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4. A long coaxial cable consists of an inner cylindrical conductor of radius a and an outer coaxial cylindrical conductor with inner radius b and outer radius c . The space between the two conductors is filled with nonmagnetic insulating material of dielectric constant K . Calculate (a) the capacitance per unit length and (b) the inductance per unit length. The cable is used as a lossless transmission line. (c) What is the characteristic impedance? (15%)

5. A source of sinusoidal electromagnetic waves radiates uniformly in all directions with an average power of 50.0 kW and frequency 100 MHz. At a distance 20.0 km from the source there is a circular loop of wire with diameter 0.150 m. (a) Calculate the amplitudes of the electric and magnetic fields at the location of the loop. (b) If the loop is perpendicular to the direction of the radiation's magnetic field, what is the maximum electromotive force induced in the wire? (20%)

($c = 3.00 \times 10^8$ m/s, $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, $\epsilon_0 = 8.85 \times 10^{-12}$ C²/N·m².)

6. An R - L - C series circuit draws an average power of 220 W from a 110-V (rms) 60.0 Hz ac line. The rms current is 2.50 A and the source voltage leads the current. (a) What is the net resistance of the circuit? (b) In order to make the whole circuit have a power factor of unity, do you connect a series capacitor or inductor to the original circuit? What value is the capacitance or inductance added? (c) What power will then be drawn from the supply line? (15%)

考試科目	近代物理	所別	應用物理研究所 (8162, 8163)	考試時間	2月26日(六)第四節
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1. [40 points] Short answer questions

- [8] (a) True or False? Justify your answers.
- (i) (T/F) X-rays travel faster than microwaves.
 - (ii) (T/F) An x-ray photon has more energy than a microwave photon.
 - (iii) (T/F) The wave equation is Galilean invariant.
 - (iv) (T/F) The kinetic energy of a particle is Lorentz invariant.
- [4] (b) Assume that a photon is scattered by an electron initially at rest. Which photon scattering angle corresponds to the largest Compton shift?
- [6] (c) Describe briefly the Stern-Gerlach experiment for the measurement of the magnetic moments of atoms, and discuss the significance of the results obtained.
- [6] (d) An electron in a hydrogen atom jumps from the $n = 5$ to $n = 3$ level. Is a photon absorbed or emitted in this process? Would this photon energy be enough to ionize a hydrogen atom in which the electron was in the $n = 6$ level? Explain.
- [4] (e) Why is the ground state of the hydrogen atom not split into two sublevels by spin-orbit coupling?
- [4] (f) Consider a two-particle wavefunction given as
- $$\psi_{ab}(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_a(x_1)\psi_b(x_2) + \psi_a(x_2)\psi_b(x_1)].$$
- Does the wavefunction display the exchange symmetry of bosons or fermions?
- [4] (g) Write out the ground state electron configuration for phosphorus ^{15}P (with atomic number $Z = 15$).
- [4] (h) What was the 2010 Nobel Prize in Physics awarded for?

2. [12 points] Spacetime and causality

In one inertial frame S , event A occurs before event B . An observer in S measures these events to be separated by a time interval Δt_{AB} and a distance Δx_{AB} .

- [4] (a) What are the conditions on Δt_{AB} and Δx_{AB} such that event A will be observed to occur before event B in all inertial frames, no matter what?
- [8] (b) Suppose A and B both occur on the x -axis (i.e. $y = z = 0$) in frame S . Consider an inertial frame S' moving at a speed v along the $+x$ direction relative to S .
- (i) Under what conditions would the two events be simultaneous in S' ?
 - (ii) Under what conditions would the two events occur at the same point in space in S' ?

請注意：背面還有試題。

考試科目	近代物理	所別	應用物理研究所 (8162, 8163)	考試時間	2月26日(六)第四節
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3. [18 points] **Schrödinger's theory of quantum mechanics**

Consider a particle confined in a harmonic potential described initially (at $t = 0$) by a wavefunction given as

$$\psi(x, t = 0) = \frac{\sqrt{3}}{2}\varphi_1(x) - \frac{1}{2}\varphi_2(x),$$

where $\varphi_n(x)$ are the normalized energy eigenfunctions, corresponding to the eigenvalues $E_n = \hbar(n + 1/2)$.

- [9] (a) Calculate the expectation value of the energy for the particle at $t = 0$.
 [9] (b) Find $\psi(x, t)$ at any later time $t > 0$.

4. [20 points] **Scattering by a step potential**

A beam of particles, each of mass m and energy $E > V_0$, is incident from the left on a step potential,

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x \geq 0, \end{cases}$$

where $V_0 > 0$.

- [6] (a) Write down the solutions in the regions $x < 0$ and $x > 0$ to the stationary Schrödinger equation. State the continuity conditions that must be satisfied by the wavefunction at $x = 0$.
 [9] (b) Calculate the fraction of particles in the incident beam that are reflected by this potential, i.e. find the reflection coefficient R .
 [5] (c) Find the transmission coefficient T .

5. [10 points] **LS coupling**

Find the possible values of the total angular-momentum quantum number j under LS coupling of two atomic electrons whose orbital quantum number are $\ell_1 = 1$ and $\ell_2 = 2$.