

3. Considering that one-dimensional motion of a particle is governed by the following nonlinear differential equation:

$$\dot{x} = \sin x \quad (\text{EQ.2})$$

where x is the position of this particle and $\dot{x}$ is the time-derivative of its position, i.e. the velocity.

- (a) Suppose that $x(t = 0) = x_0$, please find the position function $x(t)$ for this particle. (10%) (Hint: $\int \csc u du = -\ln|\csc u + \cot u| + C$ and $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)
- (b) Assuming $x_0 = \pi/4$, please plot the solution $x(t)$ qualitatively. What happens as $t \rightarrow \infty$? (10%)
- (c) Now, let's consider another thinking way for (EQ.2). Please plot (EQ.2) in the so-called phase plane, that is the position as the abscissa and the velocity as the ordinate or the $x-\dot{x}$ plane, and draw the moving directions of this imaginary particle everywhere by arrows. (10%)
- (d) For an arbitrary initial condition x_0 , what is the behavior of $x(t)$ as $t \rightarrow \infty$? (10%) (Hint: Consider those points $x = n\pi$, n is an integer)
- (e) For any one-dimensional system $\dot{x} = f(x)$ as shown in Fig. 2, could you describe the general behavior of the particle motion qualitatively as the evolution of time? (10%)

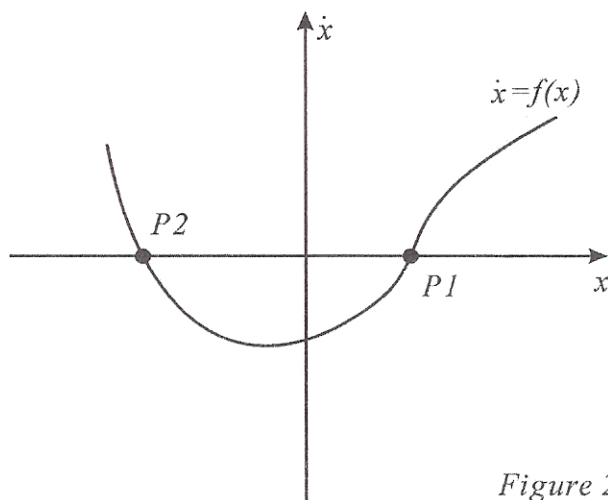


Figure 2