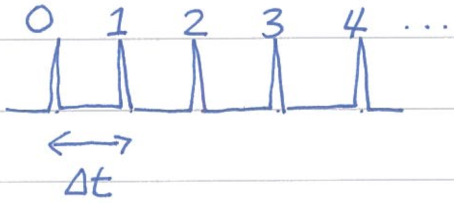
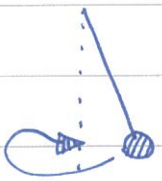
HH0081 What is time?

Time can be a very abstract concept. But, let's drop all philosophical discussions because Science is about DOABLE!

(0) Time

Time — Time is a convenient notion when comparing generic dynamics to "periodic" dynamics.





$$T = 2\pi\sqrt{\frac{l}{g}}$$

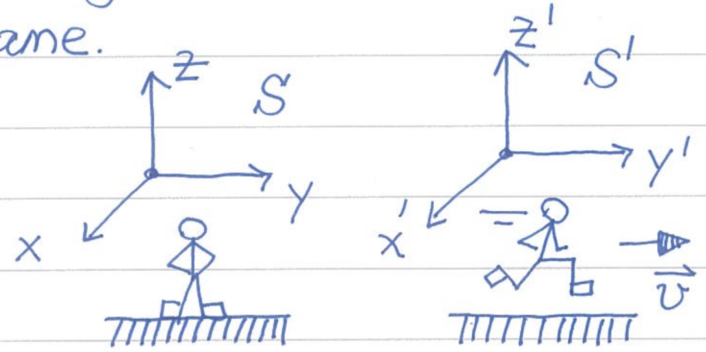
simple pendulum

We may use a simple pendulum to define the unit of time, or some other reliable means like atomic oscillations. We believe that the time series $0, \Delta t, 2\Delta t, 3\Delta t, 4\Delta t, \ldots$ are uniformly distributed because we have to believe so ☺

(1) Inertia frames.

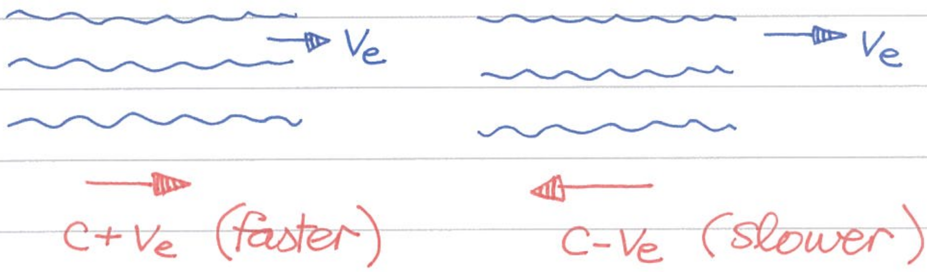
Newton's first law states that an object free from any force should maintain its motion status — at rest or at constant velocity. The definition of an inertia frame is simple — Newton's first law is valid in an inertia frame.

Once you find an inertia frame S, any other frame S' moving at constant relative velocity $\vec{v}$ is also an inertia frame.



All laws should be invariant in all inertia frames!

But, there is some confusion about electromagnetism. EM waves were assumed to propagate through medium called "ether." Assume the velocity of ether is V_e,

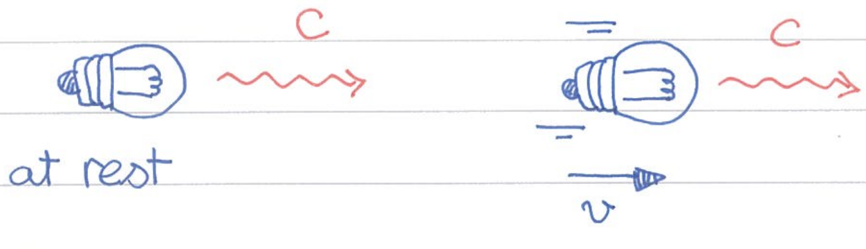


The speed of light would depends on which inertia frame we are in ... And, a special inertia frame, where ether is at rest, exists with the speed of light equal to c. It means Maxwell equations only hold in this special inertia frame. No experimental support for such thing and scientists were very confused.

Ⅱ Einstein's special relativity. To resolve the confusion, Einstein put forth the following postulates:

① The law of physics has the same form in all inertia reference frames.
② The speed of light in vacuum is constant.

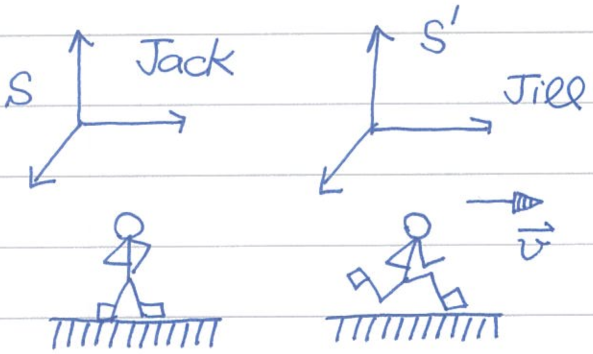
Beautifully, Einstein's special relativity explains all data without any conflict and changes our common intuitions about space and time



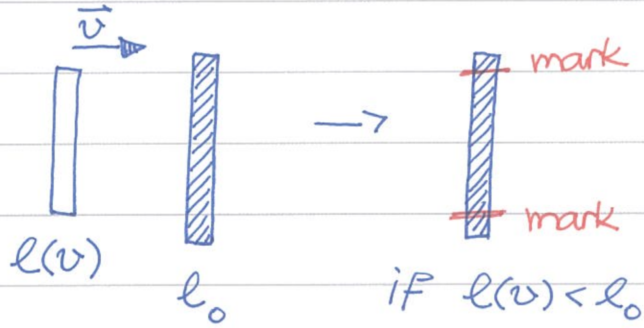
The constant c postulate is quite "weird"

Let me show you two "weird" phenomena: time dilation and length contraction.

Consider a twin, Jack and Jill, moving at a relative velocity $\vec{v} = (v, 0, 0)$. Assume both are in inertia frames.



We would like to show that the transverse length is invariant in both S, S' frames.



If the moving ruler is shorter, $\ell(v) < \ell_0$ in Jack's frame.
But, in Jill's frame,
she would draw the conclusion that $\ell_0 < \ell(-v)$.

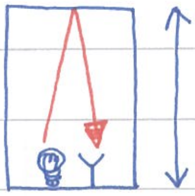
→ $\ell(-v) > \ell_0 > \ell(v)$ violates parity symmetry.

On the other hand, if the moving ruler is longer, we can repeat the same argument which leads to contradiction.
Therefore, the transverse length should be the same

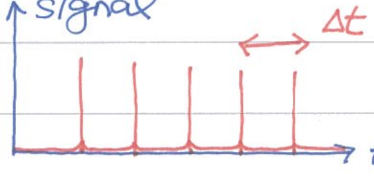
$$\ell(v) = \ell(-v) = \ell_0$$

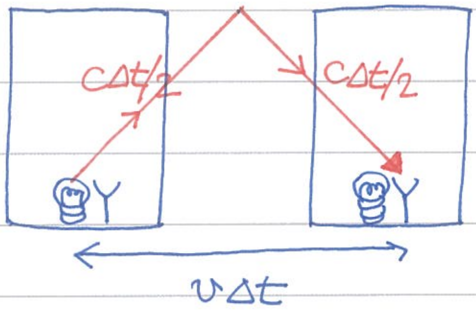
← very important result!

⊘ Time dilation: Construct a light clock as below.

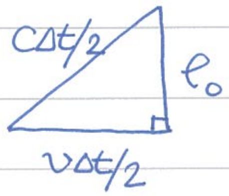


$$\Delta t_0 = \frac{2\ell_0}{c}$$





Now consider a "light clock" moving at constant velocity v. Because the transverse length is still ℓ_0,



$$(c\Delta t/2)^2 = (v\Delta t/2)^2 + \ell_0^2$$

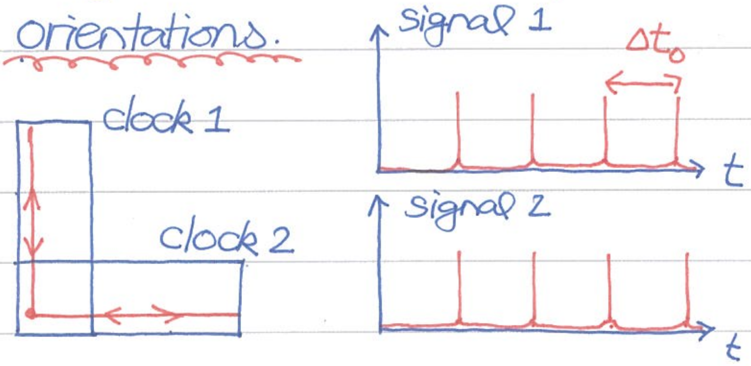
$$\rightarrow \Delta t = \frac{2\ell_0}{\sqrt{c^2 - v^2}} \neq \frac{2\ell_0}{c} \text{ anymore!}$$

Compare with light clock at rest,

$$\boxed{\Delta t = \frac{1}{\sqrt{1 - v^2/c^2}}\,\Delta t_0 = \gamma\,\Delta t_0}, \quad \text{where } \gamma = \gamma(v) \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

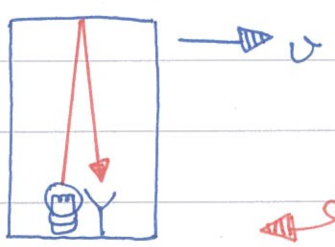
A moving clock appears to walk more slowly than a clock at rest. Time is no longer absolute!

⊘ Length contraction: From time dilation, one can derive length contraction. Compare two light clocks in different orientations.

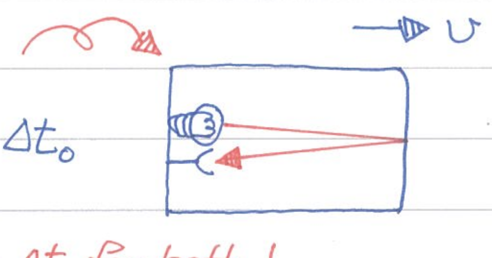


Because the signals are synchronized in the same location, they remain synchronized in all inertia frames

That is to say, moving light clocks in different orientations tick at the same rate.

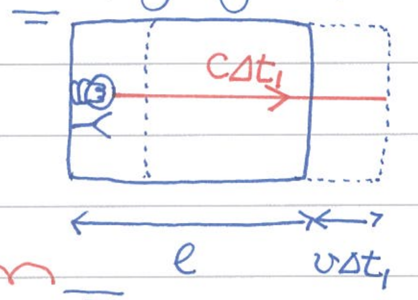


$$\Delta t = \frac{1}{\sqrt{1 - v^2/c^2}}\,\Delta t_0$$



the same Δt for both!

Let us analyze the light propagation in the moving light clock.

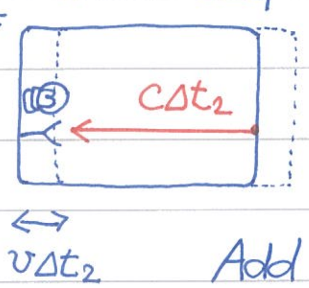


$$\ell + v\Delta t_1 = c\Delta t_1$$

$$\rightarrow \quad \Delta t_1 = \frac{\ell}{c-v}$$

$\Delta t = \Delta t_1 + \Delta t_2$

but $\Delta t_1 \neq \Delta t_2$



$$\ell - v\Delta t_2 = c\Delta t_2$$

$$\rightarrow \quad \Delta t_2 = \frac{\ell}{c+v}$$

Add Δt_1 and Δt_2 together, the total travel time is $\Delta t = \Delta t_1 + \Delta t_2$,

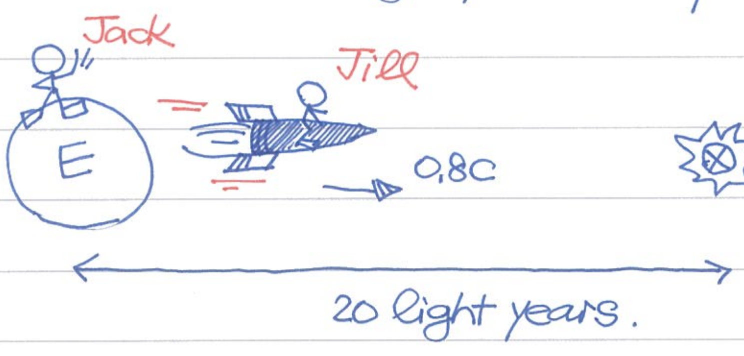
$$\Delta t = \frac{\ell}{c-v} + \frac{\ell}{c+v} = \frac{2\ell}{c}\,\frac{1}{1-v^2/c^2} \quad \leftarrow \text{equals } \gamma\Delta t_0$$

Thus, we can find the relation between ℓ and ℓ_0

$$\frac{2\ell}{c}\,\frac{1}{1-v^2/c^2} = \frac{1}{\sqrt{1-v^2/c^2}}\cdot\frac{2\ell_0}{c} \quad \rightarrow \quad \boxed{\ell = \ell_0\sqrt{1-v^2/c^2}}$$

The length of the moving light clock appears to be shorter than that of the clock at rest. Now length is not absolute either in special relativity!

⊘ Twin paradox: Consider the twins, Jack and Jill, both at age 20. Jack stays on Earth but Jill flies to the X star 20 light years away at the speed $v = 0.8c$.

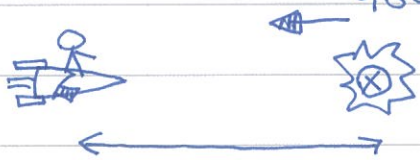


Let us analyze the trip in Jack & Jill's perspectives.

Jack's view: The trip for one way takes $\frac{20}{0.8} = 25$ years → The round trip needs 50 yrs.

But, due to time dilation, Jill's clock is slower and she just needs $50 \cdot \sqrt{1 - v^2/c^2} = 30$ yrs to complete the trip. Upon return, Jack is 70 years old but Jill is only 50 years old.

Jill's view: The X star is flying toward at velocity $-0.8c$.



Due to length contraction, the distance appears to be shorter $20\sqrt{1 - v^2/c^2} = 12$ light years. $\frac{12}{0.8} + \frac{12}{0.8} = 30$ yrs

$\ell = \ell_0 \cdot \gamma = 12$ light years

Jill finds out that she needs 30 years to complete the round trip. But, what about Jack? In Jill's view, Jack is moving and time dilation should slow down his clock.

Jack's aging $= 30 \cdot \sqrt{1 - v^2/c^2} = 18$ years ?

If so, Jack would be younger.... The aging consequences are different from Jack or Jill's views. So, who is right? Jack is right because he stays in the same inertia frame. Jill cannot explain the whole story by special relativity. She needs to learn the general theory of relativity.

清大東院
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