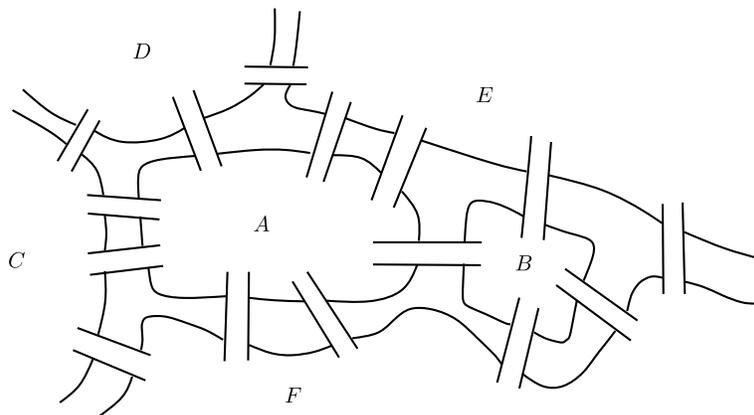


**Homework Assignment No. 5**  
**Due 5:00pm, June 9, 2021**

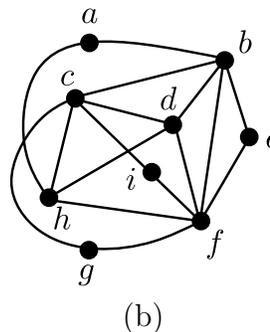
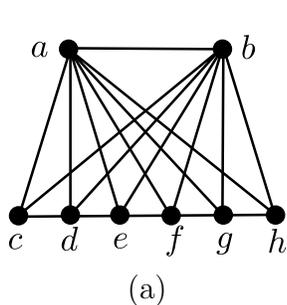
**Reading:** Grimaldi: Sections 11.3 Vertex Degree: Euler Trails and Circuits (after Example 11.12), 11.4 Planar Graphs (up to Example 11.22), 12.1 Definitions, Properties, and Examples, 12.2 Rooted Trees, 12.3 Trees and Sorting.

**Problems for Solution:**

- In his original paper on graphs Euler described an imaginary town with islands and bridges as shown in the accompanying figure.



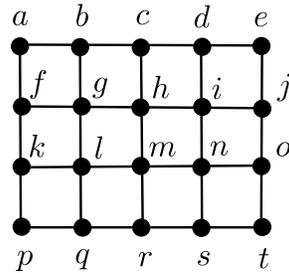
- Is there an Euler circuit in this town? Find one or explain why none exists.
  - Is there an Euler trail in this town? Find one or explain why none exists.
- Determine whether the given graph is planar. Give a planar embedding of the graph or provide an argument that none exists.



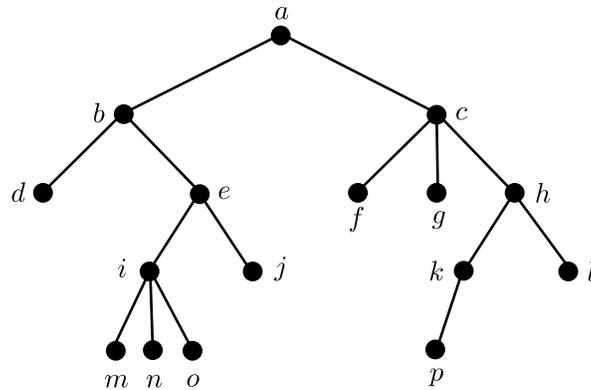
- (a) Show that in any connected planar simple graph with  $v$  vertices and  $e$  edges, where  $e > 2$ ,

$$e \leq 3v - 6.$$

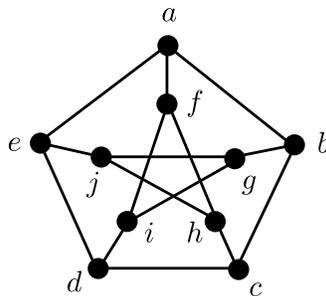
- (b) Use (a) to show that in a connected simple graph  $G$ , if all vertices have degree at least 6, then  $G$  is nonplanar.
4. If  $G = (V, E)$  is a graph (simple graph or multigraph) with  $|V| \geq 3$ , we say that  $G$  has a *Hamilton cycle* if there is a cycle in  $G$  that contains every vertex in  $V$ . Find a Hamilton cycle in the following graph.



5. (a) Show that if a connected simple graph  $G$  has 12 vertices and 12 edges, then it contains at least one cycle.
- (b) Let  $T$  be a complete  $m$ -ary tree with  $n$  vertices and  $l$  leaves. Show that  $l = \lceil (m-1)n + 1 \rceil / m$ .
6. For the rooted tree shown below, list the vertices according to a preorder traversal, a postorder traversal, and an inorder traversal.



7. Consider the Petersen graph shown below. (In this problem, please follow the alphabetical order of vertices in case of a tie.)



- (a) Find the depth-first spanning tree rooted at vertex  $a$ . What is the height of the tree?
  - (b) Find the breadth-first spanning tree rooted at vertex  $a$ . What is the height of the tree?
8. Apply merge sort to each of the following lists. Draw the splitting and merging trees for each application of the procedure.
- (a) 4, 1, 2, 5, 10, 8, 7, 9, 6, 3.
  - (b) 24, 13, 20, 7, 16, 14, 6, 15, 1, 19, 12, 8.

**Homework Collaboration Policy:** You can discuss the homework problems with any number of students currently taking the course, the teaching assistants, and the instructor. However, solutions should not be exchanged. You should make sure that you understand what you turn in, and should of course write up every word of the solution by yourself. It is OK to compare your final answer with others currently enrolled in the course, but you should fix up any error by your own effort. If these sentences are still vague, just tell yourself “*I shall not take unfair advantage of any other student*” and this should answer other policy-related questions you have in your mind. Late homework is subject to a penalty of 5% to 40% of your total points.