EECS 2060 Discrete Mathematics
Spring 2021

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Homework Assignment No. 3
Due 10:10am, April 28, 2021

Reading: Grimaldi: Sections 10.1 The First-Order Linear Recurrence Relation, 10.2 The Second-Order Linear Homogeneous Recurrence Relation with Constant Coefficients, 10.3 The Nonhomogeneous Recurrence Relation, 9.1 Introductory Examples, 9.2 Definition and Examples: Calculational Techniques, 10.4 The Method of Generating Functions.

Problems for Solution:

1. Solve the recurrence relation

$$a_{n+2} + 4a_{n+1} + 8a_n = 0, \qquad n \geq 0$$

with initial conditions $a_0 = 0$ and $a_1 = 2$.

2. Solve the recurrence relation

$$a_{n+2} - 6a_{n+1} + 9a_n = 3 \cdot 2^n + 7 \cdot 3^n, \qquad n \geq 0$$

with initial conditions $a_0 = 1$ and $a_1 = 4$.

3. In this problem the recurrence relation will be used to find a formula for $a_n =$ the sum of the first n cubes. That is, $a_1 = 1^3$, $a_2 = 1^3 + 2^3$, $a_3 = 1^3 + 2^3 + 3^3, \ldots$. Find the recurrence relation that a_n satisfies (with appropriate initial condition) and then solve for it.

4. Suppose your parents would like to get a mortgage (loan) of C dollars from the bank to buy a new house, at an *annual* interest rate r for a period of N years. The usual practice is to repay the mortgage in equal *monthly* installments of D dollars each. You, as a student of EECS 2060, should be able to compute the value of D, which is a function of C, r, and N, for your parents. Please find the value of D. (*Hint:* An annual interest rate r is equivalent to a monthly interest rate $r/12$, and currently r is around 1.3% to 1.8% for mortgage in Taiwan. There will be a total of $12N$ monthly installments for a period of N years, and N is typically 20 or 30 now in Taiwan. Let a_n represent the *unpaid balance* after n monthly payments have been made. Then just before the $(n+1)$th payment, the new balance will be $(1 + r/12) \cdot a_n$, and just after the $(n+1)$th payment the unpaid balance will be $(1+r/12)a_n - D$. Thus the sequence $\{a_n\}$ satisfies the recurrence relation: $a_{n+1} = (1 + r/12)a_n - D$.)

5. Use the generating function method to solve the recurrence relation

$$a_n - a_{n-1} - 2a_{n-2} = 2^n, \quad n \geq 2$$

with initial conditions $a_0 = 4$ and $a_1 = 12$.

6. Let F_n, $n \geq 0$, be the Fibonacci numbers. The *Lucas numbers* L_n can be defined by

$$L_n = F_{n+1} + F_{n-1}, \quad \text{for } n \geq 1$$

with $L_0 = 2$. Find the generating function for L_n.

7. Consider the following system of recurrence relations:

$$\begin{aligned} a_n &= -2a_{n-1} - 4b_{n-1} \\ b_n &= 4a_{n-1} + 6b_{n-1} \end{aligned}$$

for $n \geq 1$, with initial conditions $a_0 = 1$ and $b_0 = 0$.

   (a) Find the generating function for a_n and then solve for a_n.

   (b) Do the same for b_n.

8. Consider the system of recurrence relations in Problem 7.

   (a) Find the recurrence relation that a_n satisfies (with appropriate initial conditions).

   (b) Do the same for b_n.

Homework Collaboration Policy: I allow and encourage discussion or collaboration on the homework. However, you are expected to write up your own solution and understand what you turn in. Late homework is subject to a penalty of 5% to 40% of your total points.