94 學年度 電機領域聯合招生 系（所） ____________ 組碩士班入學考試

科目 工程數學 B 科目代碼 9903 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (18%) A fair coin is tossed until two tails occur successively. Find the expected number of the tosses required.

2. (15%) The advantage of a certain blood test is that 90% of the time it is positive for patients having a certain disease. Its disadvantage is that 20% of the time it is also positive in healthy people. In a certain location 30% of the people have the disease, and anybody with a positive blood test is given a drug that cures the disease.
   (a) If 25% of the time the drug produces a characteristic rash, what is the probability that a person from this location who has the rash had the disease in the first place?
   (b) What is the probability that a person who had the disease but was not given a drug?

3. (17%) Let X be a uniformly distributed random variable over the interval $(0,1+\theta)$, where $0<\theta<1$ is a given parameter.
   (a) (5%) Find the (cumulative) distribution function of X.
   (b) (6%) Find the probability density function of X^2.
   (c) (6%) Find a function of X, say g(X), so that its expectation E[g(X)] = θ^2.

4. (17 %) Let P_3 be the vector space of all polynomials over real numbers of degree<3.
   Let L be the operator on P_3 defined by
$$L(p(x)) = x\frac{d}{dx}p(x) - p(x)$$
   (a) (4 %)Find the matrix A representing L with respect to the standard basis $\{1, x, x^2\}$ of P_3.
   (b) (4 %)Find the matrix B representing L with respect to the basis $\{1, 1+x, 1+2x+x^2\}$.
   (c) (4 %)Find the matrix S such that $B = S^{-1}AS$.
   (d) (5 %)If $p(x) = a_0 + a_1(1+x) + a_2(1+2x+x^2)$, calculate $L^n(p(x))$ with respect to the basis $\{1, 1+x, 1+2x+x^2\}$.

5. (15%) Let A be an $n\times n$ matrix.
   (a) (7%) Prove that A is diagonalizable if and only if it has n linearly independent (LI) eigenvectors.
   (b) (8%) Please show that if A has n LI eigenvectors and we make these n eigenvectors as the columns of matrix Q, then $Q^{-1}AQ=D$ is diagonal and the jth diagonal element of D is the jth eigenvalue of A.

6. (a) (8%) Let A and B be 4×4 matrices given by $A_{i,j}=i^{j-1}$ and $B_{i,j}=(i^2+i\cdot j+j^3)\delta_{i,j}, 1\le i,j\le 4$. What is the trace of ABA^{-1} ? (For your own good, please circle your answer after you have finished this part of problem.)
   (b) (10%) Let A be an $n\times n$ matrix with entries from the field of complex numbers. If the inner product $[Ax,x]$ of Ax and x is nonnegative for all $x\in C^n$, where $[Ax,x] = \bar{x}Ax$, where $\bar{x}$ is the complex conjugate of x, show that A is Hermitian. (Please be aware that parts (a) and (b) are not related.)