科目 線性代數 類組別 032 共 1 頁第 1 頁 *請在試卷答案卷（卡）內作答

Problem 1 (30%) Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$.

(a) Find the minimal polynomial of A.

(b) Find a Jordan canonical form J for A.

(c) Find an invertible matrix $Q \in M_{4\times4}(R)$ such that $Q^{-1}AQ = J$.

Problem 2 (20%) Let A and B be two $n \times n$ matrices over a field F. Show that the characteristic polynomials of AB and BA are the same.

Problem 3 (20%) State and prove the Cramer's rule.

Problem 4 (10%) Can $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 6 \\ 1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 4 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ be the matrices of the same linear transformation in two different bases? Show your reason.

Problem 5 (20%) If V is a finite-dimensional inner product space, prove that if W is a subspace of V, then $V = W \oplus W^{\perp}$.