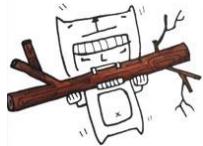


Nov. 12, 2012



EE214000 Electromagnetics, Fall
 Quiz #6, Open books, notes (21 points)

1. (3 points) What is the energy density stored in a thin parallel-plate capacitor with an electrode separation of d and a voltage of V across the two capacitor electrodes? Assume the permittivity of the dielectric between the electrodes is ϵ .

Ans: The electric field across a thin parallel-plate capacitor is $E = V/d$. The energy

$$\text{density is } \frac{\epsilon}{2} E^2.$$

2. (3 points) In “1”, the surface area of the electrode plate is S . Calculate the capacitance of the parallel-plate capacitor from the stored energy.

$$\text{Ans: the total stored energy is } \frac{\epsilon}{2} E^2 S d = \frac{1}{2} C V^2$$

$$\Rightarrow C = \frac{\epsilon S}{d}$$

3. (5 points) In “1”, the top electrode is biased at V and the bottom electrode is grounded. What is the surface polarization charge density right below the top electrode?

$$\text{Ans: } \vec{P} = \vec{D} - \epsilon_0 \vec{E} = (\epsilon - \epsilon_0) \vec{E} = (\epsilon - \epsilon_0) \frac{V}{d} (-\hat{a}_y)$$

$$\text{The surface polarization charge density is } \hat{a}_n \cdot \vec{P} = \hat{a}_y \cdot \vec{P} = -(\epsilon - \epsilon_0) \frac{V}{d}$$

4. (5 points) Calculate the capacitance per unit length of a coaxial transmission line with an inner conductor radius of R_i and outer conductor radius of R_o . Assume the permittivity of the dielectric between the two conductors is ϵ .

Ans: Assume a line charge density ρ_l on the inner conductor. The electric field at r is

$$\vec{E} = \frac{\rho_l}{2\pi r \epsilon_0} \hat{a}_r. \text{ The voltage across the two conductors is therefore}$$

$$V = - \int_{R_o}^{R_i} (\vec{E} = \frac{\rho_l}{2\pi r \epsilon_0} \hat{a}_r) \cdot \hat{a}_r dr = \frac{\rho_l}{2\pi \epsilon_0} \ln(R_o / R_i). \text{ The capacitance per unit length is}$$

$$C_l = \frac{\rho_l}{V} = \frac{2\pi\epsilon_0}{\ln(R_o/R_i)}$$