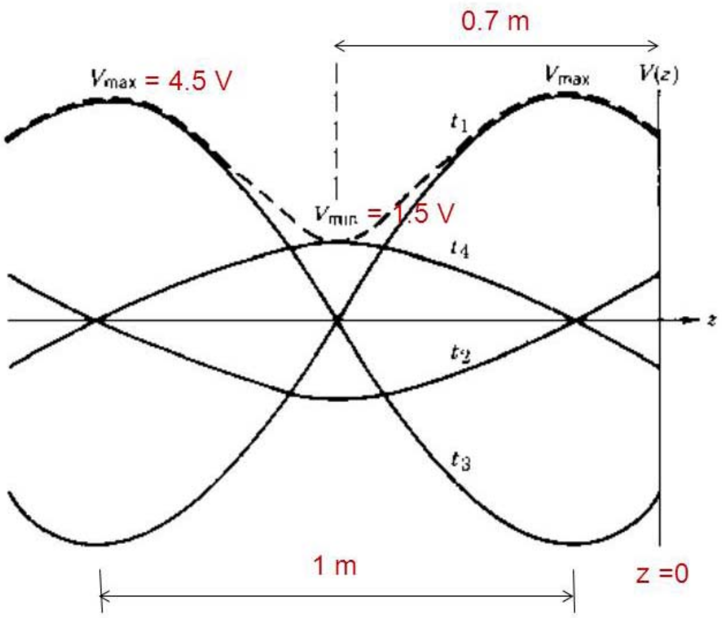
*Prof. Yen-Chieh Huang*
*Dept of Electrical Engineering*
*National Tsing-Hua University*
*office: Delta 856, HOPE 301*
*ext: 62340*
*email:ychuang@ee.nthu.edu.tw*


---

Oct. 29, 2012

EE214000 Electromagnetics, Fall 2012
Homework Assignment # 3: work out the solutions of the following midterm-exam questions
Due in class Nov. 7, 2012

1. (40%) Refer to the following voltage versus time plot on a transmission line with a characteristic impedance of 50 Ω. The maximum and minimum voltages on the standing wave envelope are 4.5 and 1.5 V, respectively. The first minimum voltage appears at z = –0.7 m, and the separation of the two adjacent voltage peaks is 1 m. The transmission line is terminated with a load impedance $Z_L$ (at $z = 0$). (1) What is the wavelength of the voltage signal on this transmission line?(3%) (2) What is the standing wave ratio on this line? (3%) (3) What is the reflection coefficient at the load, including its magnitude and phase? (6%) (4) What is the load impedance? (5%) (5) What are the locations of the first maximum and minimum currents from the load? (6%) (6) What are the values of the maximum and minimum currents on the line? (6%) (7) What is the input impedance at $z = -1$ m? (3%) (8) What is the maximum value of the total voltage measured at the load z = 0? (3%) (9) What is the average power delivered to the load? (5%)




Hints: $\hat{V}(z) = V_0^+ e^{-j\beta z} + V_0^+ |\Gamma| e^{j(\theta_\Gamma + \beta z)}) = V_0^+ e^{-j\beta z}(1 + |\Gamma| e^{j(\theta_\Gamma + 2\beta z)})$ ,

$\hat{I}(z) = I_0^+ e^{-j\beta z}(1 - |\Gamma| e^{j(\theta_\Gamma + 2\beta z)})$, $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$,

$S = \dfrac{|\hat{V}|_{\max}}{|\hat{V}|_{\min}} = \dfrac{|\hat{I}|_{\max}}{|\hat{I}|_{\min}} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$, $Z_i(-l) = Z_0 \dfrac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}$

2. (13%) A lossless 50Ω transmission line is terminated in a load with $Z_L$ = (50 – $j$100) Ω, mark on the Smith chart to find the following
   (1) The reflection coefficient Γ at the load, (2%) (2) the SWR circle on the chart, (1%) (3) the standing wave ratio, (2%) (4) the input impedance at 0.137λ from the load, (2%) (5) the input admittance at 0.137λ from the load, (2%) (6) the shortest length of the line from the load for which the input impedance is purely resistive (2%), (7) the position of the first voltage minimum from the load. (2%)

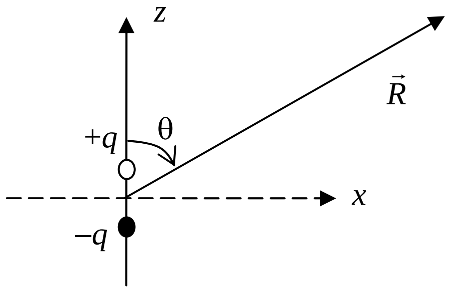
3. (10%) If the electric potential of a charge $q$, is defined to be the external work to bring a unit positive charge from the infinity to a distance $R$ from the charge $q$, prove that a minus sign has to be put in front of $\nabla V$ to form the relationship $\vec{E} = -\nabla V$ between the electric field $E$ and the electric potential $V$ associated with the charge $q$. Give explanations to the formulas and equations used in your answer.

4. (12%) A point charge Q is located at the origin of a spherical coordinate system in vacuum. (1) Use Gauss's law to calculate the electric field intensity at the position $(R_0, \theta_0, \phi_0)$. Don't forget to give the direction and magnitude of the electric field vector. (2) Calculate the work needed to take a unit positive charge from $R = \infty$ to the location $(R_0, \theta_0, \phi_0)$. (3) Calculate the work needed to take a charge $q$ from $(R_0, \theta = \pi/4, \phi = 0)$ to $(R_0, \theta = \pi/2, \phi = 0)$. (4) Calculate the work needed to take a charge $q$ from $(R_0, \theta = \pi/4, \phi = 0)$ to $(R_1, \theta = \pi/2, \phi = 0)$.

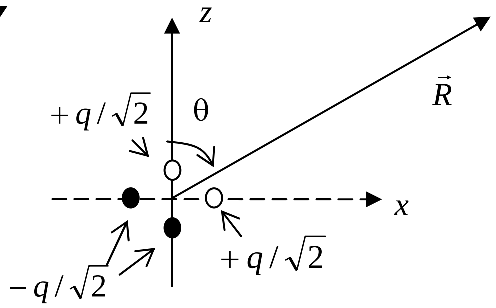
5. (25%) An electric dipole consists of two opposite charges +$q$ (open dot) and −$q$

located at $z = \pm d/2$ as shown below in (a). In the far zone $R >> d$, the electric potential was calculated to be $V = \frac{\vec{p} \cdot \hat{a}_R}{4\pi\varepsilon_0 R^2}$ in the class. (1) How is the electric dipole moment $\vec{p}$ defined? Give the magnitude and direction of it. (4%) (2) Derive and draw the equi-potential lines on the *x*-*z* plane. (6%) (3) How do you draw the electric field lines on the same plane without specifically deriving the electric dipole field? Give a physical reason for your drawing. (3%)

An electric field is generated from two electric dipoles arranged in (b), consisting of two positive charges $+q/\sqrt{2}$ located at $x = +d/2, z = +d/2$ and two negative charge of $-q/\sqrt{2}$ located at $x = -d/2, z = -d/2$. (4) Give an expression for the far zone (*R* >> *d*) electric potential. (5%) (5) Draw the equi-potential lines in the *x*-*z* plane. (4%) (6) Draw the electric field lines in the *x*-*z* plane. (3%)




(a)




(b)