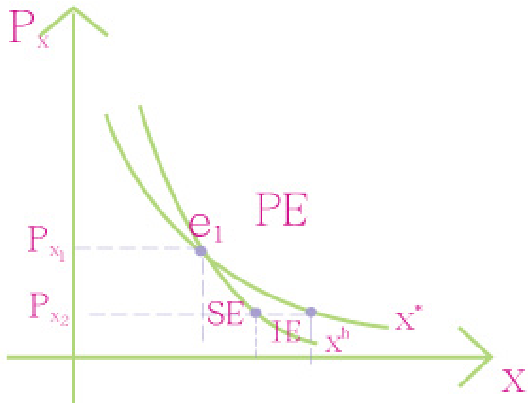
個 體 經 濟 學 一

Microeconomics (I)

Ch 5 More on Consumer Theory

＊Compensated demand functions

EX:



Ｆigure65：

$$\left.\begin{array}{ll}\min_{x,y}: & P_x x+P_y y \\ \text{s.t.} & u(x,y)=u\end{array}\right\} \text{Foc=>} \ x^h, y^h$$

Foc $\quad MRS_{xy}=\frac{Px}{Py} \quad$ ①

$\quad u(x,y)=u \quad$ ②

①=> $MRS_{xy}=\frac{Mu_x}{Mu_y}=(\frac{\alpha}{\beta}\frac{y}{x}=\frac{Px}{Py})$ ， $\beta P_x x=\alpha P_y y$ ， $P_y y=\frac{\beta}{\alpha}P_x x$

$y=\frac{\beta}{\alpha}\frac{Px}{Py}x \quad$ ①’

②=> $x^{\alpha}y^{\beta}=u$

①’⟶②

$x^{\alpha}(\frac{\beta}{\alpha}\frac{Px}{Py})^{\beta}x^{\beta}=u$

$x^{\alpha+\beta}(\frac{\beta}{\alpha}\frac{Px}{Py})^{\beta}=u$

$u(x,y)=x^{\alpha}y^{\beta}$

$x^*=\frac{\alpha}{\alpha+\beta}\frac{m}{P_x}$, $y^*=\frac{\beta}{\alpha+\beta}\frac{m}{P_y}$

$x^{\alpha+\beta} = (\frac{\beta}{\alpha}\frac{Px}{Py})^{\beta}u$

$x^{h} = x(Px, Py, u)$

$u(x^*,y^*)=(\frac{\alpha}{\alpha+\beta})^{\alpha}(\frac{m}{P_x})^{\alpha}\cdot(\frac{\beta}{\alpha+\beta})^{\beta}(\frac{m}{P_y})^{\beta}$

$=\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}\frac{m^{\alpha+\beta}}{P_x^{\alpha}P_y^{\beta}}$

$$\begin{cases} x^{h} = (\frac{\alpha}{\beta}\frac{Px}{Py})^{\frac{\beta}{\alpha+\beta}}u^{\frac{1}{\alpha+\beta}} \\ y^{h} = (\frac{\beta}{\alpha}\frac{Px}{Py})^{\frac{\alpha}{\alpha+\beta}}u^{\frac{1}{\alpha+\beta}} \end{cases}$$

max x,y : $u(x,y)$ vs. min x,y: $P_x x+P_y y$ $P_x x^h+P_y y^h$ is s function of P_x, P_y ,u

s.t. $P_x x+P_y y=m$ s.t. $u(x,y)=u$ $e(P_x,P_y,u)= P_x x^h+P_y y^h$

$=>x^*=x(P_x,P_y,m)$ $=> x^h=x(P_x,P_y,u)$ $=P_x\, x(P_x,P_y,u)+ P_y\, y(P_x,P_y,u)$

$y^*=y(P_x,P_y,m)$ $y^h=y(P_x,P_y,u)$ expenditure function 支出函數

$u(x^*,y^*)=u(x(P_x,P_y,m), y(P_x,P_y,m))=v(P_x,P_y,m)$

indirect utility function 間接效用函數

(1)

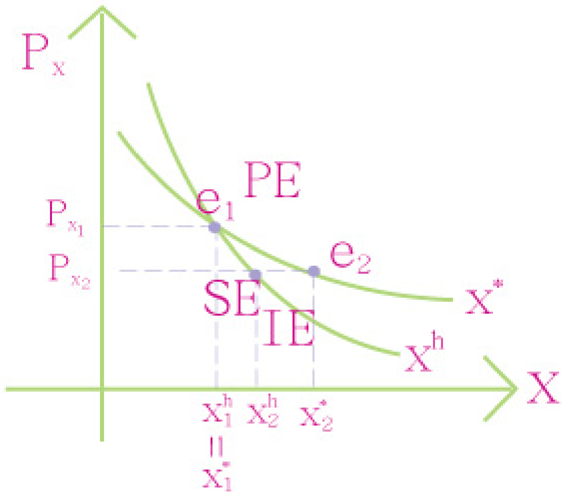


Figure66：Compensated demand functions(Hickian demand functions)

$$[\frac{dx}{dP_x}]_{PE}= [\frac{dx}{dP_x}]_{SE} + [\frac{dx}{dP_x}]_{IE}$$

slope of ordinary demand function

slope of the compensated demand function

$dm=-(dP_x)x$

⇨ $dP_x=\frac{dm}{-x}$

$[\frac{dx}{dP_x}]_{IE} = \frac{dx}{\frac{dm}{-x}} = -x\frac{dx}{dm}$

$[\frac{dx}{dP_x}]_{given\ m} = [\frac{dx}{dP_x}]_{given\ u} - x\frac{dx}{dm}$

$\frac{dx^*}{dP_x} = \frac{dx^h}{dP_x} - x\frac{dx^*}{dP_x}$ slutsky Equation

PE SE IE

$\frac{dx^*}{dP_x}\frac{P_x}{x^*} = \frac{dx^h}{dP_x}\frac{P_x}{h} - x\frac{dx^*}{dm}\frac{P_x}{x^*}$ $x^* = x^h$ at equilibrium

$-\frac{\frac{dx^*}{x^*}}{\frac{dP_x}{P_x}} = -\frac{\frac{dx^h}{x^h}}{\frac{dP_x}{P_x}} + \frac{P_x x}{m}\frac{\frac{dx^*}{x^*}}{\frac{dm}{m}}$ => $\varepsilon^P_{X^*} = \varepsilon^P_{X^h} + e_x\varepsilon^m_{X^*}$

$\varepsilon^P_{X^*} = \varepsilon^P_{X^h} + e_x \cdot \varepsilon^m_x$

(2)

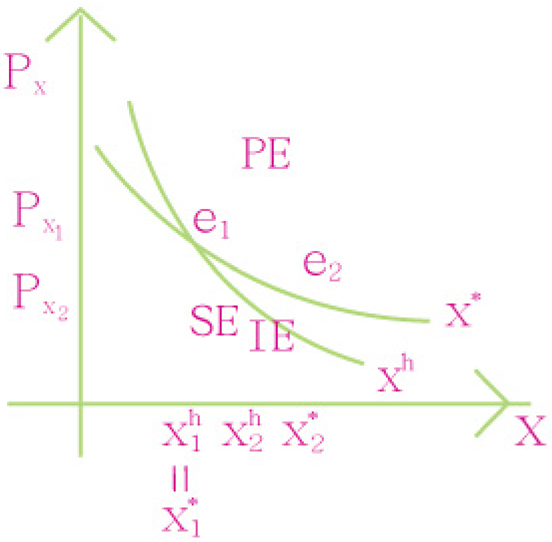


Figure67 :Compensated demand function

$[\frac{dx}{dP_x}]_{PE} = [\frac{dx}{dP_x}]_{SE} + [\frac{dx}{dP_x}]_{IE}$

slope of ordinary demand function

slope of the compensated demand function

$dm = -(dP_x)x$

⇨ $dP_x = \frac{dm}{-x}$

$[\frac{dx}{dP_x}]_{IE} = \frac{dx}{\frac{dm}{-x}} = -x\frac{dx}{dm}$

$[\frac{dx}{dP_x}]_{\text{given m}} = [\frac{dx}{dP_x}]_{\text{given } u} - x\frac{dx}{dm}$

$\frac{dx^*}{dP_x} = \frac{dx^h}{dP_x} - x\frac{dx^*}{dP_x}$ slutsky Equation

PE SE IE

$\frac{dx^*}{dP_x}\frac{P_x}{x^*} = \frac{dx^h}{dP_x}\frac{P_x}{h} - x\frac{dx^*}{dm}\frac{P_x}{x^*}$ $x^* = x^h$ at equilibrium

$-\frac{\frac{dx^*}{x^*}}{\frac{dP_x}{P_x}} = -\frac{\frac{dx^h}{x^h}}{\frac{dP_x}{P_x}} + \frac{P_x x}{m}\frac{\frac{dx^*}{x^*}}{\frac{dm}{m}}$ => $\varepsilon^P_{X^*} = \varepsilon^P_{X^h} + e_x\varepsilon^m_{X^*}$

$\varepsilon^P_{X^*} = \varepsilon^P_{X^h} + e_x \cdot \varepsilon^m_x$

(3)

$\frac{dx^*}{dP_y}$? $\frac{dx^h}{P_y} - y\frac{dx^*}{dm}$

$dm = -(dP_y)y$ => $dP_y = -\frac{dm}{y}$

$[\frac{dx}{dP_y}]_{IE} = \frac{dx}{-\frac{dm}{y}} = -y\frac{dx^*}{dm}$

$\frac{dx^*}{dP_y} = \frac{dx^h}{dP_y} - y\frac{dx^*}{dm}$

PE SE IE

$\frac{dx^*}{dP_y}\frac{P_y}{X^*} = \frac{dx^h}{dP_y}\frac{P_y}{x^h} - y\frac{dx^*}{dm}\frac{P_y}{x^*}$

$-\frac{\frac{dx^*}{x^*}}{\frac{dP_y}{P_y}} = -\frac{\frac{dx^h}{x^h}}{\frac{dP_y}{P_y}} + \frac{P_y y}{m}\frac{\frac{dx^*}{x^*}}{\frac{dm}{m}}$

$\varepsilon_{xy} = \varepsilon_{x^h y} + e_y\varepsilon^m_x$ (a)

In previous discussion,

$$\begin{cases} \frac{dx^*}{dP_y} > 0 \Rightarrow X \text{ and } Y \text{ are gross substitutes.} \\ \frac{dx^*}{dP_y} < 0 \Rightarrow X \text{ and } Y \text{ are gross complements.} \end{cases}$$

$$\begin{cases} \frac{dx^h}{dP_y} > 0 \Rightarrow X \text{ and } Y \text{ are net substitutes.} \\ \frac{dx^h}{dP_y} < 0 \Rightarrow X \text{ and } Y \text{ are net complements.} \end{cases}$$

From (a), $\varepsilon_{x^h y} < 0, > 0$ (X *and* Y *are net substitutes/complements*)

and $\varepsilon_x^m < 0, > 0$ (X *is inferior/normal good*)

=> $\varepsilon_{xy} < 0, > 0$ (X and Y are gross substitutes/complements)

compensating variation (CV) 補償變量

Equivalent variation (EV) 等值變量

與消費者滿意度有關

max x,y: $u(x,y)$ } x^*, y^* ordinary demand fct.

s.t. $P_x x + P_y y = m$ } $u(x^*, y^*) = V(P_x, P_y, m)$ indirect utility

市場上看不到，卻有存在而被估計的

min x,y: $P_x x + P_y y$ } x^h, y^h compensated demand fct.

s.t. $u(x,y)$ } $e(P_x, P_y, u) = P_x x^h + P_y y^h$ expenditure fct.

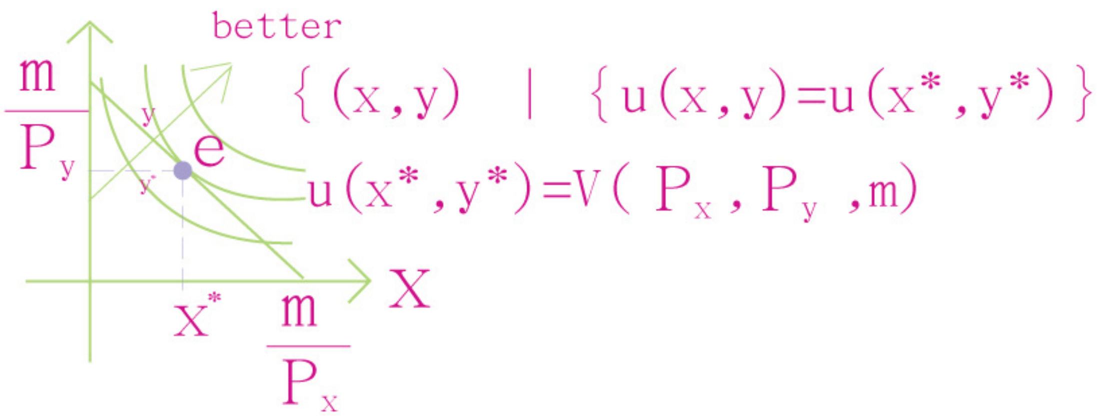


Figure68：

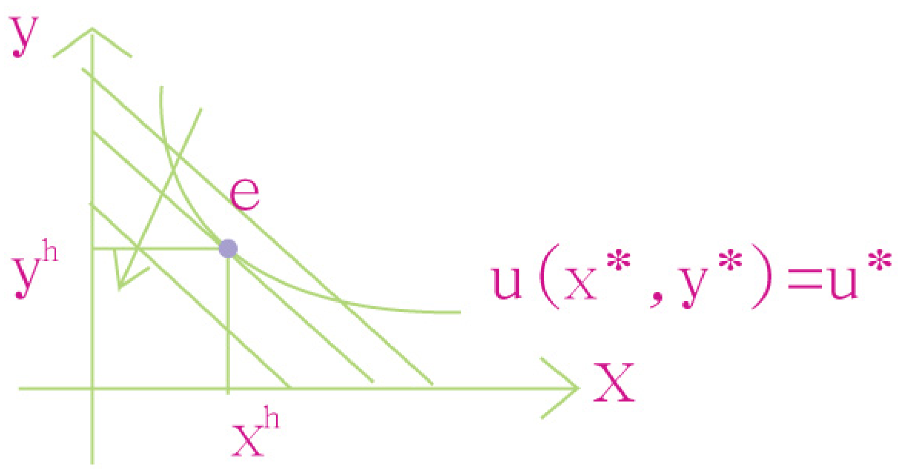


Figure69：

$x^h = x^*$

$y^h = y^*$ 為了滿足均衡而有的結果

e=$P_x x^h + P_y y^h = P_x x^* + P_y y^*$=$m$

⇨ e(P_x ,P_y ,$u(x^*, y^*)$)=m

change in P_x → change in x^*, y^*

→ change in u(x^*, y^*)

suppose ther is a tox on X => P_x↑→x^*, y^* change

how to measure the change in u(x^*, y^*) ?

補貼多少才能回到 original utility?

change in u(x^*, y^*) ~ change in income