個體經濟學二

Microeconomics (II)

Ch9. Cost

∗Short Run Cost Functions:

L: variable — price: w (wage)

K: fixed (=K_0) — price: r (capital using cost per unit)= depreciation + interest

<u>固定成本</u>：

Total fixed cost (TFC) = rK_0 —from "output" point of view

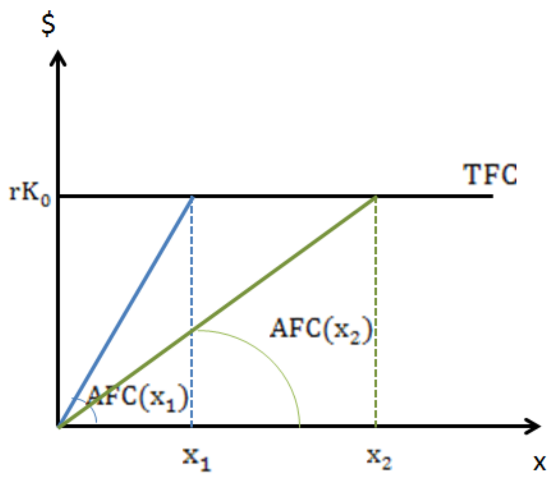
Marginal fixed cost (MFC) $= \frac{d\,TFC}{dx} = 0$

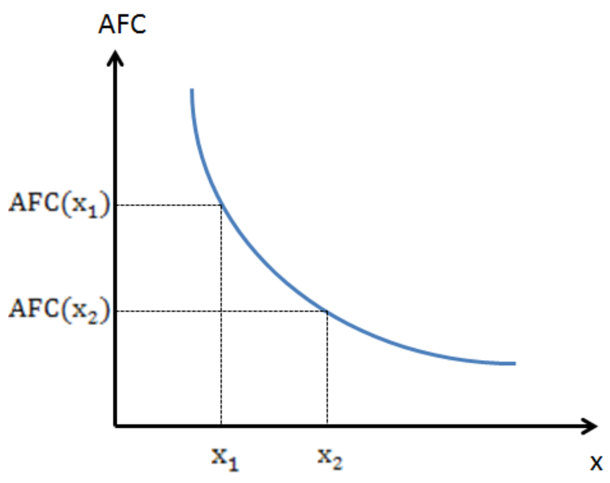
Average fixed cost (AFC) $= \frac{TFC}{x} = \frac{rK_0}{x}$ ↓ with x ↑

AFC is linear? Convex? Or concave?

$$\frac{dAFC}{dx} = (-1)\frac{TFC}{x^2} = (-1)\frac{rK_0}{x^2} < 0 \quad \therefore \text{AFC is decreasing with x}$$

$$\frac{d^2AFC}{dx^2} = (-1)(-2)\frac{TFC}{x^3} = (-1)(-2)\frac{rK_0}{x^3} > 0 \quad \therefore \text{AFC is convex}$$





<u>變動成本</u>:

TVC(x)(Total variable cost) = wL(x)

L(x): Labor requirement (to produce x units of output)

Short run production function:

$x = f(L;\ K_0)$ simply $x = f(L) \leftarrow$ given K is fixed

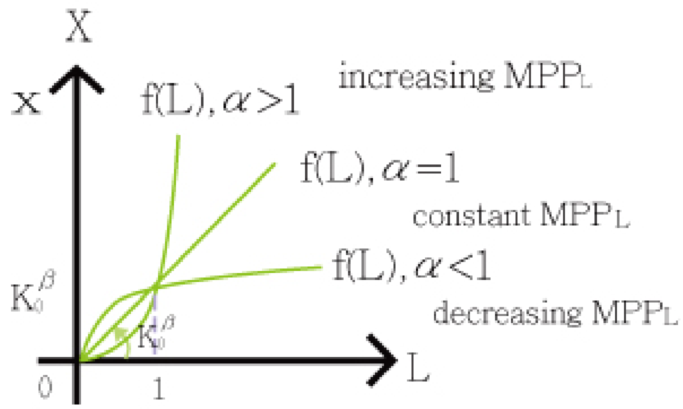
$\therefore f^{-1}(x) = L$

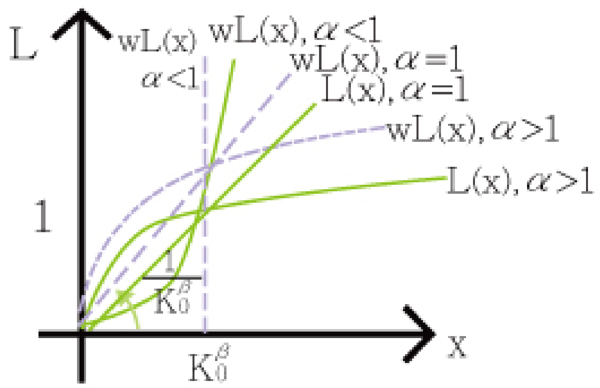
＊ Example : C-D production function

$\mathbf{x = L^{\alpha} K_0^{\beta} = f(L)}$

$L = f^{-1}(x)$

$$L^{\alpha} = \frac{x}{K_0^{\beta}} \Longrightarrow L = \left(\frac{x}{K_0^{\beta}}\right)^{\frac{1}{\alpha}} = L(x), = K_0^{\frac{-\beta}{\alpha}} x^{\frac{1}{\alpha}}，\ TVC = wL(x) = wK_0^{\frac{-\beta}{\alpha}} X^{\frac{1}{\alpha}}$$





∗ Example : L & K are perfect substitutes

$$\mathbf{x = (aL + bK)^2}$$

$K = K_0$ in the SR

$$x = (bK_0 + aL)^2$$

$$L = 0, \qquad x = f(0) = (bK_0)^2 \leftarrow \text{ intercept}$$

$$MPP_L = f'(L) = 2(aL + bK_0) \cdot a > 0$$

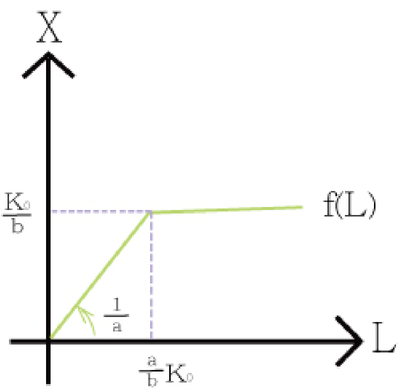
$$\frac{dMPP_L}{dL} = f''(L) = 2a^2 > 0 \;\; convex$$

∗ Example : L & K are perfect complement

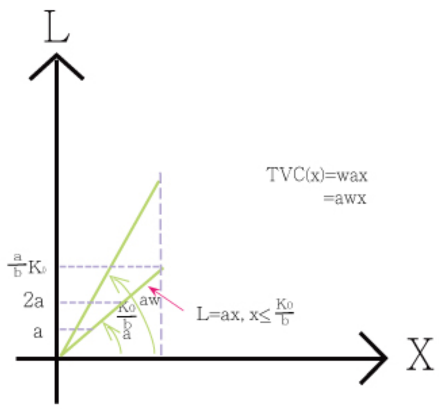
$$\mathbf{x = f(L, K) = \min\left\{\frac{L}{a}, \frac{K}{b}\right\}, \qquad K = K_0}$$

$$x = f(L) = \frac{L}{a} \qquad \text{if } \frac{L}{a} \le \frac{K_0}{b} \text{ or } L \le \frac{a}{b}K_0$$

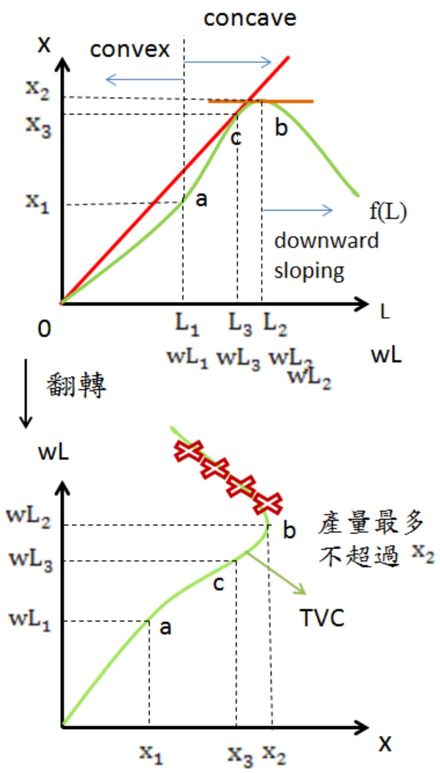
$$= \frac{K_0}{b} \qquad \text{if } \frac{L}{a} > \frac{K_0}{b} \text{ or } L > \frac{a}{b}K_0$$



F igure 59 :



F igure60 :



Short Run Total Cost：$SRTC(x) = TFC + TVC = rK_0 + wL(x)$

Short Run Marginal Cost：

$$SRMC(x) = \frac{\Delta SRTC(x)}{\Delta x} = \frac{\Delta\big(TFC + TVC(x)\big)}{\Delta x} = \frac{\Delta TVC(x)}{\Delta x} \text{ (slope of the TVC)}$$

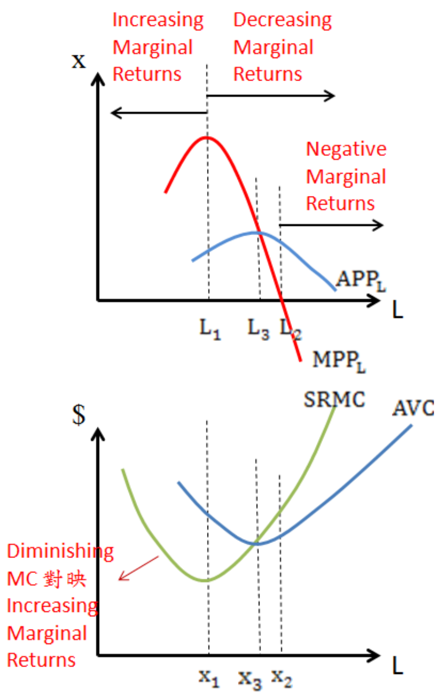
SRAC(x) (Short Run Average Cost)

$$= \frac{SRTC(x)}{x} = \frac{TFC + TVC(x)}{x} = AFC + AVC(x)$$

$0 - L_1$: f(L) convex MPP_L (slope of f(L)) ↑ (increasing marginal return)

⇓ 對應

$0 - x_1$: TVC concave SRMC(slope of TVC) ↓ (diminishing marginal cost)



$L_1 - L_2$: f(L) concave MPP_L ↓

⇓

$x_1 - x_2$: TVC convex SRMC ↑

$0 - L_3$: APP_L ↑, at L_3 $MPP_L = APP_L$ (max)

$0 - x_3$: AVC ↓, at x_3 SRMC = AVC(min)

AVC ↑↓ ? AVC vs SRMC

$$AVC(x) = \frac{TVC(x)}{x}$$

$$\frac{dAVC(x)}{dx} = \frac{d\left(\frac{TVC(x)}{x}\right)}{dx} = \frac{x \cdot \frac{dTVC(x)}{dx} - TVC(x)\frac{dx}{dx}}{x^2} = \frac{x \cdot SRMC(x) - TVC(x)}{x^2}$$

$$= \frac{SRMC(x) - AVC(x)}{x}$$

$$\frac{dAVC(x)}{dx} < 0 \text{ if } SRMC < AVC$$

$$\frac{dAVC(x)}{dx} = 0 \text{ if } SRMC = AVC$$

$$\frac{dAVC(x)}{dx} > 0 \text{ if } SRMC > AVC$$

$\therefore$ AVC(x) min $\Leftrightarrow$ SRMC = AVC

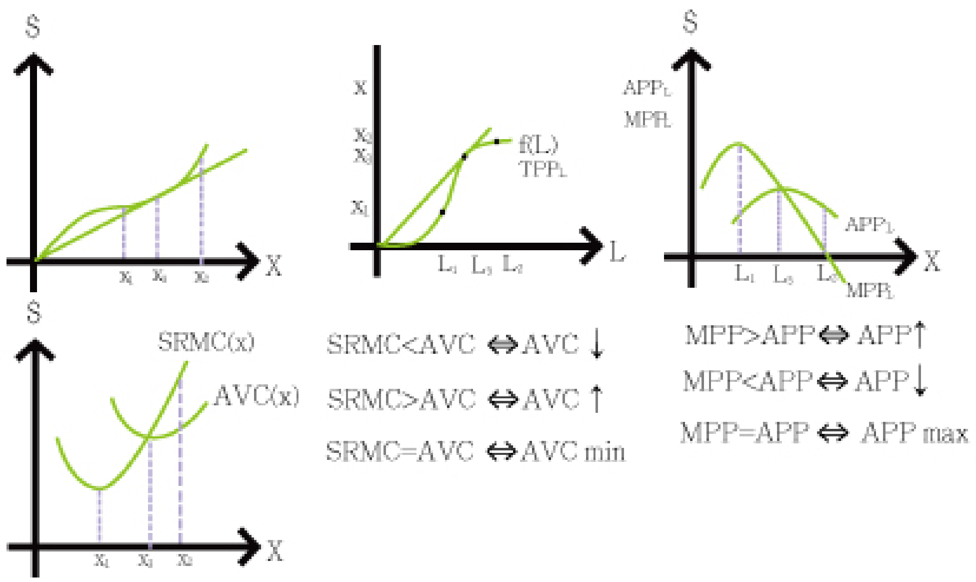


Figure63:

$SRMC < AVC \Leftrightarrow AVC \downarrow$

$SRMC > AVC \Leftrightarrow AVC \uparrow$

$SRMC = AVC \Leftrightarrow AVC \min$

$MPP > APP \Leftrightarrow APP \uparrow$

$MPP < APP \Leftrightarrow APP \downarrow$

$MPP = APP \Leftrightarrow APP \max$

$$AVC(x) = \frac{TVC(x)}{x} = \frac{wL(x)}{x} = \frac{w}{\frac{x}{L}} = \frac{w}{APP_L}$$

$$\begin{cases} APP_L \uparrow \Leftrightarrow AVC(x) \downarrow \\ APP_L \downarrow \Leftrightarrow AVC(x) \uparrow \\ APP_L \max \Leftrightarrow AVC \min \end{cases}$$

$$SRMC(x) = \frac{\Delta TVC(x)}{\Delta x} = \frac{\Delta wL(x)}{\Delta x} = w\frac{\Delta L(x)}{\Delta x} = \frac{w}{\frac{\Delta x}{\Delta L}} = \frac{w}{MPP_L}$$

$$\begin{cases} MPP_L \uparrow \Leftrightarrow SRMC(x) \downarrow \\ MPP_L \downarrow \Leftrightarrow SRMC(x) \uparrow \\ MPP_L \max \Leftrightarrow SRMC(x) \min \end{cases}$$

∗ Example : C-D production function

$$\mathbf{x = f(L, K) = L^{\alpha} K^{\beta}, K = K_0}$$

$$f(L) = L^{\alpha} K_0^{\beta} \text{(short run)}$$

$$TFC = rK_0$$

$$L = \left(\frac{x}{K_0^{\beta}}\right)^{\frac{1}{\alpha}}$$

$$TVC = wL = w\left(\frac{x}{K_0^{\beta}}\right)^{\frac{1}{\alpha}}$$

$$SRMC = \frac{dTVC}{dx} = \frac{w}{\alpha}\left(\frac{x}{K_0^{\beta}}\right)^{\frac{1}{\alpha}-1}\frac{1}{K_0^{\beta}} = \frac{w}{\alpha}\left(\frac{1}{K_0^{\beta}}\right)^{\frac{1}{\alpha}} x^{\frac{1}{\alpha}-1}$$

$$\frac{dSRMC}{dx} = \frac{w}{\alpha}\left(\frac{1}{K_0^{\beta}}\right)^{\frac{1}{\alpha}}\left(\frac{1}{\alpha} - 1\right)x^{\frac{1}{\alpha}-2}$$

$$\frac{w}{\alpha}\left(\frac{1}{K_0^{\beta}}\right)^{\frac{1}{\alpha}} > 0, x^{\frac{1}{\alpha}-2} > 0$$

$$\therefore \frac{dSRMC}{dx} = \frac{w}{\alpha}\left(\frac{1}{K_0^{\beta}}\right)^{\frac{1}{\alpha}}\left(\frac{1}{\alpha} - 1\right)x^{\frac{1}{\alpha}-2} \quad \begin{array}{l} > 0 \text{ if } \alpha < 1 \\ = 0 \text{ if } \alpha = 1 \\ < 0 \text{ if } \alpha > 1 \end{array}$$

↕ 對應

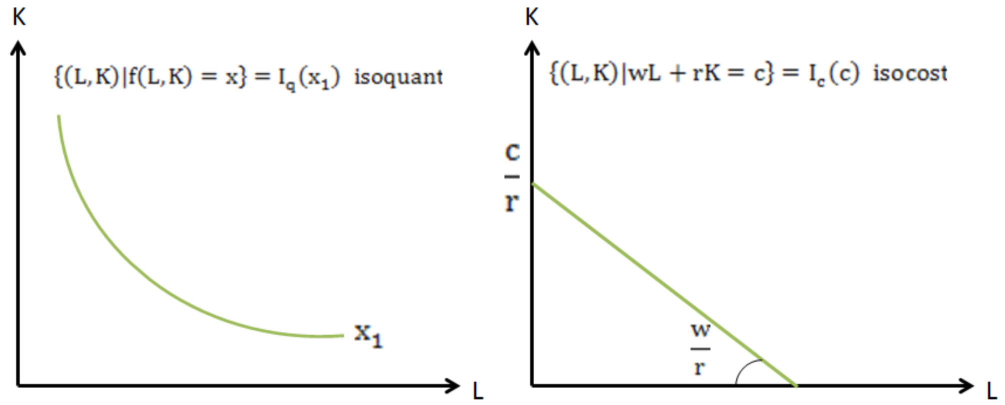
Note that $\begin{array}{l} \alpha < 1 \text{ if } MPP_L \downarrow \text{(diminishing } MPP_L) \\ \alpha = 1 \text{ if } MPP_L \text{ is constant} \\ \alpha > 1 \text{ if } MPP_L \uparrow \text{(inreasing } MPP_L) \end{array}$

∗Long Run Cost Function(L, K are variable)

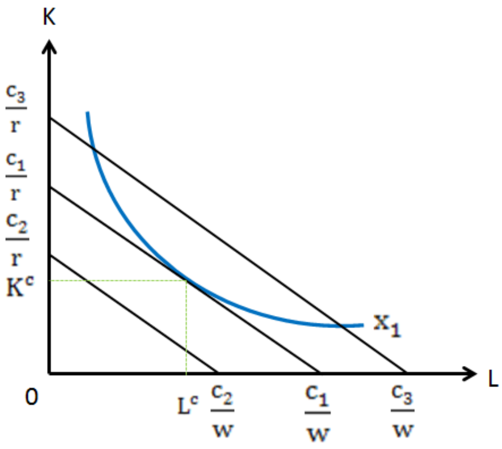
$(L^0, K^0)$is economically efficient if it minimizes cost of producing output x

that is (L^0, K^0) solves

$$\begin{cases} \min wL + rK \\ s.t.\ f(L, K) = x \end{cases}$$



找離原點最近又能滿足 output (isoquant)的 costline (isocost)



$(L^0, K^0)$is economically efficient

$\Longrightarrow MRTS_{LK}$ (slope of an isoquant)

$= \frac{w}{r}$ (slope of isocost) at (L^0, K^0)

Diminishing $MRTS_{LK}$ S.O.C.

(L^0, K^0) solves

$$\left.\begin{array}{r} \min wL + rK \\ s.t.\ f(L,K) = x \end{array}\right\} \Longrightarrow F.O.C. \begin{cases} MRTS_{LK} = \frac{w}{r} \\ f(L,K) = x \end{cases}$$

$$\Longrightarrow \left.\begin{array}{l} L^0 = L(w,r,x) \\ K^0 = K(w,r,x) \end{array}\right\}$$ conditional input demand function

** In equilibrium, Isocost and Isoquant touch(tangent) to each other.

Slope of an isocost = slope of an isoquant

$\frac{w}{r} = MRTS_{LK}$

another necessary condition: $f(L,K) = x$

∗ Corresponding Lagrangian

$$\mathcal{L}(L,K,\lambda) = (wL + rK) + \lambda(x - f(L,K))$$

Foc: $\mathcal{L}_L = w - \lambda\frac{\partial f(L,K)}{\partial L} = 0$ ① $\Rightarrow w = \lambda\frac{\partial f(L,K)}{\partial L}$ ①′

$\mathcal{L}_K = r - \lambda\frac{\partial f(L,K)}{\partial K} = 0$ ② $\Rightarrow r = \lambda\frac{\partial f(L,K)}{\partial K}$ ②′

$$\mathcal{L}_\lambda = x - f(L, K) = 0$$

$$\frac{①'}{②'} = \frac{w}{r} = \frac{\frac{\partial f(L,K)}{\partial L}}{\frac{\partial f(L,K)}{\partial K}} = \frac{MPP_L}{MPP_K} = MRTS_{LK}$$

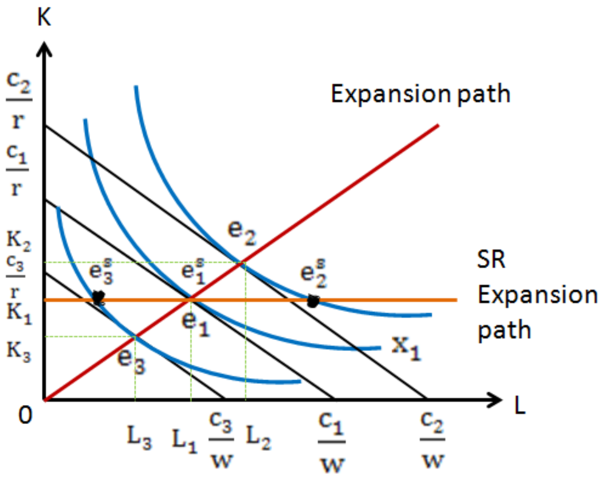
$$F.O.C. \Longrightarrow \left.\begin{array}{l} L^0 = L(w,r,x) \\ K^0 = K(w,r,x) \end{array}\right\}$$ conditional (on x) input demand functions

＊Comparative Static Analysis

x, (w, r) change

x, $\frac{w}{r}$ change

x changes



at e_2 cost $= wL_2 + rK_2 = C_2$

$\Longrightarrow LRTC(x_2)$

at e_1 cost $= wL_1 + rK_1 = C_1$

$\Longrightarrow LRTC(x_1)$

at e_3 cost $= wL_3 + rK_3 = C_3$

$\Longrightarrow LRTC(x_3)$

除了e_1^s外, 其他點的 SR 成本高於 LR

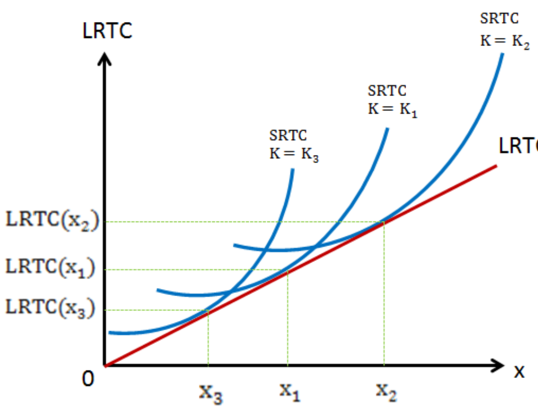
＊Compare LRTC & SRTC

same x, compare LRTC(x) and SRTC(x)

suppose $K = K_1$ fixed in the SR

$LRTC(x) \leq SRTC(x)$ for every given K
(只有一條)　(每一個 K 對映一條 SRTC)

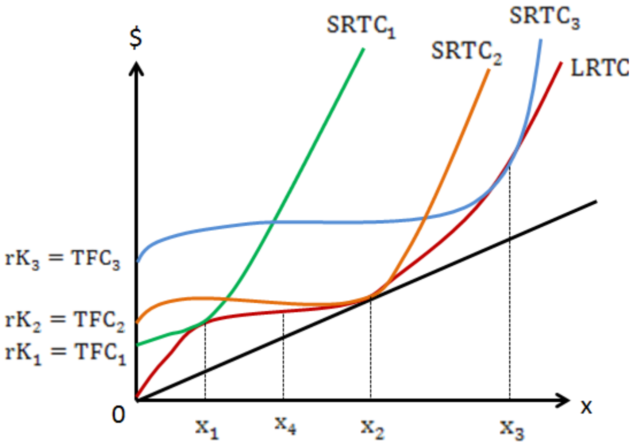
*LRTC 之圖形應如下圖，在這為簡化



畫成直線

＊Envelope curve

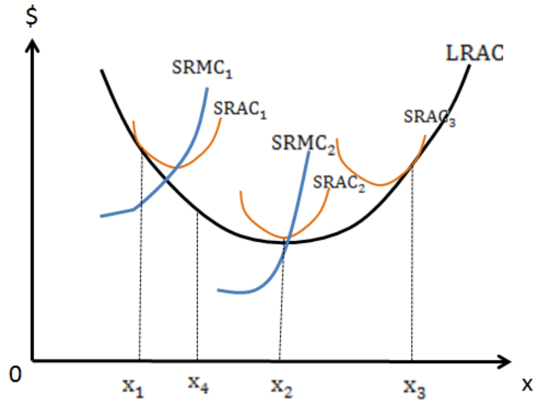
LRTC is the envelope of the SRTC curves



$$LRAC(x) = \frac{LRTC(x)}{x}$$

$$LRMC(x) = \frac{\Delta LRTC(x)}{\Delta x}$$

＊ LRAC is the envelope of SRAC



＊ Firm's problem

$$\begin{cases} \min_{L,K} wL + rK \\ s.t.\ f(L,K) = x \end{cases}$$

<u>Static Analysis:</u>

F. O. C. :

$$MRTS_{LK} \left(= \frac{MPP_L}{MPP_K}\right) = \frac{w}{r}$$ isoquant 和 isocost 相切

$$f(L,K) = x$$

$$\Longrightarrow L^0 = L(w,r,x)$$

$$K^0 = K(w,r,x)$$

<u>Comparative Analysis:</u>

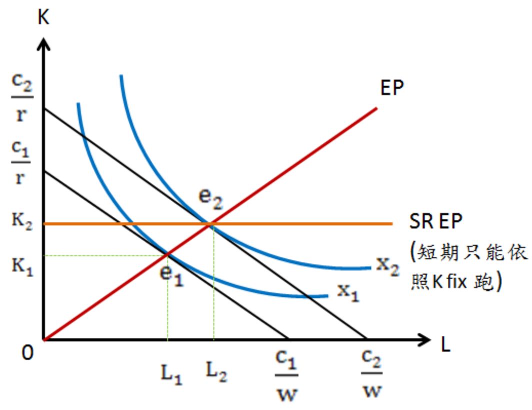
(1) x changes $\Longrightarrow$ expansion path

$\mathrm{LRTC(x)} \le \mathrm{SRTC(x)}$ for every given K

$\frac{\mathrm{LRTC(x)}}{\mathrm{x}}$(割線斜率) $\le \frac{\mathrm{SRTC(x)}}{\mathrm{x}}$
for every given K

$\Rightarrow \mathrm{LRAC(x)} \le \mathrm{SRAC(x)}$ for every given K

∴ LRAC is the envelope of SRAC curves

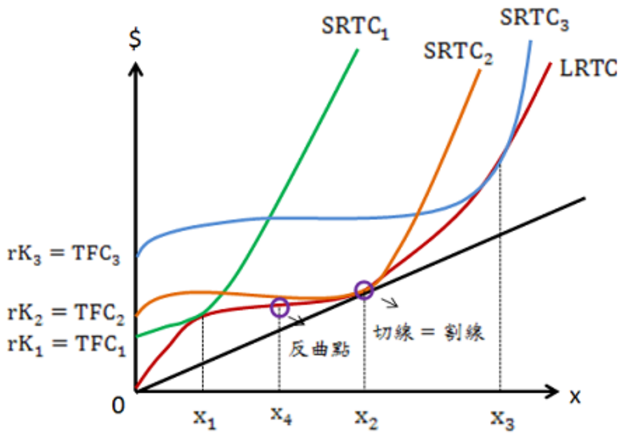


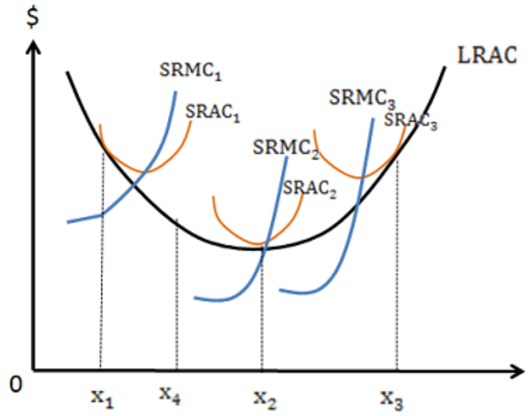
$\mathrm{C_1 = wL_1 + rK_1}$

$\mathrm{LRTC(x) = w \cdot L^c(x;w,r) + r \cdot K^c(x;w,r)}$

at x_1 $\mathrm{LRAC}(x_1) = \mathrm{SRAC}(x_1)$

at x_2 $\mathrm{LRAC}(x_2) = \mathrm{SRAC}(x_2)$



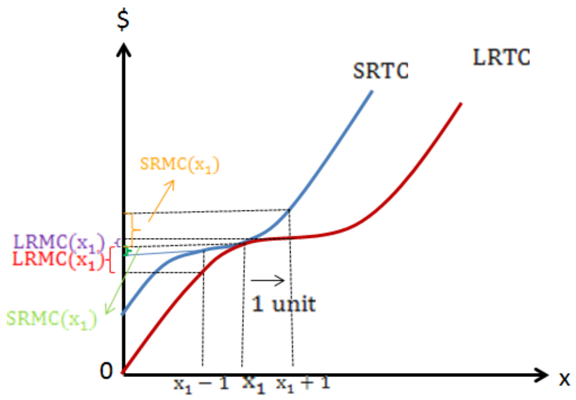


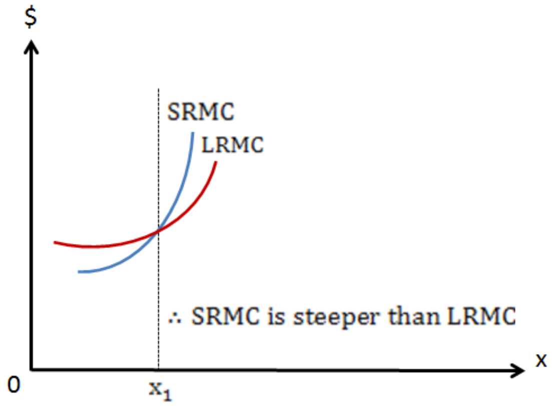
$\text{LRMC}(x) > SRMC(x)$ for $x < x_1$

$\text{LRMC}(x) < \text{SRMC}(x)$ for $x > x_1$

$\text{LRMC}(x) < \text{SRMC}(x)$ for $x > x_1$

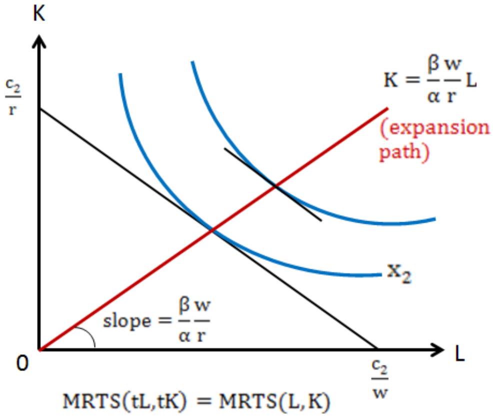
$\Longrightarrow$ LRMC is not the envelope of the SRMC curve





Example : Cobb-Douglas production function

$f(L,K) = L^{\alpha}K^{\beta}$



$$\left.\begin{array}{l}\min\limits_{L,K} wL + rK \\ s.t \quad L^{\alpha}K^{\beta} = x\end{array}\right\}$$

$$F.O.C.:\ MRTS_{LK} = \frac{w}{r}$$

$$\Rightarrow \frac{MPP_L}{MPP_K} = \frac{\alpha L^{\alpha-1}K^{\beta}}{\beta\ L^{\alpha}K^{\beta-1}} = \frac{\alpha}{\beta}\frac{K}{L} = \frac{w}{L}$$

$$\Rightarrow K = \frac{\beta}{\alpha}\frac{w}{r}L \Rightarrow \text{expansion path} \quad ①$$

$L^{\alpha}K^{\beta} = x$ (isoquant) ②

① 代入②

$$\Rightarrow L^{\alpha}\left(\frac{\beta}{\alpha}\frac{w}{r}L\right)^{\beta} = x \quad , \quad \left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\beta}L^{\alpha+\beta} = x$$

$$\Rightarrow L^{\alpha+\beta} = \left(\frac{\alpha}{\beta}\frac{r}{w}\right)^{\beta}x$$

$$\Rightarrow L^{c} = \left(\frac{\alpha}{\beta}\frac{r}{w}\right)^{\frac{\beta}{\alpha+\beta}}x^{\frac{1}{\alpha+\beta}}$$

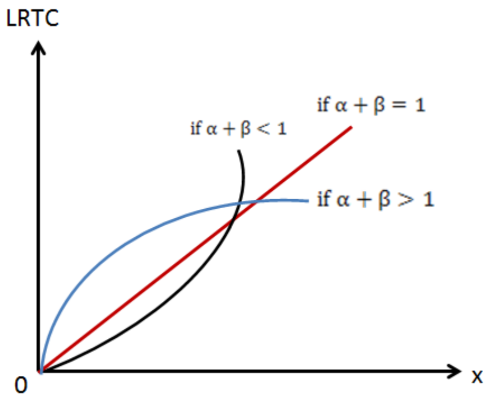
$$\Rightarrow K^{c} = \left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{\frac{\alpha}{\alpha+\beta}}x^{\frac{1}{\alpha+\beta}}$$

$$LRTC(x) = wL^{c} + rK^{c}$$

$$= \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}r^{\frac{\beta}{\alpha+\beta}}w^{\frac{\alpha}{\alpha+\beta}}x^{\frac{1}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}r^{\frac{\beta}{\alpha+\beta}}w^{\frac{\alpha}{\alpha+\beta}}x^{\frac{1}{\alpha+\beta}}$$

$$= \left[\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right]r^{\frac{\beta}{\alpha+\beta}}w^{\frac{\alpha}{\alpha+\beta}}x^{\frac{1}{\alpha+\beta}}$$

$$= c(w,r,\alpha,\beta) \cdot x^{\frac{1}{\alpha+\beta}}$$



$\alpha+\beta > 1$ $LRTC(x)$ concave

$\alpha+\beta < 1$ $LRTC(x)$ convex

$\alpha+\beta = 1$ $LRTC(x)$ linear

∗Example : $\mathbf{f(L, K) = min\left\{\frac{L}{a}, \frac{K}{b}\right\}}$

Most efficient: $\frac{L}{a} = \frac{K}{b} \Rightarrow K = \frac{b}{a}L$ (not only most efficient, but also EP)

$$x = \frac{L}{a} = \frac{K}{b} \;\Rightarrow\; L^0 = ax \text{ (no w)}, \qquad K^0 = bx \text{ (no r)}$$

$$LRTC(x) = wL^0 + rK^0 = w\,ax + r\,bx = (aw + br)x$$

$$LRAC = aw + br$$

$$LRMC = aw + br$$

SR cost:

$$K = K_0$$

$$TFC = rK_0$$

$$TVC = wL(x)$$

$$x = f(L, K) = \min\left\{\frac{L}{a}, \frac{K_0}{b}\right\}$$

$$\frac{L}{a} \geq \frac{K_0}{b}, \quad x = \frac{K_0}{b} \text{(fixed)}$$

$$x \leq \frac{K_0}{b} \text{(capacity constraint)}$$

$$x > \frac{K_0}{b} \text{(infeasible)} \;\Rightarrow\; SRTC \to \infty \text{ if } \; x > \frac{K_0}{b}$$

$$\frac{L}{a} < \frac{K_0}{b}, \quad x = \frac{L}{a} (= f(L))$$

$$x \leq \frac{K_0}{b} \text{(capacity constraint)}$$

$$x = \frac{L}{a} \;\Rightarrow\; L = ax$$

$$TVC = wL = wax \text{ (note that } TFC = rK_0)$$

$$SRTC = TFC + TVC = rK_0 + awx$$

when $x = \frac{K_0}{b}$

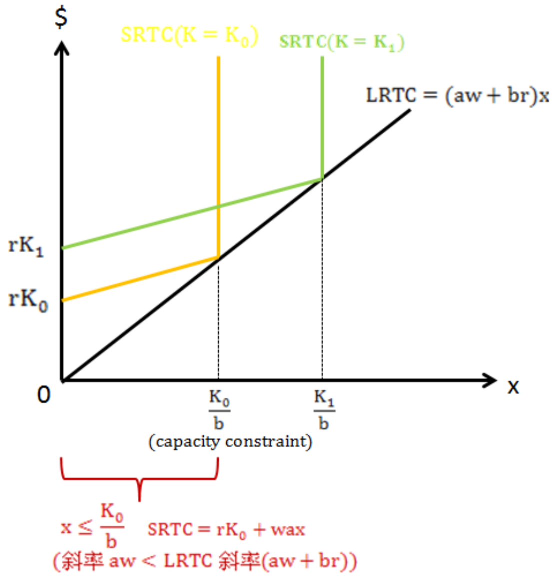
$$\Rightarrow SRTC = rK_0 + awx = rK_0 + aw\frac{K_0}{b}$$

$$LRTC\left(x = \frac{K_0}{b}\right) = (aw + br)x$$

$$= (aw + br)\frac{K_0}{b}$$

$$= rK_0 + aw\frac{K_0}{b}$$

$$= SRTC\left(x = \frac{K_0}{b}\right)$$



<u>Another comparative statics analysis:</u>

(2) w (or r) changes (change in price of input)

the firm's problem is:

$$\left.\begin{array}{l} \min_{L,K} wL + rK \\ s.t\ f(L, K) = x \end{array}\right\}$$

$$\Rightarrow F.O.C.:\ MRTS_{LK} = \frac{w}{r}, f(L, K) = x$$

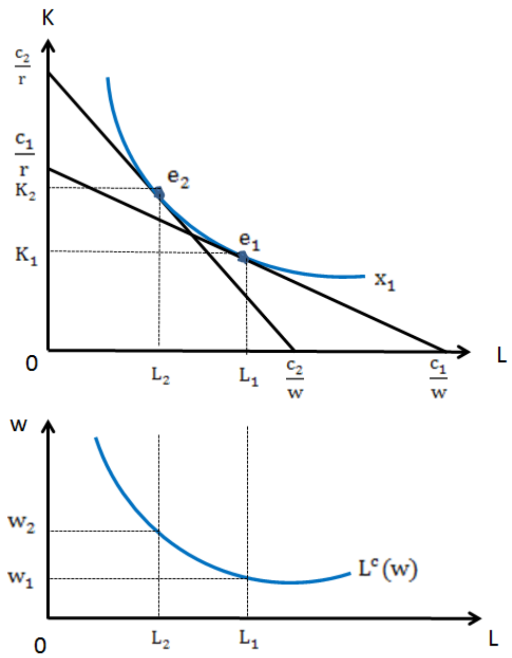
$$\Rightarrow L^0, K^0$$

$$\Rightarrow LRTC = wL^0 + rK^0$$

x fixed(x_1)

r fixed

$w \uparrow \Rightarrow w_1 \rightarrow w_2, w_2 > w_1$



$\therefore e_1 \rightarrow e_2$ (both e_1 and e_2 are on isoquant x_1)

Locus of equilibrium with respect to change in w = isoquant x_1

<u>w, r change in the same proportion</u>

＊Example : problem： $\min_{L,K} \omega L + rK$ s.t. $f(L,K)=x$

$(\omega,r)\rightarrow(t\omega,tr)$

Foc $MRTS_{LK} = {}^{\omega}/_{r} \rightarrow MRTS_{LK} = {}^{t\omega}/_{tr} = {}^{\omega}/_{r}$ ， unchanged

$f(L,K) = x \leftarrow$ no (ω, r) $\Rightarrow L^o, K^o$ don't change

i.e. $L^o=L(\omega,r,x)=L(t\omega,tr,x)$

$K^o=K(\omega,r,x)=K(t\omega,tr,x)$

→ $L(\omega,r,x)$ and $K(\omega,r,x)$ are homogeneous of degree 0 in ω and r

Conditional input demand functions are homogeneous of degree 0 in ω and r

$LRTC(x;\omega,r)=\omega L^o+rK^o$

$LRTC(x;t\omega,tr)=t\omega L^o+trK^o=tLRTC(x;\omega,r)$

→LRTC is homogeneous of degree 1 in ω and r

***Comparative Statics Analysis**

w, r change ⇒

L^o , K^o change, $L^\circ = L(w,r,x)$

$K^\circ = K(w,r,x)$

$LRTC(x;w,r) = wL^\circ + rK^\circ = wL(w,r,x) + rK(w,r,x)$

L°, K° are homogeneous of degree 0 in w and r

$L(tw,tr,x) = t^0L(w,r,x)$

$K(tw,tr,x) = t^0K(w,r,x)$

$LRTC(x;tw,tr) = t^1LRTC(x;w,r)$

LRTC is homogeneous of degree 1 in w and r

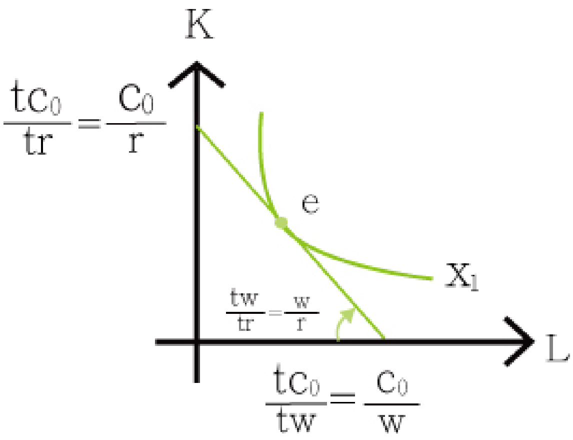


Figure70:

$L°, K°$ depend on $\frac{w}{r}$, i.e. $L°, K°$ are functions of $\frac{w}{r}$ and x

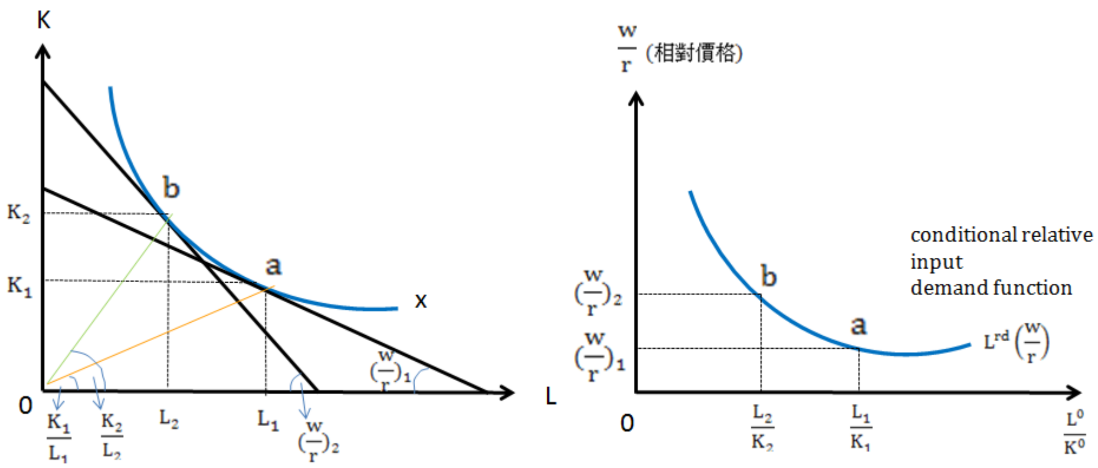
$$L° = L(w, r, x) = L(\frac{w}{r}, 1, x)$$

$$K° = K(w, r, x) = K(\frac{w}{r}, 1, x)$$

* Conditional relative input demand function

$\frac{L^0}{K^0}\left(\frac{w}{r}\right)$: conditional relative labor demand function (conditional 是指 conditional on x(fixed))

$$\frac{L^0}{K^0} = \frac{L(w, r, x)}{K(w, r, x)} = \frac{L\left(\frac{w}{r}, x\right)}{K\left(\frac{w}{r}, x\right)} = L^{rd}\left(\frac{w}{r}\right)$$



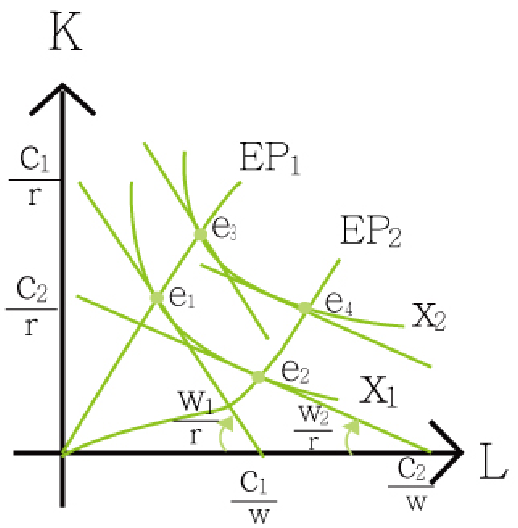
* ε^{rd}, price elasticity of conditional relative input demand

$$\varepsilon^{rd} = \left|\frac{\frac{\Delta(\frac{L^0}{K^0})}{\frac{L^0}{K^0}}}{\frac{\Delta\frac{w}{r}}{\frac{w}{r}}}\right| = \left|\frac{d(\frac{L^0}{K^0})}{d(\frac{w}{r})}\frac{\frac{w}{r}}{\frac{L^0}{K^0}}\right| = \left|\frac{d\ln\left(\frac{L^0}{K^0}\right)}{d\ln\left(\frac{w}{r}\right)}\right| = -\frac{d\ln\left(\frac{L^0}{K^0}\right)}{d\ln\left(\frac{w}{r}\right)}$$

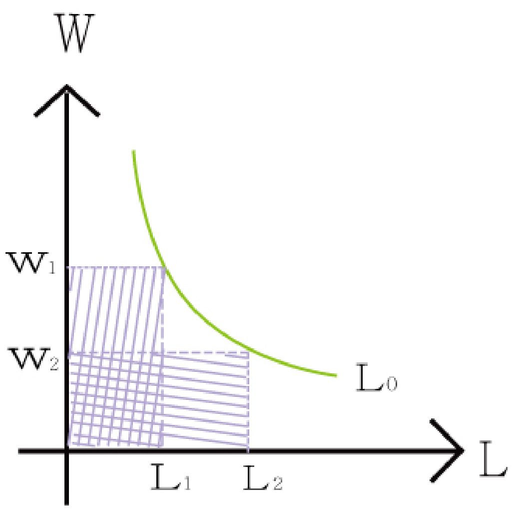
note that in equilibrium $MRTS_{LK} = \frac{w}{r}$

$$\varepsilon^{rd} = -\frac{d\ln\frac{L}{K}}{d\ln(MRTS_{LK})} = -\left(-\frac{d\ln\frac{K}{L}}{d\ln(MRTS_{LK})}\right) = \frac{d\ln\frac{K}{L}}{d\ln(MRTS_{LK})}$$

$$= \sigma \text{ (Elasticity of substition)}$$



F igure73:



F igure74:

w changes (r is given)

$w\downarrow$, $w_1 \to w_2$, $w_2 < w_1$

①$e_1 \to e_2$, given x= e_1

② $EP_1 \to EP_2$

In each EP, w, r are given, and x is free to change

① $w\downarrow$, $w_1 \rightarrow w_2$, $w_2 < w_1$

Given $x = x_1, \frac{w}{r} \downarrow \left(\frac{w_2}{r} < \frac{w_1}{r}\right)$

$\Rightarrow L\uparrow, K\downarrow (L_1 \rightarrow L_2, K_1 \rightarrow K_2;\ L_1 < L_2, K_1 > K_2)$

$\Rightarrow$ L° is downward sloping

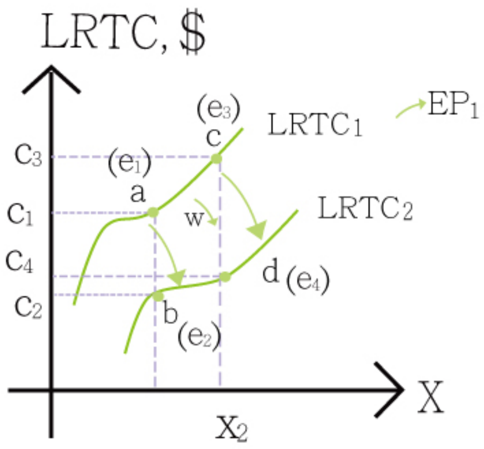


Figure75:

$w\downarrow \Rightarrow$ LRTC↑↓?

$e_2\ LRTC(x_1, w_2, r) \overset{>}{\underset{<}{=}} e_1 LRTC(x_1, w_1, r)$

From Figure 73, $\frac{C_2}{r} < \frac{C_1}{r}$ (r fixed)

$$\Rightarrow C_1\big(\text{i.e } LRTC(x_1, w_1, r)\big) > C_2(LRTC(x_1, w_2, r))$$

From EP_1 and EP_2, $w_1 \rightarrow w_2 (w\downarrow) \Rightarrow LRTC_1 \rightarrow LRTC_2$ (LRTC shifts downward)

Change in w $\Rightarrow$ shift of LRTC

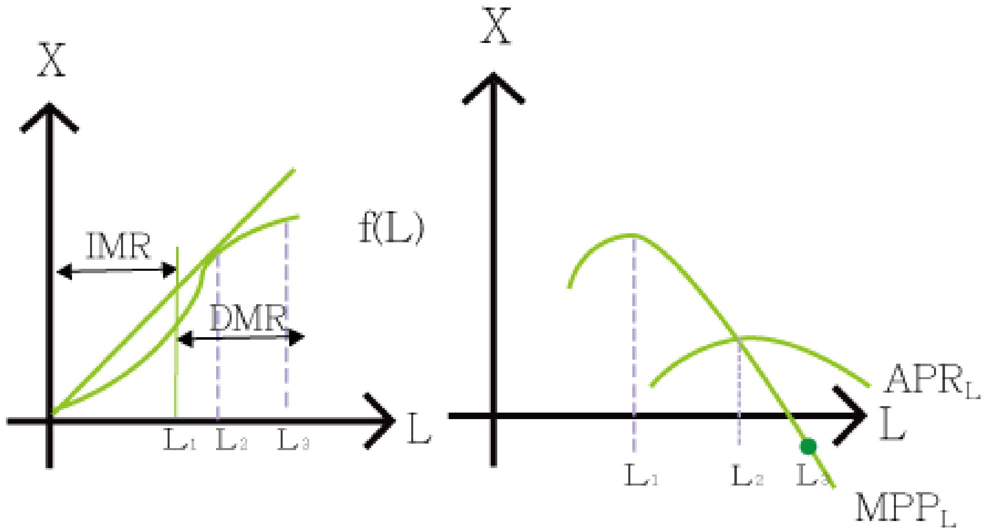
$w\uparrow \Rightarrow$ upward shift of LRTC

$w\downarrow \Rightarrow$ downward shift of LRTC

$r\uparrow \Rightarrow$ upward shift of LRTC

$r\downarrow \Rightarrow$ downward shift of LRTC

Ling Run Cost Curves



F igure76:

In the SR, K fixed

IMR $\Rightarrow$ Diminishing marginal cost

DMR $\Rightarrow$ Increasing marginal cost

In the LR L&K are variable

L^0, K^0 is economically efficient, if L^0, K^0 solves $\min\limits_{L,K}\ wL + rK$ s.t. $f(L, K) = x$

$\Rightarrow L° = L(w, r, x), K° = K(w, r, x)$

$LRTC(x) = wL° + rK° = wL(w, r, x) + rK(w, r, x)$

*Returns to scale (指的是生產技術 f(x)): x= f(L, K)

$f(tL, tK) > tf(L,K)$, $t>1$ $\Rightarrow$ Increasing returns to scale

$f(tL, tK) = tf(L,K)$, $t>0 \Rightarrow$ Constant returns to scale

$f(tL, tK) < tf(L,K)$, $t>1$ $\Rightarrow$ Decreasing returns to scale

* **Economy of scale(指的是 cost): LRAC(x) is decreasing with output**

* **Diseconomy of scale: LRAC is increasing with output**

Suppose f(L,K) is homogenous of degree k in L and K

$f(tL, tK) = t^k f(L,K)$

k>1, f(L,K) is increasing returns to scale

($t^k > t$ for $k>1$ ⇒ $f(tL, tK) > tf(L,K)$)

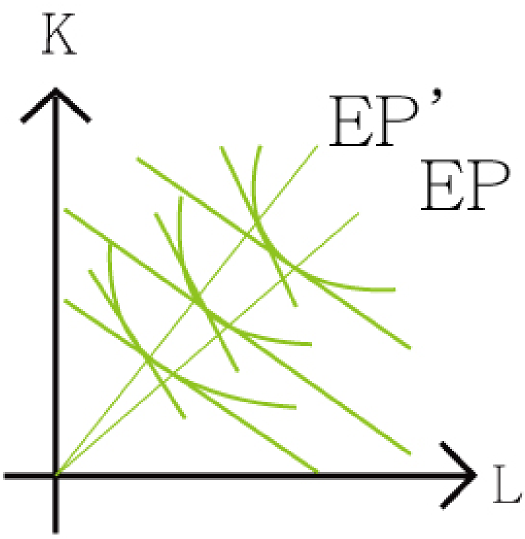
k<1, f(L,K) is decreasing returns to scale

($t^k < t$ for $k<1$ ⇒ $f(tL, tK) < tf(L,K)$)

k=1, $f(tL, tK) = tf(L,K)$ constant returns to scale

note that: f(L,K) is constant returns to scale

⇒f(L,K) must be homogeneous of degree 1 in L&K



F igure77:

* **Expansion path at a homogeneous production function is a straight line through the origin**

i.e. $MRTS_{LK}$ is a function of $\frac{K}{L}$

Suppose (L°, K°) is the cost minimum input combination to produce **x** unit of output

$$L^0, K^0 \text{ solves } \begin{array}{c} \min\limits_{L,K} \; wL + rK \\ \text{s.t. } f(L,K) = x \end{array}$$

$$\Rightarrow LRTC = wL^\circ + rK^\circ$$

Suppose we would like to produce an output of **tx**, $t > 1$ $(tx > x)$

$f(L,K)$ is homogenous of degree k

$$f(tL, tK) = t^k f(L,K)$$

$$K \begin{array}{c} = \\ > \\ < \end{array} 1 \quad f(L,K) \text{ is } \begin{array}{c} \text{constant} \\ \text{increasing} \\ \text{decreasing} \end{array} \text{ return to scale}$$

$$f(tL, tK) = t^k f(L,K) \begin{array}{c} = \\ > \\ < \end{array} t f(L,K) = tx$$

$$\Longrightarrow f(t'L, t'K) = t f(L,K) = tx$$

$$\Longrightarrow \begin{array}{ccc} t' & = & t \\ t' & < & t \\ t' & > & t \end{array} \text{ and } L^o, K^o \text{ is optimal to produce output } x^o$$

$$(t'L, t'K) \text{ sovles } \begin{array}{c} \min\limits_{L,K} wL + rK \\ \text{s.t. } f(L,K) = tx \end{array}$$

$$LRTC(tx) = w \times t'L^0 + r \times t'K^0 = t'(wL^0 + rK^0) = tLRTC(x)$$

$$LRAC(x) = \frac{wL^o + rK^o}{x}$$

LRAC(x)

$$LRAC(tx) = \frac{w \times t'L^o + r \times t'K^o}{tx} = \frac{t'(wL^o + rK^o)}{tx} \begin{array}{c} = \\ < \\ > \end{array} \frac{wL^o + rK^o}{x}$$

$$\text{Since } tx > x \Rightarrow LRAC(x) \text{ is } \begin{array}{c} \text{fixed} \\ \text{decreasing} \\ \text{increasing} \end{array} \text{ with } x \Longrightarrow \begin{array}{c} \text{Economy of scale} \\ \text{Diseconomy of scale} \end{array}$$

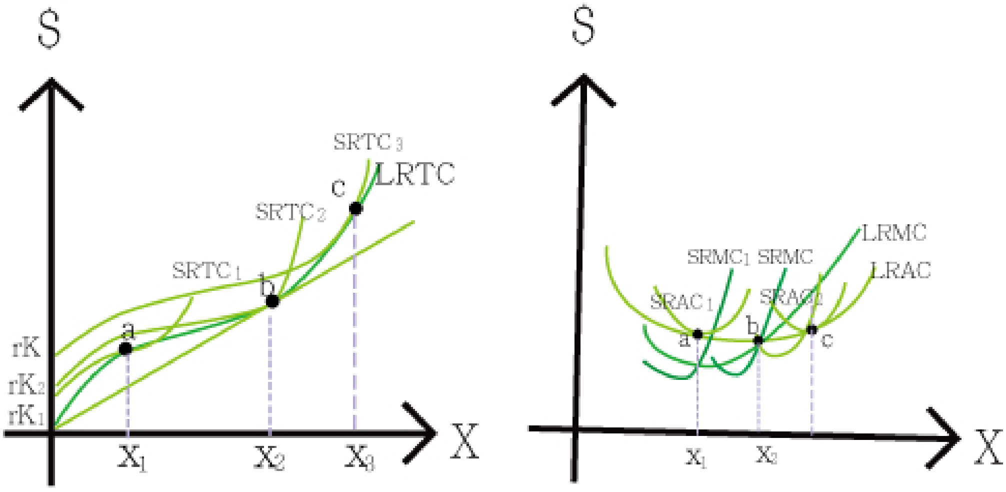


Figure78:

f(L, K) is a homogeneous function

$$f(L,K)\text{ is }\begin{array}{c}\text{increasing}\\\text{constant}\\\text{decreasing}\end{array}\text{ returns to scale }\begin{array}{c}\Leftrightarrow\\\Leftrightarrow\\\Leftrightarrow\end{array}\text{ the LRAC(x) is }\begin{array}{c}\text{decreasing}\\\text{constant}\\\text{increasing}\end{array}\text{ in x}$$

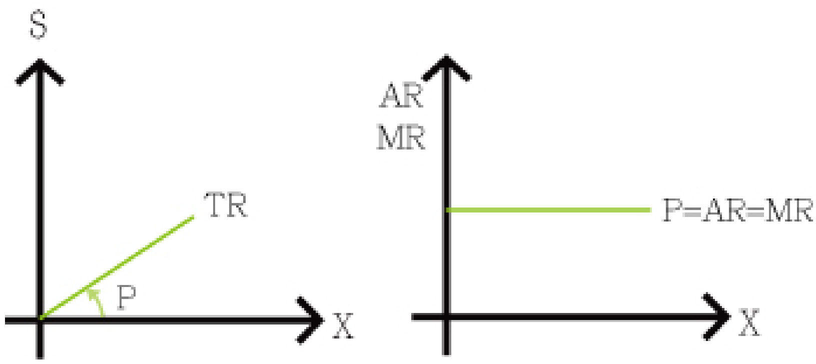


Figure79:

$$(L^o, K^o) \text{ solves} \begin{cases} \min\limits_{L,K} \; wL + rK \\ \text{s.t. } f(L,K) = x \end{cases} \quad (*)$$

$$LRTC(x) = wL^o + rK^o$$

$$\text{F.O.C.} \begin{cases} MRTs_{LK}(L^o, K^o) = \frac{w}{r} \\ f(L^o, K^o) = x \end{cases}$$

$$LRAC(x) = \frac{LRTC(x)}{x}$$

$$x \to tx, t > 1 \Rightarrow \begin{cases} \min\limits_{L,K} \; wL + rK \\ \text{s.t. } f(L,K) = x \end{cases} \quad (**)$$

$$\text{note that: } f(tL, tK) = t^k f(L,K)$$

$$t = (t^{\frac{1}{k}})^k$$

$$\Rightarrow f\left(t^{\frac{1}{k}}L, t^{\frac{1}{k}}K\right) = tf(L,K) = tx$$

$$(L^o, K^o) \to x$$

$\left(t^{\frac{1}{k}}L^o, t^{\frac{1}{k}}K^o\right) \to tx$ 符合 $f(L^o, K^o) = x$

$$\left(t^{\frac{1}{k}}L^o, t^{\frac{1}{k}}K^o\right) = tx$$

e_2 satisfies 2nd FOC for $(**)$

$$MRTS_{LK}\left(t^{\frac{1}{k}}L^o, t^{\frac{1}{k}}K^o\right)$$
$$= MRTs_{LK}(L^o, K^o)$$
$$= \frac{w}{r} \to \text{EP is a straight line through the origin}$$

First FOC is also satisfied

e_2 solves (**)

$(t^{\frac{1}{k}}L^0, t^{\frac{1}{k}}K^0)$

$$\Rightarrow \mathrm{LRTC(tx)} = w\left(t^{\frac{1}{k}}L^0\right) + r(t^{\frac{1}{k}}K^0)$$

$$= t^{\frac{1}{k}}(wL^0 + rK^0)$$

$$= t^{\frac{1}{k}}\mathrm{LRTC(x)}$$

$$\mathrm{LRAC(tx)} = \frac{\mathrm{LRTC(tx)}}{\mathrm{tx}} = \frac{t^{\frac{1}{k}}\mathrm{LRTC(x)}}{\mathrm{tx}} = t^{\frac{1}{k}-1}\mathrm{LRAC(x)}$$

1. K=1 (constant returns to scale)

$\Rightarrow \mathrm{LRAC(tx)} = \mathrm{LRAC(x)}$, for all $t > 0$ constant LRAC

2. K>1 (increasing returns to scale) $\rightarrow \frac{1}{k} - 1 < 0$

$\Rightarrow t^{\frac{1}{k}-1} = \frac{1}{t^{1-\frac{1}{k}}} < 1$, for all t>1

$\Rightarrow \mathrm{LRAC(tx)} < LRAC(x), for\ all\ t > 1\ (tx > x)$

LRAC is decreasing in x (economy of scale)

3. K<1 (decreasing returns to scale) $\rightarrow \frac{1}{k} - 1 > 0$

$\Rightarrow t^{\frac{1}{k}-1} > 1$, for all t>1

$\Rightarrow \mathrm{LRAC(tx)} > LRAC(x), for\ all\ t > 1\ (tx > x)$

LRAC is increasing in x (diseconomy of scale)

Suppose f(L,K) is not a homogeneous function

increasing returns to scale $\leftrightarrow$ economy of scale ?

decreasing returns to scale $\leftrightarrow$ diseconomy of scale ?

sure,

f(L,K) is constant returns to scale

$f(tL, tK) = tf(L, K) \leftrightarrow f(L, K)$ is homogeneous of degree one in L and K

*** increasing returns to scale**

$f(tL,tK) > tf(L,K) = tx$

(L^0,K^0) solves

$\min_{L,K} wL+rK$

s.t $f(L,K)= x$

$LRTC(x)=WL^0 +rK^0$

(tL^0, tK^0) costs $tLRTC(x)$

$(tL^0, tK^0) \to tx$

$f(tL^0, tK^0) > t f(tL^0, tK^0) = tx$

$$LRTC(x') \neq \frac{w(tL^0) + r(tk^0)}{x'} < \frac{w(tL^0) + r(tk^0)}{tx} = LRAC(x)$$

Since $f(L,K)$ is not a homogeneous function

$MRTS_{LK}(tL^0,tK^0) \neq MRTS_{LK}(L^0,K^0)$

$(t L^0,t K^0)$ doesn't solve $\begin{cases} \min_{L,K} wL + rK \quad (**) \\ \quad s.t.\ f(L,K) = x' \end{cases}$

There is another (L', K') solves (**) and costs less,

$LRTC(x') = wL' + rK' < w(tL^0)+r(tK^0)$

$\Rightarrow$ increasing returns to scale $\to$ decreasing LRAC(x) (economy scale)

increasing LRAC(x)(diseconomy of scale) $\to$ decreasing returns to scale

∗ Example : CES production

$$f(L,K) = (L^\rho + K^\rho)^{\frac{\varepsilon}{\rho}} \quad \rho < 1, \varepsilon > 0, \rho \neq 0$$

$$f(tL,tK) = ((tL)^\rho + (tK)^\rho)^{\frac{\varepsilon}{\rho}} = t^\varepsilon (L^\rho + K^\rho)^{\frac{\varepsilon}{\rho}}$$

FOC:

$$MRTS_{LK} = \frac{w}{r}$$

$$MRTS_{LK} = \frac{\frac{\varepsilon}{\rho}(L^\rho + K^\rho)^{\frac{\varepsilon}{\rho}-1}\rho\, L^{\rho-1}}{\frac{\varepsilon}{\rho}(L^\rho + K^\rho)^{\frac{\varepsilon}{\rho}-1}\rho\, K^{\rho-1}} = (\frac{L}{K})^{\rho-1} = \frac{w}{r}$$

$$\Rightarrow \frac{L}{K} = (\frac{w}{r})^{\frac{1}{\rho-1}} \qquad \therefore L = (\frac{w}{r})^{\frac{1}{\rho-1}} K$$

another FOC: $(L^\rho + K^\rho)^{\frac{\varepsilon}{\rho}} = x$

$$\left[(\left(\frac{w}{r}\right)^{\frac{1}{\rho-1}} K)^\rho + K^\rho \right]^{\frac{\varepsilon}{\rho}} = x$$

$$\left[(1 + \left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}}) K^\rho \right]^{\frac{\varepsilon}{\rho}} = x$$

$$(1 + \left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}})^{\frac{\varepsilon}{\rho}} \cdot K^\varepsilon = x$$

$$K^\varepsilon = (1 + \left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}})^{-\frac{\varepsilon}{\rho}} x$$

$$K^0 = (1 + \left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}})^{-\frac{1}{\rho}} x^{\frac{1}{\varepsilon}} = c_K \cdot x^{\frac{1}{\varepsilon}}$$

$$L^0 = (\frac{w}{r})^{\frac{1}{\rho-1}} (1 + \left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}})^{-\frac{1}{\rho}} x^{\frac{1}{\varepsilon}} = c_L \cdot x^{\frac{1}{\varepsilon}}$$

$$LRTC(x) = wL^0 + rK^0 = wc_L x^{\frac{1}{\varepsilon}} + rc_K x^{\frac{1}{\varepsilon}}$$

$$= (wc_L + rc_K) x^{\frac{1}{\varepsilon}} = A(w,r) x^{\frac{1}{\varepsilon}}$$

$$\text{LRAC} = \frac{\text{LRTC}}{x} = A(w, r)x^{\frac{1}{\varepsilon}-1}$$

$\varepsilon > 1$ increasing returns to scale

$\frac{1}{\varepsilon} < 1, \frac{1}{\varepsilon} - 1 < 0$ LRAC ↓ with x (economy of scale)

$\varepsilon < 1$ decreasing returns to scale

$\frac{1}{\varepsilon} > 1, \frac{1}{\varepsilon} - 1 > 0$ LRAC ↑ with x (diseconomy of scale)

$\varepsilon = 1$ constant returns to scale

$$\text{LRAC}(x) = A(w, r)x^{\frac{1}{1}-1} = A(w, r)$$