Chapter 8 Applications of Plane Stress (Pressure vessels, Beams, and Combined Loadings)

8.1 Introduction

Plane stress conditions : buildings, machines, vehicles, and aircraft

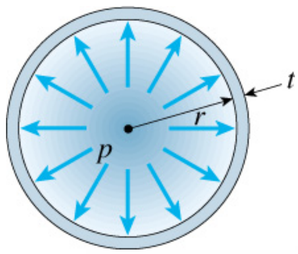
In this chapter, the structures to be discussed are

pressure vessels

stress in beams : principle stresses, maximum shear stress

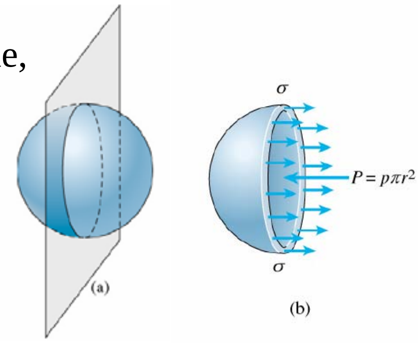
structures subjected to combined loadings

8.2 Spherical Pressure Vessels



pressure vessels are classified as shell structures, it is subjected to internal pressure p

to determine the stresses in the wall, let us cut through the sphere on a vertical diametral plane, the resultant pressure force is



$$P = p(\pi r^2)$$

where r is the inner radius of the sphere

the resultant of the tensile stresses σ in the wall is

$$F = \sigma(2\pi r_m t)$$

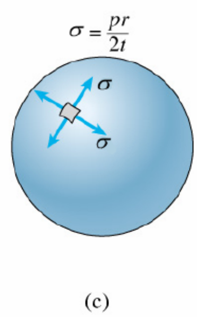
where t is the thickness of the wall and r_m is its mean radius

$$r_m = r + t/2$$

equation of equilibrium in the horizontal direction

$$\Sigma F_{\text{horiz}} = 0$$

$$\sigma (2 \pi r_m t) - p (\pi r^2) = 0$$



(c)

the tensile stresses in the wall is

$$\sigma = \frac{p r^2}{2 r_m t}$$

for $r >> t \quad (r > 10\,t), \quad r_m \simeq r$ then

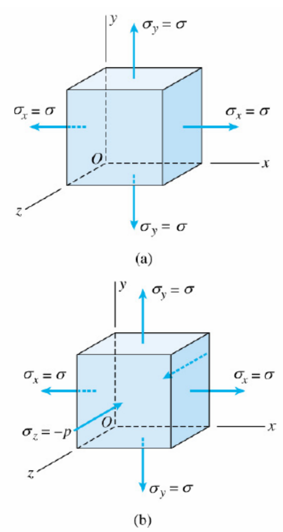
$$\sigma = \frac{p r}{2 t}$$

when we cut through the center of the sphere in any direction, we can conclude that the sphere is subjected to uniform tensile stress σ in all directions, they are known as membrane stresses

Stresses at the outer surface

$$\sigma_1 = \sigma_2 = \frac{p r}{2 t} \qquad \sigma_3 = 0$$

then $$\tau_{max} = \frac{\sigma}{2} = \frac{p r}{4 t}$$



(a)

(b)

Stresses at the inner surface

$$\sigma_1 = \sigma_2 = \frac{p r}{2 t} \qquad \sigma_3 = - p$$

then $$\tau_{max} = \frac{\sigma + p}{2} = \frac{p r}{4 t} + \frac{p}{2} = \frac{p}{2} (\frac{r}{2t} + 1)$$

Limitations of thin-shell theory

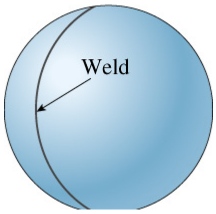
1. $r > 10\,t$ or more

2. internal pressure must exceed external pressure

3. only pressure loading is considered for stress calculation

4. stress concentrations are not considered in the formula derived

Example 8-1



$d = 450$ mm $\quad t = 7$ mm

a. $\sigma_{allow} = 115$ MPa, $\quad p_a = ?$

b. $\tau_{allow} = 40$ MPa $\quad p_b = ?$

c. $\varepsilon_{allow} = 0.0003 \quad E = 210$ GPa

$\nu = 0.28 \quad p_c = ?$

d. $T_{failure} = 1.5$ MN/m $\quad n = 2.5 \quad p_d = ?$

e. $p_{allow} = ?$

a. $\sigma = pr/2t$

$$p_a = \frac{2\,t\,\sigma_{allow}}{r} = \frac{2 \times 7 \times 115}{225} = 7.16 \text{ MPa}$$

b. $\tau = pr/4t$

$$p_b = \frac{4\,t\,\tau_{allow}}{r} = \frac{4 \times 7 \times 40}{225} = 4.98 \text{ MPa}$$

c. $\varepsilon_x = (\sigma_x - \nu\,\sigma_y) / E$

for $\sigma_x = \sigma_y = \sigma = pr / 2t$

then $$\varepsilon_x = \frac{\sigma}{E}(1 - \nu) = \frac{p\,r}{2t\,E}(1 - \nu)$$

thus $$p_c = \frac{2t\,E\,\varepsilon_{allow}}{r\,(1 - \nu)} = \frac{2 \times 7 \times 210 \times 10^3 \times 0.0003}{225\,(1 - 0.28)} = 5.44 \text{ MPa}$$

d. $T_{allow} = T_{failure} / n = 1.5/2.5 = 0.6$ MN/m $= 600$ N/m

$\sigma_{allow} = T_{allow} / t = 600 / 7 = 85.7$ MPa

$$p_d = \frac{2\, t\, \sigma_{allow}}{r} = \frac{2 \text{ x } 7 \text{ x } 85.7}{225} = 5.3 \text{ MPa}$$

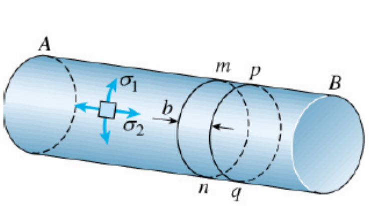
e. $p_{allow} = \min[p_a, \; p_b, \; p_c$ and $p_d] = 4.9$ MPa

for this p_{allow}, the tensile stresses in the shell are

$$\sigma = \frac{p\, r}{2\, t} = \frac{4.9 \text{ x } 225}{2 \text{ x } 7} = 78.8 \text{ MPa}$$

8.3 Cylindrical Pressure Vessels

consider a thin-walled circular tank AB subjected to internal pressure

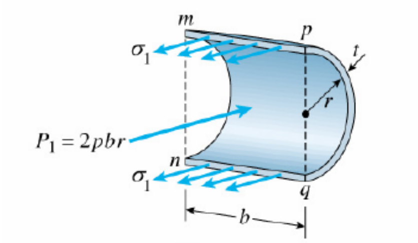


(a)

σ_1 : circumferential stress or hoop stress

σ_2 : longitudinal stress or axial stress

from the free body *mpqn*, we have the equation of equilibrium as



(b)

$$\sigma_1 (2\, b\, t) \; - \; 2\, p\, b\, r = 0$$

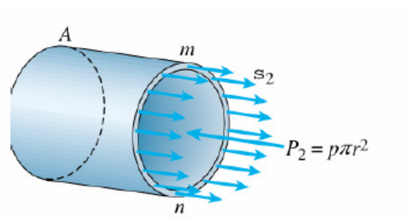
then $$\sigma_1 = \frac{p\, r}{t}$$

and the equation of equilibrium in longitudinal direction

$$\sigma_2 (\, 2\, \pi\, r\, t) \; - \; p\, \pi\, r^2 = 0$$

then $$\sigma_2 = \frac{p\, r}{2\, t}$$

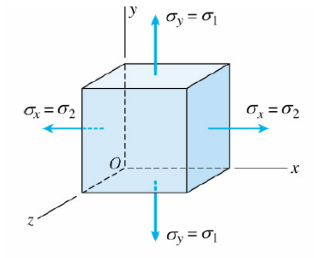
now, we have $\sigma_1 = 2\, \sigma_2$



(c)

Stress at the outer surface

$\sigma_1 = pr / t \qquad \sigma_2 = pr / 2t \qquad \sigma_3 \;=\; 0$



(a)

$$(\tau_{max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{p\,r}{4\,t}$$

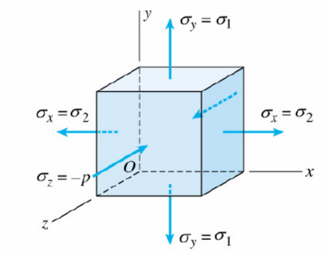
$$(\tau_{max})_x = \frac{\sigma_1}{2} = \frac{p\,r}{2\,t} \qquad (\tau_{max})_y = \frac{\sigma_2}{2} = \frac{p\,r}{4\,t}$$

thus $\quad \tau_{max} = \dfrac{\sigma_1}{2} = \dfrac{p\,r}{2\,t}$

this stress occurs on a plane that has been rotated 45° about the x axis

Stress at the inner surface

$\sigma_1 = pr / t \qquad \sigma_2 = pr / 2t \qquad \sigma_3 = - p$



(b)

$$(\tau_{max})_x = \frac{\sigma_1 - \sigma_3}{2} = \frac{p\,r}{2\,t} + \frac{p}{2}$$

$$(\tau_{max})_y = \frac{\sigma_2 - \sigma_3}{2} = \frac{p\,r}{4\,t} + \frac{p}{2}$$

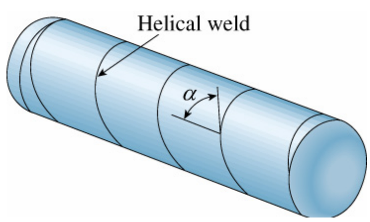
$$(\tau_{max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{p\,r}{4\,t}$$

Example 8-2

consider a cylindrical pressure vessel

$r \;=\; 1.8$ m $\qquad t \;=\; 20$ mm

weld angle $\alpha \;=\; 55°$



$E \;=\; 200$ GPa $\qquad \nu \;=\; 0.30 \qquad p \;=\; 800$ kPa

determine :

a. σ_1 and σ_2

b. the maximum in-plane and out-of-plane shear stress

c. ε_1 and ε_2

d. σ_w and τ_w in the welded seam

a. $\sigma_1 \;=\; p\,r\,/\,t \;=\; 800 \text{ x } 1800 / 20 \;=\; 72 \text{ MPa}$

$\sigma_2 \;=\; p\,r\,/\,2\,t \;=\; 800 \text{ x } 1800 / 2 \text{ x } 20 \;=\; 36 \text{ MPa}$

b.

$$(\tau_{max})_z = \frac{\sigma_1 - \sigma_2}{2} = \frac{p\,r}{4\,t} = 18 \text{ MPa} \quad \text{(in-plane shear)}$$

$$(\tau_{max})_x = \frac{\sigma_1}{2} = \frac{p\,r}{2\,t} = 36 \text{ MPa} \quad \text{(out-of-plane shear)}$$

c.

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\,\sigma_2) \qquad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\,\sigma_1)$$

$\sigma_1 \;=\; p\,r\,/\,t \qquad \sigma_1 \;=\; p\,r\,/\,2\,t$

then

$$\varepsilon_1 = \frac{\sigma_1}{2\,E}(2 - \nu) = \frac{72\,(2 - 0.3)}{2 \text{ x } 200 \text{ x } 10^3} = 306 \text{ x } 10^{-6}$$

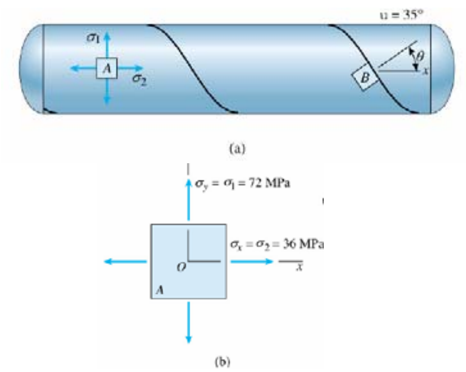
$$\varepsilon_2 = \frac{\sigma_2}{E}(1 - 2\,\nu) = \frac{36\,(1 - 2 \text{ x } 0.3)}{200 \text{ x } 10^3} = 72 \text{ x } 10^{-6}$$

d. $\theta \;=\; 90 - \alpha \;=\; 35^o$

$\sigma_x = \sigma_2 \;= pr\,/\,2t = 36$ MPa

$\sigma_y = \sigma_1 = pr\,/\,t = 72$ MPa

$\tau_{xy} \;=\; 0$



$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

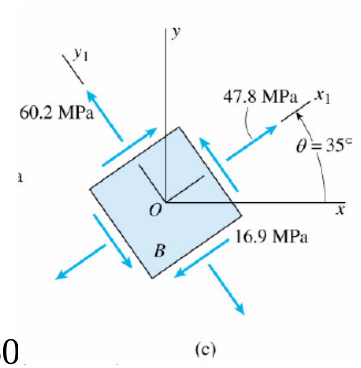
$$\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

then

$$\sigma_{x1} = \frac{p\,r}{4\,t}(3 - \cos 2\theta) = 47.8 \text{ MPa} = \sigma_w$$

$$\tau_{x1y1} = \frac{p\,r}{4\,t} \sin 2\theta = 16.9 \text{ MPa} = \tau_w$$

$$\sigma_{y1} = \sigma_1 + \sigma_2 - \sigma_{x1} = 60$$



(c)

Mohr's circle

$$R = \frac{72 - 36}{2} = 18 \text{ MPa}$$

$$\frac{\sigma_1 + \sigma_2}{2} = \frac{27 + 36}{2} = 54 \text{ MPa}$$

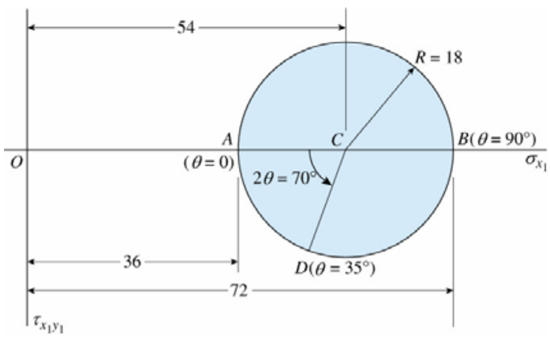
$$\theta = 35^o \qquad 2\theta = 70^o$$

$$\sigma_{x1} = 54 - R \cos 70^o = 47.8 \text{ MPa}$$

$$\tau_{x1y1} = R \sin 70^o = 16.9 \text{ MPa}$$

same results as earlier

Mohr's Circle :



8.4 Maximum Stresses in Beams

8.5 Combined Loadings