Chapter 2 Axial Loaded Members

2.1 Introduction

Axial loaded member : structural components subjected only to tension or compression, such as trusses, connecting rods, columns, etc.

change in length for prismatic bars, nonuniform bars are determined, it will be used to solve the statically indeterminate structures,

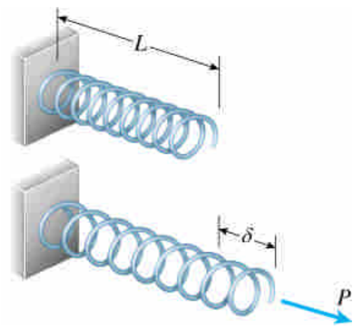
change in length by thermal effect is also considered

stresses on inclined sections will be calculated

several additional topics of importance in mechanics of materials will be introduced, such as strain energy, impact loading, fatigue, stress concentrations, and nonlinear behavior, etc.

2.2 Changes in Length of Axial Loaded Members

consider a coil spring with natural length L subjected to an axial load P



if the material of the spring is linear elastic, then

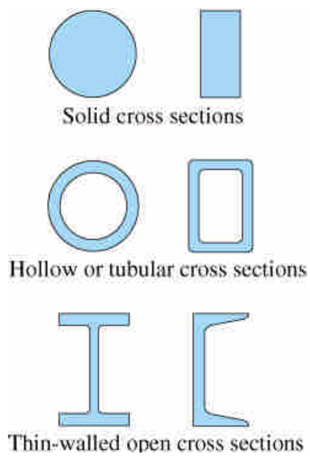
$$P = k\delta \qquad \text{or} \qquad \delta = fP$$

k : stiffness (spring constant)

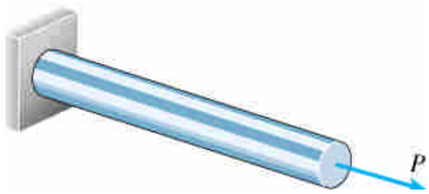
f : flexibility (compliance)

with $kf = 1$

some cross-sectional shapes are shown



prismatic bar : a member having straight longitudinal axis and constant cross section

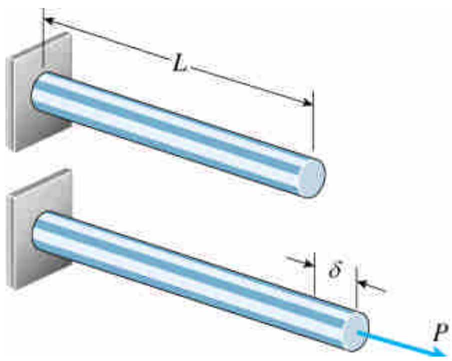


consider a prismatic bar with cross-sectional area A and length L subjected to an axial load P

then $\sigma = P / A$

and $\varepsilon = \delta / L$

material is elastic $\sigma = E \varepsilon$



$$\therefore \quad \delta = \varepsilon L = \frac{\sigma L}{E} = \frac{P L}{E A}$$

$E A$: axial rigidity of the bar

compare with $P = k \delta$ we have

$$k = \frac{E A}{L} \quad \text{or} \quad f = \frac{L}{E A}$$

Cable : used to transmit large tensile forces

the cross-section area of a cable is equal to the total cross-sectional area of the individual wires, called effective area, it is less than the area of a circle having the same diameter

also the modulus of elasticity (called the effective modulus) of a cable is less than the modulus of the material of which it is made

Example 2-1

a *L*-shape frame *ABC* with

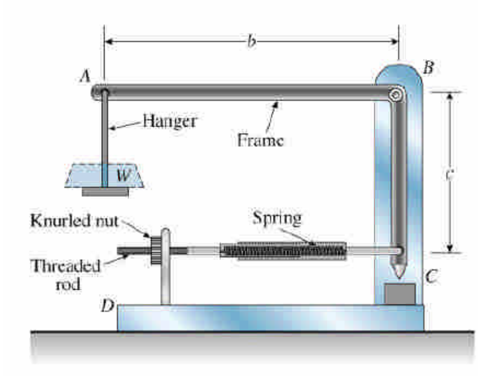
b = 10.5 in c = 6.4 in

spring constant k = 4.2 lb/in

pitch of the threads p = 1/16 in

if W = 2 lb, how many revolutions of the nut are required to bring the pointer back to the mark ?

(deformation of *ABC* are negligible)

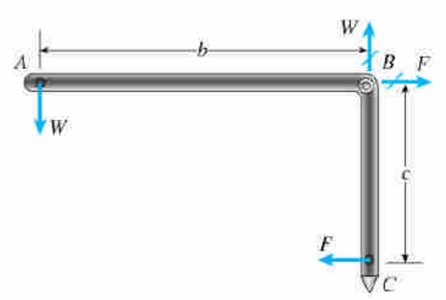


(a)

$$\Sigma M_B = 0 \Rightarrow F = W\,b\,/\,c$$

the elongation δ of the spring is

$$\delta = F/k = W\,b\,/\,c\,k = n\,p$$



(b)

Then

$$n = \frac{W\,b}{c\,k\,p} = \frac{(2\text{ lb})\,(10.5\text{ in})}{(6.4\text{ in})\,(4.2\text{ lb/in})\,(1/16\text{ in})} = 12.5\text{ revolutions}$$

Example 2-2

the contraption shown in figure

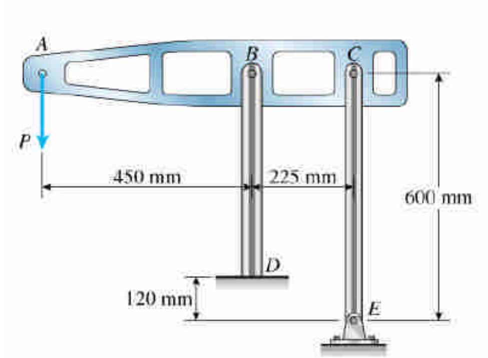
AB = 450 mm BC = 225 mm

BD = 480 mm CE = 600 mm

A_{BD} = 1,020 mm^3 A_{CE} = 520 mm^3

E = 205 GPa δ_A = 1 mm

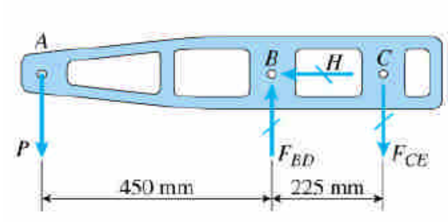
P_{max} = ?   *ABC* is rigid



(a)

take the free body *ABC*,

$\Sigma M_B = 0$ and $\Sigma F_y = 0$, we have



(b)

$F_{CE} = 2\,P \qquad F_{BD} = 3\,P$

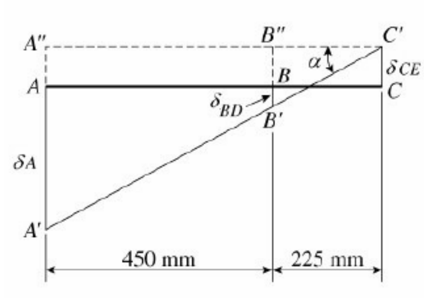
the shortening of BD is

$$\delta_{BD} = \frac{F_{BD}\,L_{BD}}{E\,A_{BD}} = \frac{(3\,P)\,(480\text{ mm})}{(205\text{ GPa})\,(1020\text{ mm}^2)} = 6.887\,P \times 10^{-6} \qquad (P : \text{N})$$

and the lengthening of CE is

$$\delta_{CE} = \frac{F_{CE}\,L_{CE}}{E\,A_{CE}} = \frac{(2\,P)\,(600\text{ mm})}{(205\text{ GPa})\,(520\text{ mm}^2)} = 11.26\,P \times 10^{-6} \qquad (P : \text{N})$$

a displacement diagram showing the beam is deformed from ABC to $A'B'C'$ using similar triangles, we can find the relationships between displacements



(c)

$$\frac{A'A''}{A''C'} = \frac{B'B''}{B''C'} \quad \text{or} \quad \frac{\delta_A + \delta_{CE}}{450 + 225} = \frac{\delta_{BD} + \delta_{CE}}{225}$$

or $$\frac{\delta_A + 11.26\,P \times 10^{-6}}{450 + 225} = \frac{6.887\,P \times 10^{-6} + 11.26\,P \times 10^{-6}}{225}$$

substitute for $\delta_A = 1$ mm and solve the equation for P

$$P = P_{max} = 23{,}200\text{ N} = 23.2\text{ kN}$$

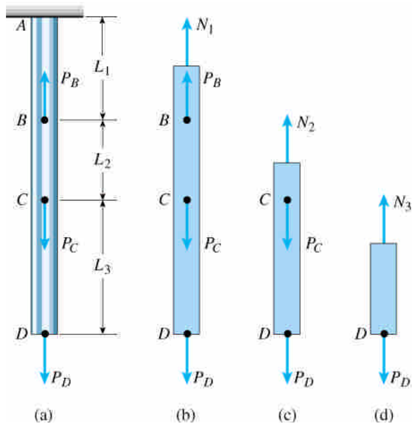
also the rotation of the beam can be calculated

$$\tan\alpha = \frac{A'A''}{A''C'} = \frac{\delta_A + \delta_{CE}}{675\text{ mm}} = \frac{(1 + 0.261)\text{ mm}}{675\text{ mm}} = 0.001868$$

$$\alpha = 0.11^\circ$$

2.3 Changes in Length Under Nonuniform Conditions

consider a prismatic bar is loaded by one or more axial loads, use the free body diagrams, the axial forces in each segment can be calculated



$$N_1 = -P_B + P_C + P_D$$

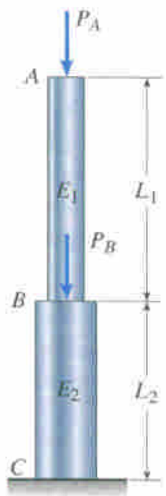
$$N_2 = P_C + P_D \qquad N_3 = P_D$$

the changes in length of each segment are

$$\delta_1 = \frac{N_1 L_1}{EA} \qquad \delta_2 = \frac{N_2 L_2}{EA} \qquad \delta_3 = \frac{N_3 L_3}{EA}$$

and the change in length of the entire bar is

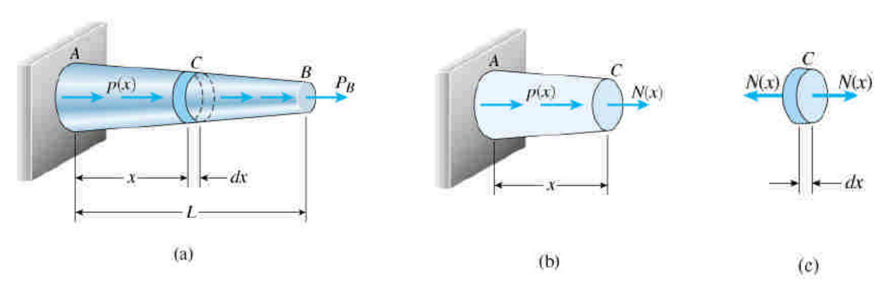
$$\delta = \delta_1 + \delta_2 + \delta_3$$



the same method can be used when the bar consists of several prismatic segments, each having different axial forces, different dimensions, and different materials, the change in length may be obtained

$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$

when either the axial force N or the cross-sectional area A vary continuously along the bar, the above equation no longer suitable consider a bar with varying cross-sectional area and varying axial force



for the element dx, the elongation is

$$d\,\delta = \frac{N(x)\,dx}{E\,A(x)}$$

the elongation of the entire bar is obtained by integrating

$$\delta = \int_0^L d\,\delta = \int_0^L \frac{N(x)\,dx}{E\,A(x)}$$

in the above equation, $\sigma = P/A$ is used, for the angle of the sides is 20°, the maximum error in normal stress is 3% as compared to the exact stress, for α small, error is less, for α large, more accurate methods may be needed

Example 2-3

L_1 = 20 in A_1 = 0.25 in^2

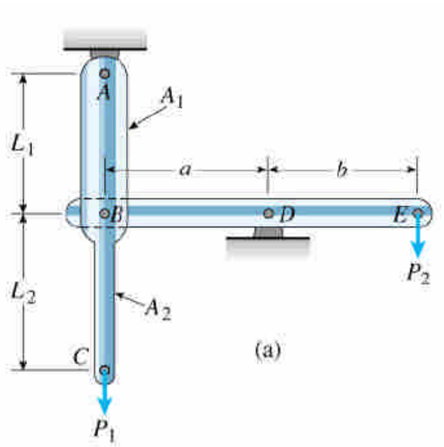
L_2 = 34.8 in A_2 = 0.15 in^2

E = 29 x 10^6 psi

a = 28 in b = 25 in

P_1 = 2100 lb P_2 = 5600 lb

calculate δ_C at point C

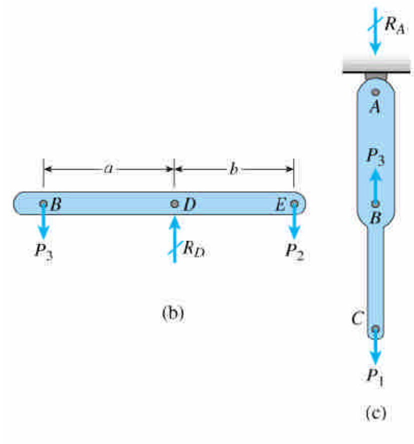


taking moment about D for the free body BDE

$$P_3 \quad = \quad P_2\, b \,/\, a \quad = 5600 \times 25 / 28 = 5000 \text{ lb}$$

on free body ABC

$$R_A \quad = \quad P_3 - P_1 \quad = 5000 - 2100 = 2900 \text{ lb}$$



then the elongation of ABC is

$$\delta \quad = \quad \sum_{i=1}^{n} \frac{N_i\, L_i}{E_i\, A_i} \quad = \quad \frac{N_1\, L_1}{E\, A_1} \quad + \quad \frac{N_2\, L_2}{E\, A_2}$$

$$= \quad \frac{(-2900 \text{ lb})\,(20 \text{ in})}{(29 \times 10^6 \text{ psi})\,(0.25 \text{ in}^2)} \quad + \quad \frac{(2100 \text{ lb})\,(34.8 \text{ in})}{(29 \times 10^6 \text{ psi})\,(0.15 \text{ in}^2)}$$

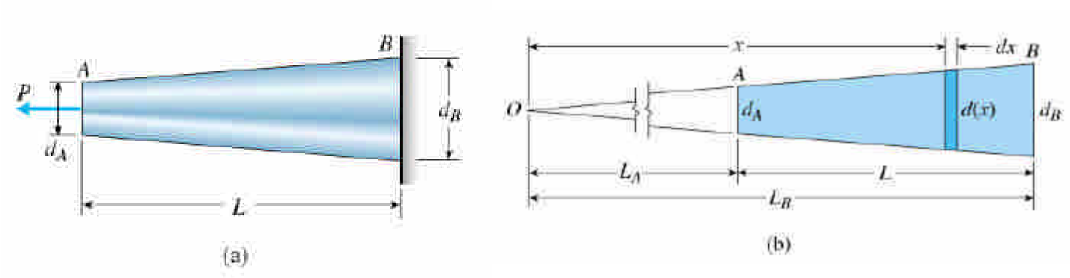
$$= \quad -\,0.0080 \text{ in} \quad + \quad 0.0168 \text{ in} \quad = \quad 0.0088 \text{ in}$$

$$\delta \quad = \quad \delta_C \quad = \quad 0.0088 \text{ in} \quad (\downarrow)$$

this displacement is downward

Example 2-4

a tapered bar AB of solid circular cross section with length L is supported to a tensile load P, determine δ



$$\frac{L_A}{L_B} = \frac{d_A}{d_B} \qquad \frac{d(x)}{d_A} = \frac{x}{L_A} \qquad d(x) = \frac{d_A\, x}{L_A}$$

the cross-sectional area at distance x is

$$A(x) = \frac{\pi [d(x)]^2}{4} = \frac{\pi d_A^2 x^2}{4 L_A^2}$$

then the elongation of the bar is

$$\delta = \int \frac{N(x)\, dx}{E A(x)} = \int_{L_A}^{L_B} \frac{P\, dx\, (4L_A^2)}{E (\pi d_A^2 x^2)} = \frac{4 P L_A^2}{\pi E d_A^2} \int_{L_A}^{L_B} \frac{dx}{x^2}$$

$$\delta = \frac{4 P L_A^2}{\pi E d_A^2} \left[- \frac{1}{x}\right]_{L_A}^{L_B} = \frac{4 P L_A^2}{\pi E d_A^2} \left(\frac{1}{L_A} - \frac{1}{L_B} \right) = \frac{4 P L_A^2}{\pi E d_A^2} \frac{L_B - L_A}{L_A L_B}$$

$$= \frac{4 P L}{\pi E d_A^2} \left(\frac{L_A}{L_B} \right) = \frac{4 P L}{\pi E d_A d_B}$$

for a prismatic bar $d_A = d_B = d$

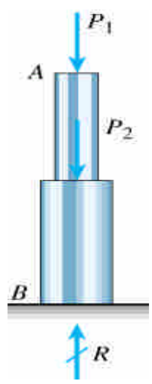
$$\delta = \frac{4 P L}{\pi E d^2} = \frac{P L}{E A}$$

2.4 Statically Indeterminate Structures

flexibility method (force method) [another method is stiffness method (displacement method)]

consider an axial loaded member

equation of equilibrium



$$\Sigma F_y = 0 \qquad R_A - P + R_B = 0$$

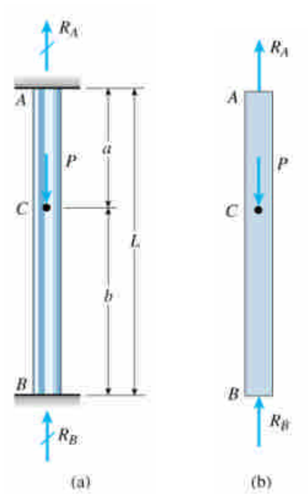
one equation for two unknowns

[statically indeterminate]

∵ both ends A and B are fixed, thus

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0$$

this is called **equation of compatibility**



elongation of each part can be obtained

$$\delta_{AC} = \frac{R_A\,a}{E\,A} \qquad \delta_{BC} = -\frac{R_B\,b}{E\,A}$$

thus, we have

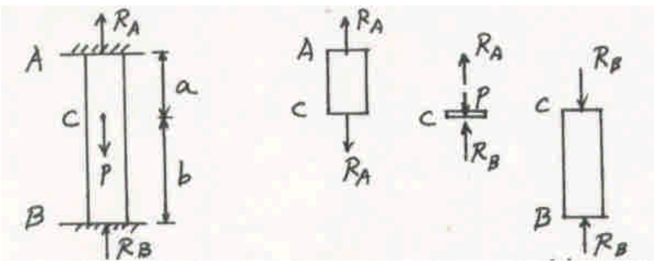
$$\frac{R_A\,a}{E\,A} - \frac{R_B\,b}{E\,A} = 0$$

then $\quad R_A = P\,b\,/\,L \qquad R_B = P\,a\,/\,L$

and $\quad \delta_C = \delta_{AC} = \dfrac{R_A\,a}{E\,A} = \dfrac{P\,a\,b}{L\,E\,A}$

summarize of flexibility method : take the force as unknown quantity, and the elongation of each part in terms of these forces, use the equation of compatibility of displacement to solve the unknown force

stiffness method to solve the same problem



the axial forces R_A and R_B can be expressed in terms of δ_C

$$R_A = \frac{E\,A}{a}\delta_C \qquad R_B = \frac{E\,A}{b}\delta_C$$

equation of equilibrium

$$R_A + R_B = P$$

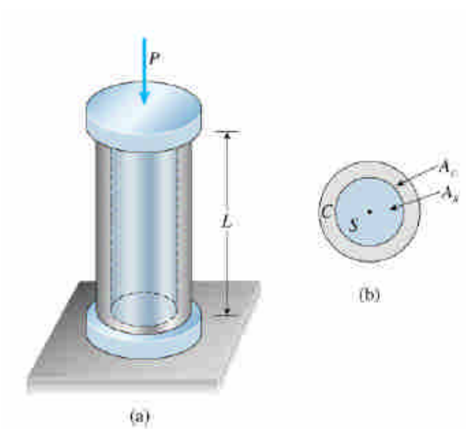
$$\frac{E\,A}{a}\delta_C + \frac{E\,A}{b}\delta_C = P \quad \Rightarrow \quad \delta_C = \frac{P\,a\,b}{E\,A\,L}$$

and $\quad R_A = P\,b\,/\,L \qquad R_B = P\,a\,/\,L$

summarize of stiffness method : to select a suitable displacement as unknown quantity, and the unknown forces in terms of these displacement, use the equation of equilibrium to solve the displacement

Example 2-5

a solid circular steel cylinder S is encased in a hollow circular copper C subjected to a compressive force P



for steel : E_s, A_s

for copper : E_c, A_c

determine P_s, P_c, σ_s, σ_c, δ

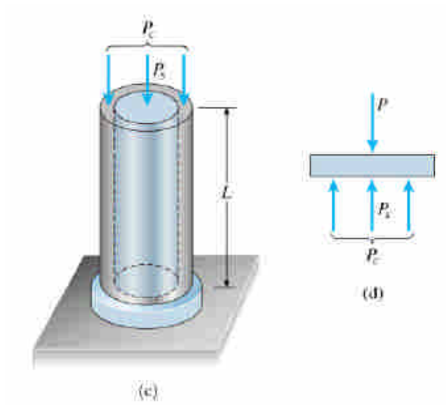
P_s : force in steel, P_c : force in copper

force equilibrium

$$P_s + P_c = P$$

flexibility method

for the copper tube



$$\delta_c = \frac{P_c\,L}{E_c\,A_c} = \frac{P\,L}{E_c\,A_c} - \frac{P_s\,L}{E_c\,A_c}$$

for the steel cylinder

$$\delta_s = \frac{P_s\,L}{E_s\,A_s}$$

$$\delta_s = \delta_c \qquad \frac{P_s L}{E_s A_s} = \frac{P L}{E_c A_c} - \frac{P_s L}{E_c A_c}$$

$$P_s = \frac{E_s A_s}{E_s A_s + E_c A_c} P$$

$$P_c = P - P_s = \frac{E_c A_c}{E_s A_s + E_c A_c} P$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{P E_s}{E_s A_s + E_c A_c} \qquad \sigma_c = \frac{P_c}{A_s} = \frac{P E_c}{E_s A_s + E_c A_c}$$

the shortening of the assembly δ is

$$\delta = \frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c} = \frac{P L}{E_s A_s + E_c A_c}$$

stiffness method : P_s and P_c in terms of displacement δ

$$P_s = \frac{E_s A_s}{L} \delta \qquad P_c = \frac{E_c A_c}{L} \delta$$

equation of equilibrium

$$P_s + P_c = P$$

$$\frac{E_s A_s}{L} \delta + \frac{E_c A_c}{L} \delta = P$$

it is obtained $\delta = \dfrac{P L}{E_s A_s + E_c A_c}$ same result as above

Example 2-6

a horizontal bar AB is pinned at end A and supported by two wires at points D and F

a vertical load P acts at end B

(a) $(\sigma_{\text{all}})_{\text{CD}} = \sigma_1$ $(\sigma_{\text{all}})_{\text{EF}} = \sigma_2$

wire CD : E_1, d_1; wire EF : E_2, d_2

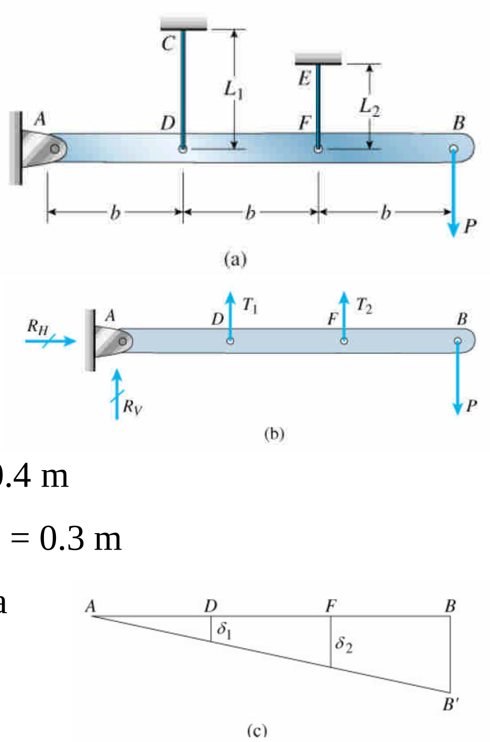
$P_{all} = ?$

(b) E_1 = 72 GPa (Al), d_1 = 4 mm, L_1 = 0.4 m

E_2 = 45 GPa (Mg), d_2 = 3 mm, L_2 = 0.3 m

$\sigma_1 =$ 200 MPa $\sigma_2 =$ 125 MPa

$P_{all} = ?$



(a) (b) (c)

take the bar AB as the free body

$$\Sigma M_A = 0 \Rightarrow T_1 b + T_2 (2b) - P(3b) = 0$$

i.e. $T_1 + 2T_2 = 3P$

assume the bar is rigid, the geometric relationship between elongations is

$$\delta_2 = 2\delta_1$$

$$\delta_1 = \frac{T_1 L_1}{E_1 A_1} = f_1 T_1 \qquad \delta_2 = \frac{T_2 L_2}{E_2 A_2} = f_2 T_2$$

$f = L / E A$ is the flexibility of wires, then we have

$$f_2 T_2 = 2 f_1 T_1$$

thus the forces T_1 and T_2 can be obtained

$$T_1 = \frac{3 f_2 P}{4 f_1 + f_2} \qquad T_2 = \frac{6 f_1 P}{4 f_1 + f_2}$$

the stresses of the wires are

$$\sigma_1 = \frac{T_1}{A_1} = \frac{3\,P}{A_1}\left(\frac{f_2}{4\,f_1 + f_2}\right) \quad => \quad P_1 = \frac{\sigma_1 A_1 (4\,f_1 + f_2)}{3\,f_2}$$

$$\sigma_2 = \frac{T_2}{A_2} = \frac{6\,P}{A_2}\left(\frac{f_1}{4\,f_1 + f_2}\right) \quad => \quad P_2 = \frac{\sigma_2 A_2 (4\,f_1 + f_2)}{6\,f_2}$$

$$P_{allow} = \text{minimum}\,(P_1, P_2)$$

(b) numerical calculation

$A_1 = \pi\, d_1^2 / 4 = 12.57\ \text{mm}^2$ $\qquad A_2 = \pi\, d_2^2 / 4 = 7.069\ \text{mm}^2$

$f_1 = L_1 E_1 / A_1 = 0.442 \times 10^{-6}$ m/N

$f_2 = L_2 E_2 / A_2 = 0.9431 \times 10^{-6}$ m/N

with $\sigma_1 = 200$ MPa and $\sigma_2 = 125$ MPa

we can get $P_1 = 2.41$ kN and $P_2 = 1.26$ kN

then $P_{allow} = 1.26$ kN

at this load, $\sigma_{Mg} = 175$ MPa,

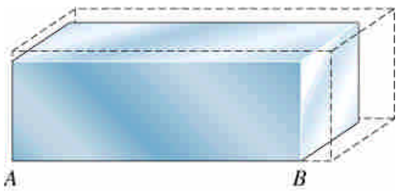
at that time $\sigma_{Al} = 200\,(1.26/2.41) = 105$ MPa $<$ 200 MPa

2.5 Thermal Effects, Misfits and Prestrains

temperature change => dimension change => thermal stress and strain

for most materials, thermal strain ε_T is proportional to the temperature change $\triangle T$

$$\varepsilon_T = \alpha \triangle T$$



α : thermal expansion coefficient

(1/°C or 1/°F)

$\triangle T$: increase in temperature

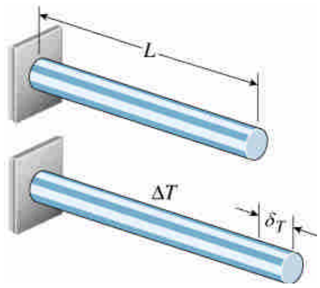
thermal strain usually are reversible, expand when heard and contract when cooled

no stress are produced for a free expansion body

but for some special material do not behave in the customary manner, over certain temperature range, they expand when cooled and contract when heated (internal structure change), e.g. water : maximum density at 4°C

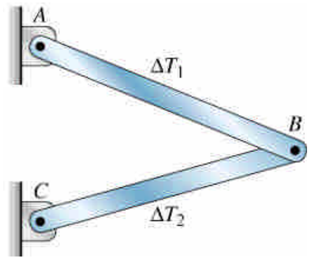
for a bar with length L, its elongation δ_t due to temperature change $\triangle T$ is

$$\delta_t = \varepsilon_t L = \alpha (\triangle T) L$$

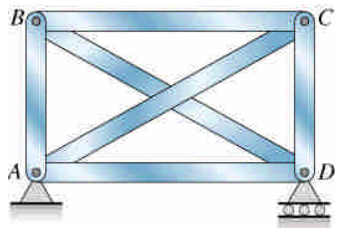


this is the temperature-displacement relation

no stress are produced in a statically determinate structure when one or more members undergo a uniform temperature change



temperature change in a statically indeterminate structure will usually produce stress in members, called thermal stress



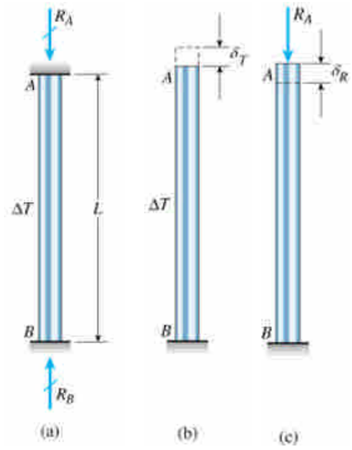
for the statically indeterminate structure, free expansion or contraction is no longer possible

thermal stress may also occurs when a member is heated in a nonuniform manner for structure is determinate or indeterminate

Example 2-7

a prismatic bar AB of length L

the temperature is raised uniformly by $\triangle T$



$\Sigma F_y = 0 \qquad R_A = R_B = R$

displacement at A due to

$\triangle T: \quad \delta_t = \alpha (\triangle T) L \qquad (\uparrow)$

$R: \ \delta_R = R L / E A \qquad (\downarrow)$

$\delta_A = \delta_t - \delta_R = 0$

$$\therefore \ \alpha (\triangle T) L = \frac{R L}{E A}$$

$R = E A \alpha (\triangle T)$ and $\sigma = R / A = E \alpha (\triangle T)$

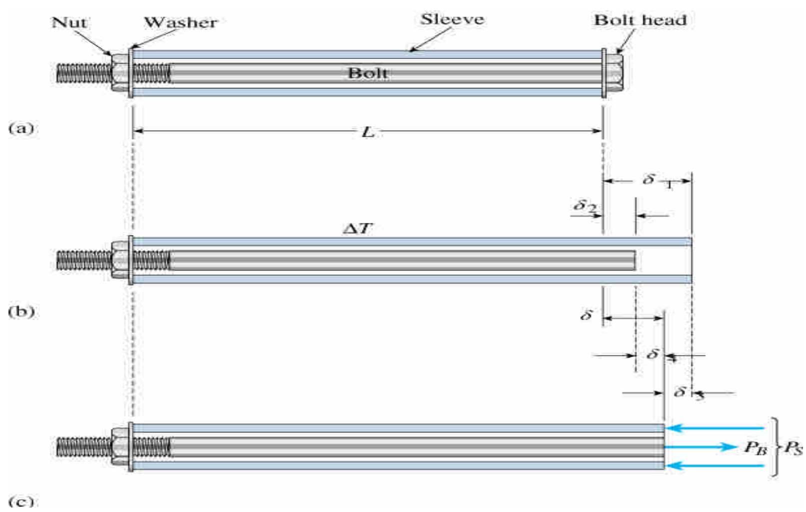
the stress is compressive when the temperature of the bar increases

Example 2-8

a sleeve and the bolt of the same length L are made of different materials

sleeve : A_s, α_s bolt : A_b, α_b $\alpha_s > \alpha_b$

temperature raise $\triangle T$, σ_s, σ_b, δ = ?



take a free body as remove the head of the bolt

for temperature raise $\triangle T$

$$\delta_1 = \alpha_s (\triangle T) L \qquad \delta_2 = \alpha_b (\triangle T) L$$

if $\alpha_s > \alpha_b \quad => \quad \delta_1 > \delta_2$

the force existing in the sleeve and bolt, until the final elongation of the sleeve and bolt are the same, then

$$\delta_3 = \frac{P_s L}{E_s A_s} \qquad \delta_4 = \frac{P_b L}{E_b A_b}$$

equation of compatibility

$$\delta = \delta_1 - \delta_3 = \delta_2 + \delta_4$$

$$\alpha_s (\triangle T) L - \frac{P_s L}{E_s A_s} = \alpha_b (\triangle T) L + \frac{P_b L}{E_b A_b}$$

equation of equilibrium

$$P_b = P_s$$

it is obtained

$$P_b = P_s = \frac{(\alpha_s - \alpha_b)(\triangle T) E_s A_s E_b A_b}{E_s A_s + E_b A_b}$$

the stresses in the sleeve and bolt are

$$\sigma_s = \frac{P_s}{A_s} = \frac{(\alpha_s - \alpha_b)(\triangle T) E_s E_b A_b}{E_s A_s + E_b A_b}$$

$$\sigma_b = \frac{P_b}{A_s} = \frac{(\alpha_s - \alpha_b)(\triangle T) E_s A_s E_b}{E_s A_s + E_b A_b}$$

and the elongation of the sleeve and bolt is

$$\delta = \frac{(\alpha_s E_s A + \alpha_b E_b A_b)(\triangle T) L}{E_s A_s + E_b A_b}$$

partial check :

if $\alpha_s = \alpha_b = \alpha$, then $P_b = P_s = 0$, and $\delta = \alpha (\triangle T) L$ (O.K.)

stiffness method : choose the final displacement δ as an unknown quantity

$$P_s = \frac{E_s A_s}{L} [\alpha_s (\triangle T) L - \delta]$$

$$P_b = \frac{E_b A_b}{L} [\delta - \alpha_b (\triangle T) L]$$

$\because$ $P_s = P_b$, it is obtained

$$\delta = \frac{(\alpha_s E_s A + \alpha_b E_b A_b)(\triangle T) L}{E_s A_s + E_b A_b}$$ same result

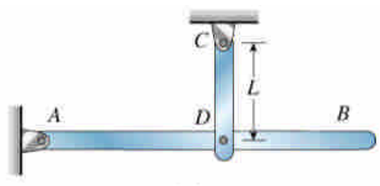
Misfits and Prestrains

For the length of the bars slightly different due to manufacture

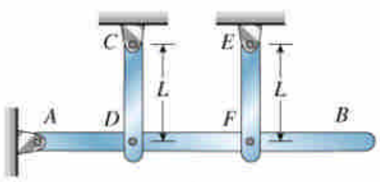
if the structure is statically determinate, no prestrains and prestress

if the structure is statically indeterminate, it is not free to adjust to misfits, prestrains and prestresses will be occurred

if CD is slightly longer, CD is in compression and EF is in tension



(a)

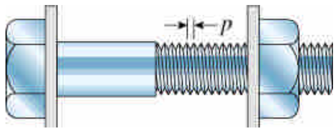


(a)

if $\boldsymbol{P}$ is added, additional strains and stresses will be produced

Bolts and Turnbuckles

for a bolt, the distance δ traveled by the nut is

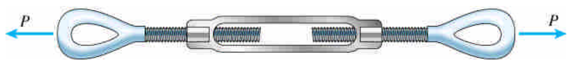


$$\delta \quad = \quad n\,p$$

where p is the pitch of the threads

for a double-acting turnbuckle, the shorten δ is

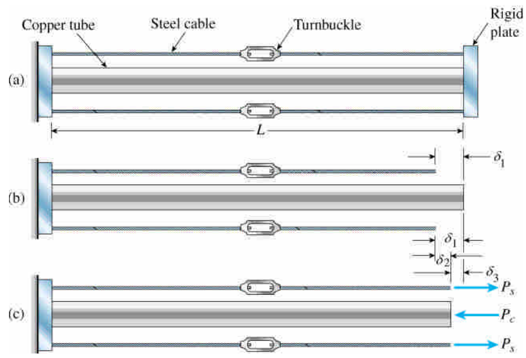
$$\delta \quad = \quad 2\,n\,p$$



Example 2-9

(a) determine the forces in tube and cables when the buckle with n turns

(b) determine the shorten of the tube



$$\delta_1 \quad = \quad 2\,n\,p$$

$$\delta_2 \quad = \quad P_s\,L\,/\,E_s\,A_s$$

$$\delta_3 \quad = \quad P_c\,L\,/\,E_c\,A_c$$

eq. of compatibility $\quad \delta_1 - \delta_2 \quad = \quad \delta_3$

$$2\,n\,p \;-\; \frac{P_s\,L}{E_s\,A_s} \;=\; \frac{P_c\,L}{E_c\,A_c} \qquad (1)$$

eq. of equilibrium $\qquad 2\,P_s \quad = \quad P_c \qquad (2)$

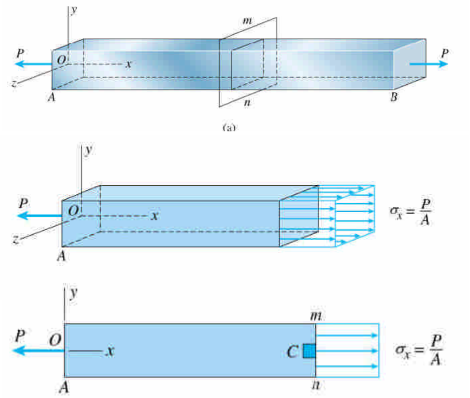
(1) and (2)

$$P_s \;=\; \frac{2\,n\,p\,E_c\,A_c\,E_s\,A_s}{L\,(E_c\,A_c + 2\,E_s\,A_s)} \qquad\qquad P_c \;=\; \frac{4\,n\,p\,E_c\,A_c\,E_s\,A_s}{L\,(E_c\,A_c + 2\,E_s\,A_s)}$$

Shorten of the tube is

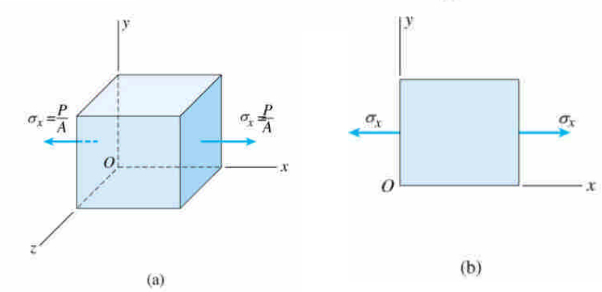
$$\delta_3 = \frac{P_c\,L}{E_c\,A_c} = \frac{4\,n\,p\,E_s\,A_s}{E_c\,A_c + 2\,E_s\,A_s}$$

2-6 Stresses on Inclined Sections



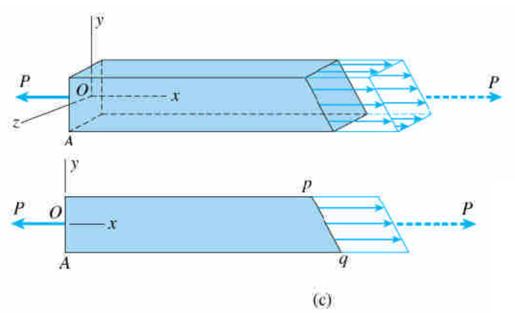
consider a prismatic bar subjected to an axial load P

the normal stress $\sigma_x = P / A$ acting on mn in 3-D and 2-D views are shown



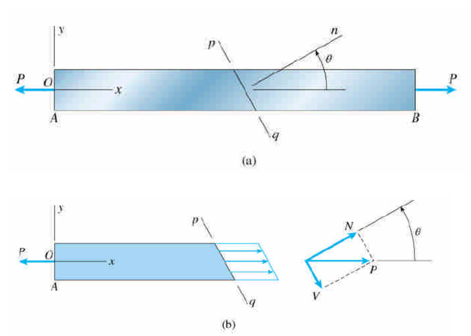
(a) (b)

also the stress element in 3-D and 2-D views are presented (the dimensions of the element are assumed to be infinitesimally small)



(c)

we now to investigate the stress on the inclined sections pq, the 3-D and 2-D views are shown



the normal and shear forces on pq are calculated

$N = P \cos\theta \quad V = P \sin\theta$

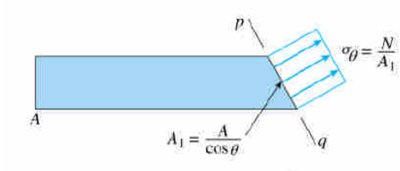
the cross-sectional area of pq is

$A_1 = A / \cos\theta$

thus the normal and shear stresses on pq

are

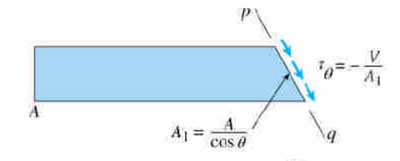
$$\sigma_\theta = \frac{N}{A_1} = \frac{P\cos\theta}{A/\cos\theta} = \sigma_x \cos^2\theta$$



(c)

$$\tau_\theta = -\frac{V}{A_1} = -\frac{P\sin\theta}{A/\cos\theta} = -\sigma_x \sin\theta\cos\theta$$

sign convention : positive as shown in figure



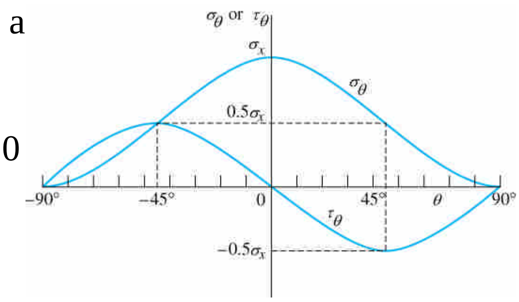
(d)

also using the trigonometric relations, we get

$$\sigma_\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

$$\tau_\theta = -\frac{\sigma_x}{2} \sin 2\theta$$

it is seen that the normal and shear stresses are changed with the angle θ a shown in figure



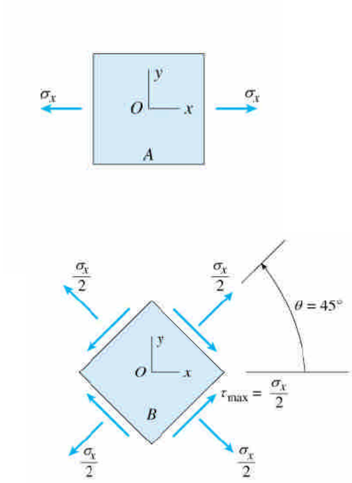
maximum normal stress occurs at $\theta = 0$

$$\sigma_{max} = \sigma_x$$

maximum shear stress occurs at $\theta = \pm 45^o$

$$\tau_{max} = \sigma_x / 2$$

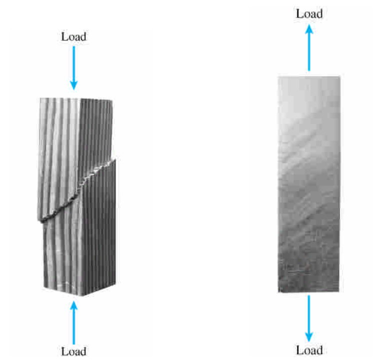
the shear stress may be controlling stress if the material is much weaker then in tension, such as



(c)

short block of wood in compression

mild steel in tension (Luder's bands)



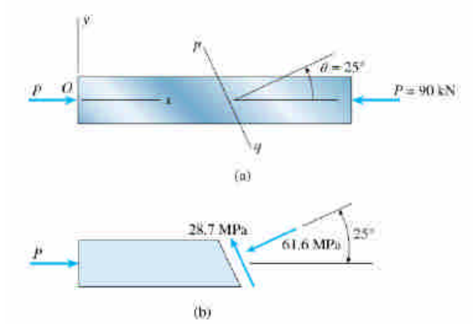
Example 2-10

a prismatic bar, $A = 1200\ mm^2$

$P = 90$ kN $\qquad \theta = 25^o$

determine the stress state at pq section

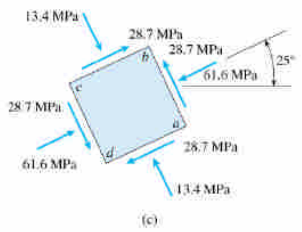
show the stresses on a stress element



$$\sigma_x = -\frac{P}{A} = -\frac{90\ \text{kN}}{1200\ \text{mm}^2} = -75\ \text{MPa}$$

$$\sigma_\theta = \sigma_x \cos^2\theta = (-75\ \text{MPa})(\cos 25^o)^2 = -61.6\ \text{MPa}$$

$$\tau_\theta = -\sigma_x \sin\theta\cos\theta = 28.7\ \text{MPa}$$



to determine the complete stress state

on face ab, $\theta = 25^o$, the stresses are calculated

on face ad, $\theta = 115^o$, the stresses are

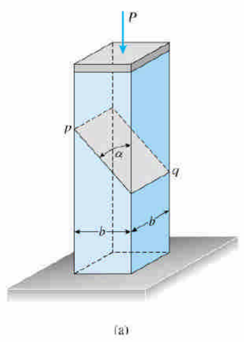
$$\sigma_\theta = \sigma_x \cos^2\theta = -75\cos^2 115^o = -13.4\ \text{MPa}$$

shear stress is the same as on face ab, the complete stress state is shown in figure

Example 2-11

a plastic bar with square cross section of side b is connected by a glued joint along plane pq

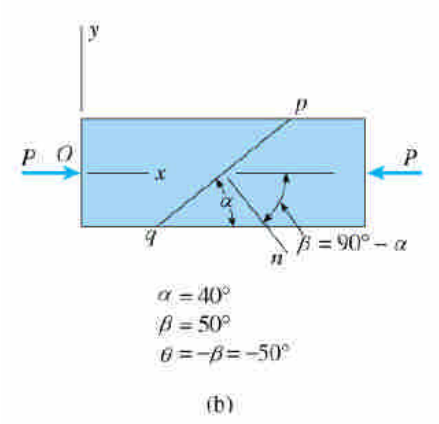
$P = 8000$ lb $\quad \alpha = 40^o$



σ_{all} = 1100 psi τ_{all} = 600 psi

$(\sigma_{glude})_{all}$ = 750 psi $(\tau_{glude})_{all}$ = 500 psi

determine minimum width b



$$A = P / \sigma_x$$

$\because \quad \alpha = 40^o \qquad \therefore \quad \theta = -\beta = -50^o$

$$\sigma_x = \frac{\sigma_\theta}{\cos^2\theta} \qquad \sigma_x = -\frac{\tau_\theta}{\sin\theta\cos\theta}$$

(a) based on the allowable stresses in the glued joint

σ_θ = - 750 psi θ = - 50° ==> σ_x = - 1815 psi

τ_θ = - 500 psi θ = - 50° ==> σ_x = - 1015 psi

(b) based on the allowable stresses in the plastic

σ_x = - 1100 psi

τ_{max} = 600 psi occurs on the plane at 45° = $\sigma_x / 2$

=> σ_x = - 1200 psi

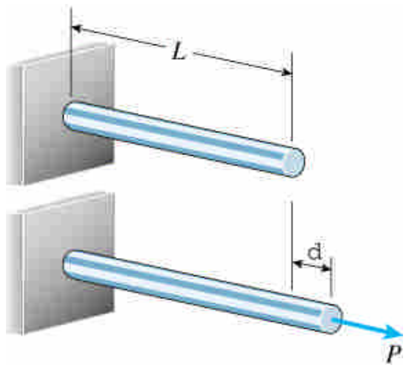
(c) minimum width of the bar, choose σ_x = - 1015 psi, then

$$A = \frac{8000 \text{ lb}}{1015 \text{ psi}} = 7.88 \text{ in}^2$$

$b_{min} = \sqrt{A} = \sqrt{7.88 \text{ in}^2} = 2.81$ in, select b = 3 in

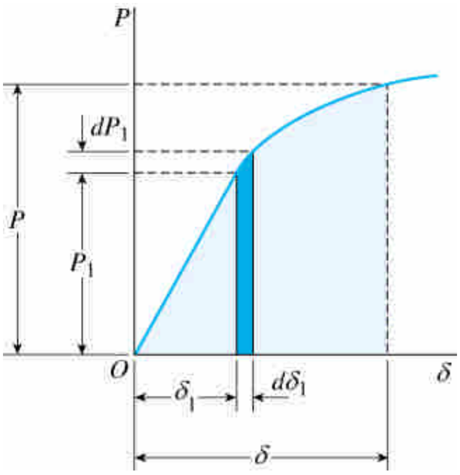
2.7 Strain Energy

the concept of strain energy principles are widely used for determining the response of machines and structures to both



static and dynamic loads

consider a prismatic bar of length L subjected to tension force P, which is gradually increases from zero to maximum value P, the load-deflection diagram is plotted



after P_1 is applied, the corresponding elongation is δ, additional force dP_1 produce $d\delta_1$, the work done by P_1 is

$$dW = P_1 \, d\delta_1$$

and the total work done is

$$W = \int_0^{\delta} P_1 \, d\delta_1$$

the work by the load is equal the area under the load-deflection curve

strain energy : energy absorbed by the bar during the load process

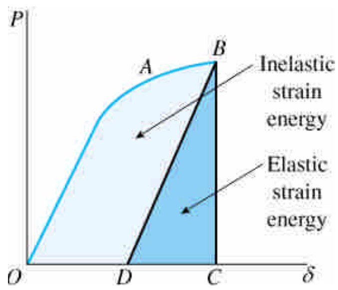
thus the strain energy U is

$$U = W = \int_0^{\delta} P_1 \, d\delta_1$$

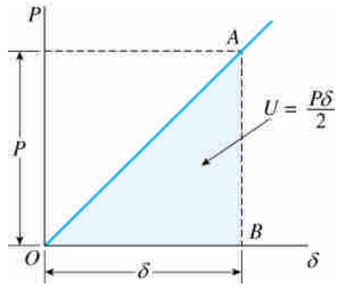
U referred to as internal work

the unit of U and W is J (J = N · m) [SI], ft-lb [USCS]

during unloading, some or all the strain energy of the bar may be recovered



if P is maintained below the linear elastic range

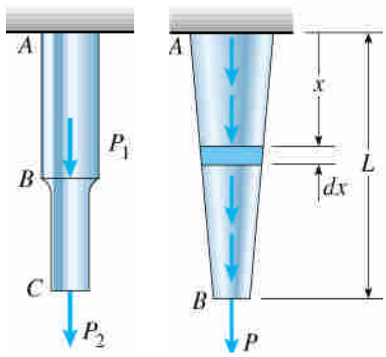


$$U = \frac{P\,\delta}{2} = W$$

$$\because \quad \delta = \frac{P\,L}{E\,A} \quad \therefore \quad U = \frac{P^2\,L}{2\,E\,A} = \frac{E\,A\,\delta^2}{2\,L}$$

$$\text{also} \quad k = \frac{E\,A}{L} \quad \text{thus} \quad U = \frac{P^2}{2\,k} = \frac{k\,\delta^2}{2}$$

the total energy of a bar consisting of several segments is

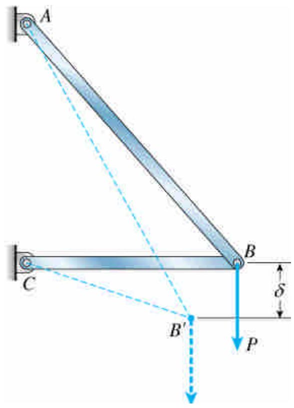


$$U = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{N_i^2\,L_i}{2\,E_i\,A_i}$$

the strain energy for a nonprismatic bar or a bar with varying axial force can be written as

$$U = \int_0^L \frac{P_x^2\,dx}{2\,E\,A_x}$$

displacements caused by a single load



$$U = W = \frac{P\,\delta}{2} \quad ==> \quad \delta = \frac{2\,U}{P}$$

$$U = U_{AB} + U_{BC}$$

strain energy density u is the total strain energy U per unit volume for linear elastic behavior

$$u = \frac{U}{V} = \frac{U}{A\,L} = \frac{P^2\,L}{2\,E\,A}\,\frac{1}{A\,L} = \frac{\sigma_x^{\;2}}{2\,E} = \frac{E\,\varepsilon^2}{2} = \frac{\sigma_x\,\varepsilon}{2}$$

modulus of resilience u_r

$$u_r = \frac{\sigma^2_{pl}}{2E}$$

σ_{pl} : proportional limit

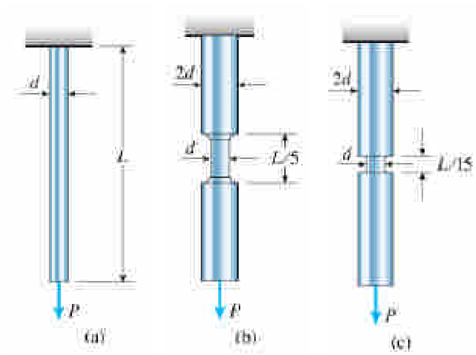
resilience represents the ability of the material to absorb and release energy within the elastic range

modulus of toughness u_t is the area under the stress-strain curve when fracture, u_t represents the maximum energy density can be absorbed by the material

strain energy (density) is always a positive quantity

Example 2-12

3 round bars having same L but different shapes as shown



when subjected to the same load P

calculate the energy stored in each bar

$$U_1 = \frac{P^2 L}{2EA}$$

$$U_2 = \sum_{i=1}^{n} \frac{N_i^2 L_i}{2E_iA_i} = \frac{P^2 (L/5)}{2EA} + \frac{P^2 (4L/5)}{2E(4A)} = \frac{P^2 L}{5EA} = \frac{2U_1}{5}$$

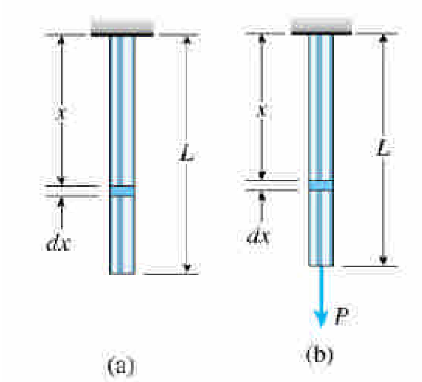
$$U_3 = \sum_{i=1}^{n} \frac{N_i^2 L_i}{2E_iA_i} = \frac{P^2 (L/15)}{2EA} + \frac{P^2 (14L/15)}{2E(4A)} = \frac{3P^2 L}{20EA} = \frac{3U_1}{10}$$

the third bar has the least energy-absorbing capacity, it takes only a small amount of work to bring the tensile stress to a high value

when the loads are dynamic, the ability to absorb energy is important, the presence of grooves is very damaging

Example 2-13

determine the strain energy of a prismatic bar subjected to (a) its own weight (b) own weight plus a load P



(a) consider an element dx

$$N(x) = \gamma A (L - x)$$

γ : weight density

$$U = \int_0^L \frac{[N(x)]^2 dx}{2 E A(x)} = \int_0^L \frac{[\gamma A(L - x)]^2 dx}{2 E A} = \frac{\gamma^2 A L^3}{6 E}$$

it can be obtained from the energy density

$$\sigma = \frac{N(x)}{A} = \gamma (L - x)$$

$$u = \frac{\sigma^2}{2 E} = \frac{\gamma^2 (L - x)^2}{2 E}$$

$$U = \int u \, dV = \int_0^L u \, (A dx) = \int_0^L \frac{[\gamma A(L - x)]^2 dx}{2 E A} = \frac{\gamma^2 A L^3}{6 E}$$

same result as above

(b) own weight plus P

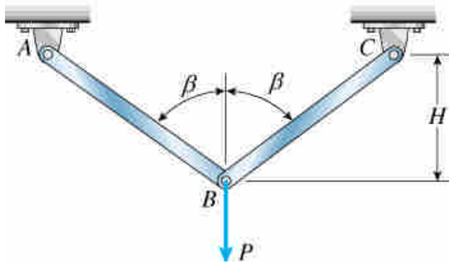
$$N(x) = \gamma A (L - x) + P$$

$$U = \int_0^L \frac{[\gamma A(L - x) + P]^2 dx}{2 E A} = \frac{\gamma^2 A L^3}{6 E} + \frac{\gamma P L^2}{2 E} + \frac{P^2 L}{2 E A}$$

note that the strain energy of a bar subjected to two loads is not equal to the sum of the strain energies produced by the individual loads

Example 2-14

determine the vertical displacement δ_B of the joint B, both bar have the same axial rigidity $E A$



equation of equilibrium in vertical direction, it is obtained

$$F = \frac{P}{2 \cos \beta}$$

the strain energy of the two bars is

$$U = 2 \cdot \frac{F^2 L_1}{2 E A} = \frac{P^2 H}{4 E A \cos^3\beta} \qquad L_1 = H / \cos \beta$$

the work of force P is

$$W = P \delta_B / 2$$

equating U and W and solving for δ_B

$$\delta_B = \frac{P H}{2 E A \cos^3\beta}$$

this is the energy method to find the displacement, we did not need to draw a displacement diagram at joint B

Example 2-15

a cylinder and cylinder head are clamped by bolts as shown

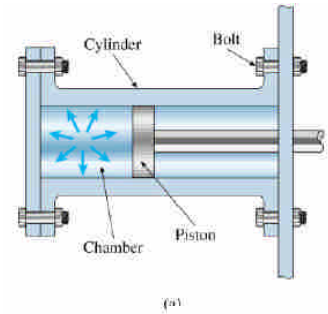
d = 0.5 in d_r = 0.406 in g = 1.5 in t = 0.25 in L = 13.5 in

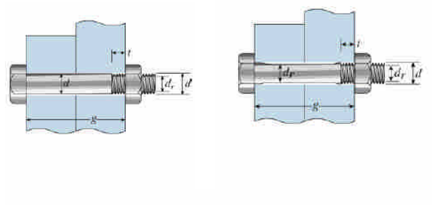
compare the energy absorbing of the three bolt configurations

(a) original bolt

$$U_1 = \sum_{i=1}^{n} \frac{N_i^2 L_i}{2 E_i A_i} = \frac{P^2 (g - t)}{2 E A_s} + \frac{P^2 t}{2 E A_r}$$

$$A_s = \frac{\pi d^2}{4} \qquad A_r = \frac{\pi d_r^2}{4}$$



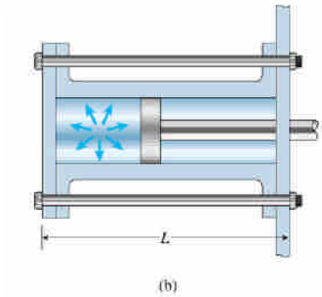


thus U_1 can be written as

$$U_1 = \frac{2 P^2 (g - t)}{\pi E d^2} + \frac{2 P^2 t}{\pi E d_r^2}$$

(b) bolt with reduced shank diameter

$$U_2 = \frac{P^2 g}{2 E A_r} = \frac{2 P^2 g}{\pi E d_r^2}$$



the ratio of strain energy U_2 / U_1 is

$$\frac{U_2}{U_1} = \frac{g d^2}{(g - t) d_r^2 + t d^2} = \frac{1.5 \cdot 0.5^2}{(1.5 - 0.25) 0.406^2 + 0.25 \times 0.5} = 1.40$$

(c) long bolts

$$U_3 = \frac{2 P^2 (L - t)}{\pi E d^2} + \frac{2 P^2 t}{\pi E d_r^2}$$

the ratio of strain energy $U_3 / 2 U_1$ is

$$\frac{U_3}{2 U_1} = \frac{(L - t) d_r^2 + t d^2}{2 [(g - t) d_r^2 + t d^2]}$$

$$= \frac{(13.5 - 0.25) 0.406^2 + 0.25 \cdot 0.5^2}{2 [(1.5 - 0.25) 0.406^2 + 0.25 \cdot 0.5^2]} = 4.18$$

thus, the long bolts increase the energy-absorbing capacity when designing bolts, designers must also consider the maximum tensile stresses, maximum bearing stresses, stress concentration, and other matters

2.8 Impact Loading

2.9 Repeated Loading and Fatigue

2.10 Stress Concentrations

2.11 Nonlinear Behavior

2.12 Elastoplastic Analysis