Midterm for Thermal Physics (II)
Date: Mar 26, 2012

(1) Please do not flip the sheet until instructed.
(2) Please try to be as neat as possible so that I can understand your answers without ambiguity.
(3) While it is certainly your rights to make wild guesses or memorize irrelevant details, I would truly appreciate if you try to make your answers logical.
(4) Good luck for all hard-working students!

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1. Photon Carnot engine (20%) Consider a Carnot engine that uses a photon gas as the working substance between two thermal reservoirs of temperatures τ_h and τ_l. Starting from the minimal volume V_1, the photon gas changes its volume to V_2, V_3, V_4 and back to V_1 when going through isothermal and isentropic expansions, followed by isothermal and isentropic compressions. The internal energy for the photon gas follows the relation,

$$U = a\, V \tau^4,$$

where a is some constant. The entropy can be computed from the definition $(\partial\sigma/\partial U)_V = 1/\tau$. Compute the heat Q_h absorbed from the high-temperature τ_h reservoir and Q_l emitted to the low-temperature τ_l one. Verify that it indeed agrees with the Carnot efficiency derived in class.

2. Carnot efficiency (20%) The second law states that heat can not flow from low temperature τ_l to high temperature τ_h spontaneously. Making use of the above statement, draw appropriate heat-flow diagram and derive the inequality between the efficiency η of an irreversible Carnot engine and that of a reversible one η_C.

3. Thermal ionization of hydrogen (20%) Consider thermal ionization of a charge-neutral deuterium (isotope of hydrogen) gas with number concentration $n_D \equiv N_D/V$ at temperature τ. The atomic reaction $e^- + D^+ \leftrightarrow D$ leads to relation between the concentrations of the reactants in equilibrium. Neglect the spins of all particles. Given the quantum concentration for electrons $n_Q \equiv (m_e\tau/2\pi\hbar^2)^{3/2}$ and the ionization energy ϵ_I for atomic deuterium, estimate the number concentration n_{D^+} of the ionized deuterium.

Bonus The deuterium atom has internal structure due to the excitation spectrum and gives rise to an internal partition function Z_{int}. Why can we ignore these contributions in the above calculations?

4. Superconducting transition (20%) For a type-I superconductor, the free energy F_N in the normal state and F_S in the superconducting state are related to the critical magnetic field $B_c(\tau)$ that destroys superconductivity,

$$F_N - F_S = \frac{1}{2\mu_0} V B_c^2(\tau),$$

where $B_c(\tau)$ decreases to zero as $\tau \to \tau_c$. Below the critical temperature, applying magnetic field brings the superconducting state to the normal. Is the phase transition first-order or second-order? If first-order, compute the latent heat L. If second order, compute the jump in specific heat $\Delta C = C_S - C_N$.

5. Landau theory of phase transition (20%) Consider a second-order phase transition described by the following Landau free energy,

$$F_L(\xi) = \frac{a}{2}(\tau - \tau_c)\,\xi^2 + \frac{b}{4}\,\xi^4,$$

where τ_c is the critical temperature, ξ is the order parameter and a, b are positive constants. Compute the order parameter $\xi_0(\tau)$, the free energy $F(\tau) = F_L(\xi_0, \tau)$ and its derivatives in the vicinity of the phase transition. Explain the crucial properties of the phase transition and why it is called second order in glory details.

6. Phase transition in a ferromagnet (Bonus 20%) The Landau free energy for a ferromagnet at temperature τ and under external magnetic field h is

$$F_L(m; h, \tau) = -(h - h_c)m + \frac{a}{2}(\tau - \tau_c)\, m^2 + \frac{b}{4}\, m^4,$$

where the order parameter m is the magnetization and the critical magnetic field is $h_c = 0$. Investigate the profile of the Landau free energy and answer the following questions. No explicit calculations are required.

(a) In the absence of magnetic field $h = 0$, explain the nature of the phase transition driven by varying temperature τ.

(b) Below the critical temperature $\tau < \tau_c$, explain the nature of the phase transition driven by varying magnetic field h.

(c) Above the critical temperature $\tau > \tau_c$, explain the nature of the phase transition driven by varying magnetic field h.

Collect the results in (a), (b), (c) and plot the phase diagram for the ferromagnet in the $\tau - h$ plane. Compare your result to the liquid-gas phase diagram in $\tau - p$ plane.