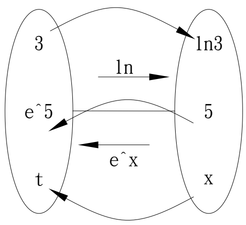
L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

Thm: $(e^x)' = e^x$, $\forall\ x \in \mathbb{R}$.

pf:

$\because\ (\ln x)' = 1/x \neq 0$, $\forall\ x \in \mathbb{R}^+$.

$\therefore\ e^x$ is diff. on $\mathbb{R}$.

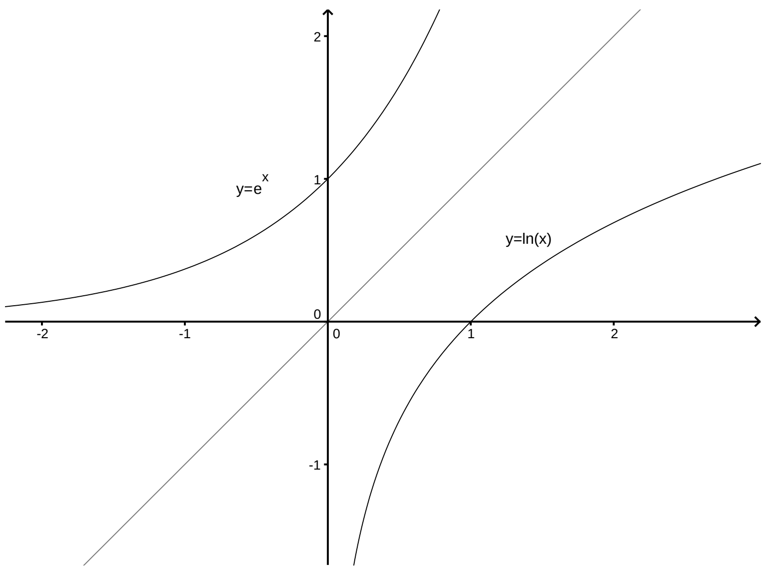


Moreover $(e^x)' = 1/(1/t)$, where $t = e^x$

$= t = e^x$.

Q:它可不可微取決於對數函數的什麼？
A:可微且微分不為 0。
Q:lnx 的微分是？
A:1/x。
Q:1/t 是那個函數的對應域還定義域？
A:定義域。
Q: $(f^{-1})'(x) = ?$
A: $1/f'(f^{-1}(x))$

Rmk:



我們知道 lnx 的圖形，一定知道 $\ln^{-1}x$ 的圖形。

$e^x > 0$、$e^0 = 1$、遞增、開口朝上、

從圖形可知 $\lim(x \to -\infty)\ e^x = 0$、$\lim(x \to \infty)\ e^x = \infty$

L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

Thm: $\int e^{x}dx = e^{x} + C$

Thm: $\int f'(x)e^{f(x)}dx = e^{f(x)} + C$

pf:Let u=f(x), then du=f'(x)dx

eg.

① $(xe^{-x^2})' = e^{-x^2} + x\cdot e^{-x^2}\cdot(-2x)$

② $(\sqrt{1+e^{x^3}})' = \frac{1}{2}\frac{e^{x^3}\cdot 3x^2}{\sqrt{1+e^{x^3}}}$

③ $\int 9e^{3x}dx = 3e^{3x} + C$

④ $\int_1^2 \frac{e^{\sqrt{2x}}}{\sqrt{x}}dx = \sqrt{2}\int_*^{**} e^{u}du = \sqrt{2}e^{\sqrt{2x}}\Big|_1^2 = \sqrt{2}(e^2 - e^{\sqrt{2}})$

Let u= $\sqrt{2x}$, then du= $\frac{2}{2\sqrt{2x}}dx = \frac{dx}{\sqrt{2x}}$

⑤ $\int_0^1 e^{x}(1+e^{x})dx = \frac{1}{2}(1+e^{x})^2\Big|_0^1 = \frac{1}{2}(1+e)^2 - 2$

Let u=1+e^x, then du=e^x

Ex:P362(18.24.31.40.41.47.49.72)

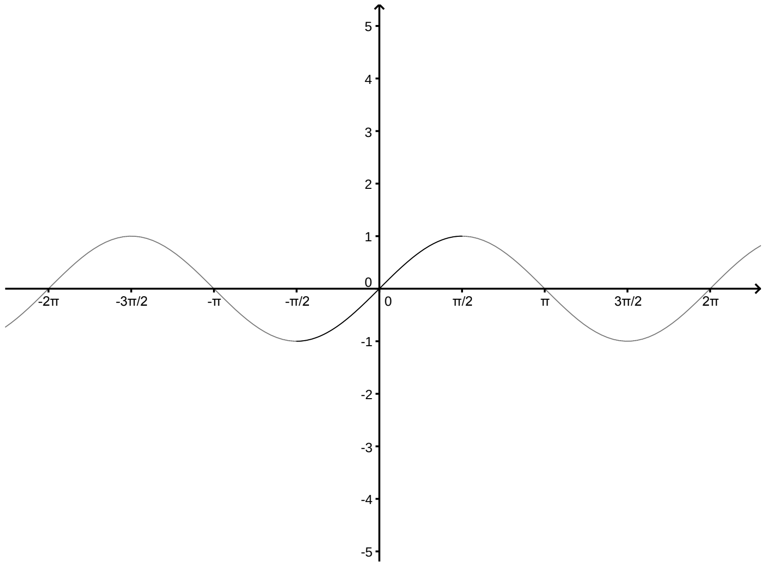
§ 7.6 先不講

L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

§ 7.7 The inverse Triogonometric functions

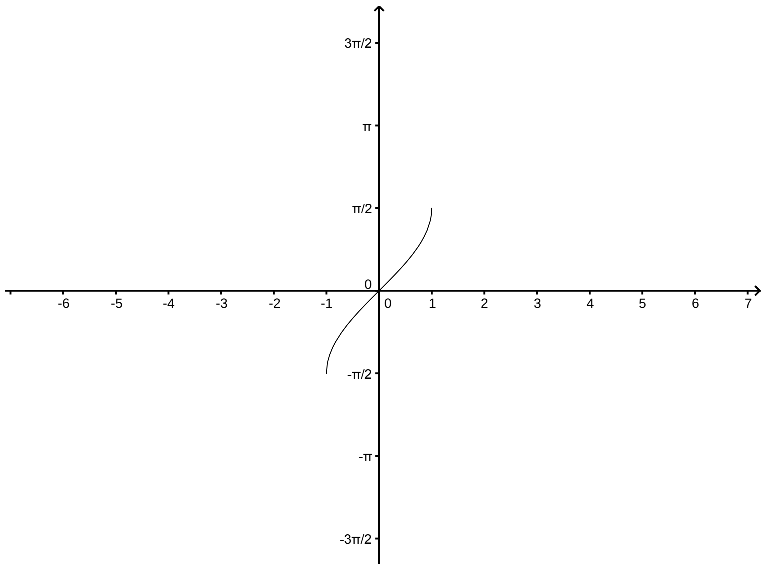
Q:首先六個三角函數是一對一嗎？
A:不是，有周期。
要限制定義域，使得成爲一對一。
第一個如果這個函數有定義域在 0 點，就一定要取，有正一定取正。要把定義域縮小，縮小成一對一的性質，一對一的範圍要最大。

(i) $\sin^{-1}x$. (唸 arcsin)



∵ $\sin:[-\pi/2,\pi/2]\to[-1,1]$ is one-to-one

∴ $\sin^{-1}:[-1,-1]\to[-\pi/2,\pi/2]$ exists



其圖形爲

L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

Question:$(\sin^{-1}x)'=?$

Q:siny，y 落在哪裡？A: [-π/2, π/2]

Answer:Let y= $\sin^{-1}x$. Then siny=x, y∈[-π/2, π/2].

$(\sin^{-1}x)'=1/\sin'y=1/\cos y=\pm 1/(\sqrt{1-\sin^2 y})$ ∵ y=(-π/2, π/2) ∴ cosy>0

$\cos y=\pm 1/(\sqrt{1-\sin^2 y})$，因爲定義域(-π/2, π/2)，所以 cosy 大於零，因爲在-π/2 和π/2 會使得分母爲零，所以拿掉。

Thm:$(\sin^{-1}x)'=1/(\sqrt{1-x^2})$, x∈(-1,1)

這的東西一出來，它的相關題目，圈入和積分。它的定義域、對應域、和如何對應。構成函數的三大要素：定義域、對應域、如何對應。

eg. $[\sin^{-1}(3x^2-5)]'=\dfrac{6x}{\sqrt{1-(3x^2+5)^2}}$

Thm: Let a>0 $\int\dfrac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\dfrac{x}{a}+C$

pf: $\int\dfrac{dx}{\sqrt{a^2-x^2}}=\dfrac{1}{a}\dfrac{dx}{\sqrt{1-(\frac{x}{a})^2}}=\int\dfrac{du}{\sqrt{1-u^2}}=\sin^{-1}\dfrac{x}{a}+C$

Let u=x/a, then du=1/adx

By the way~請你不要再特殊解法，國際共通語言。
有些同學微分人家用 fˊ、他用 fˋ。

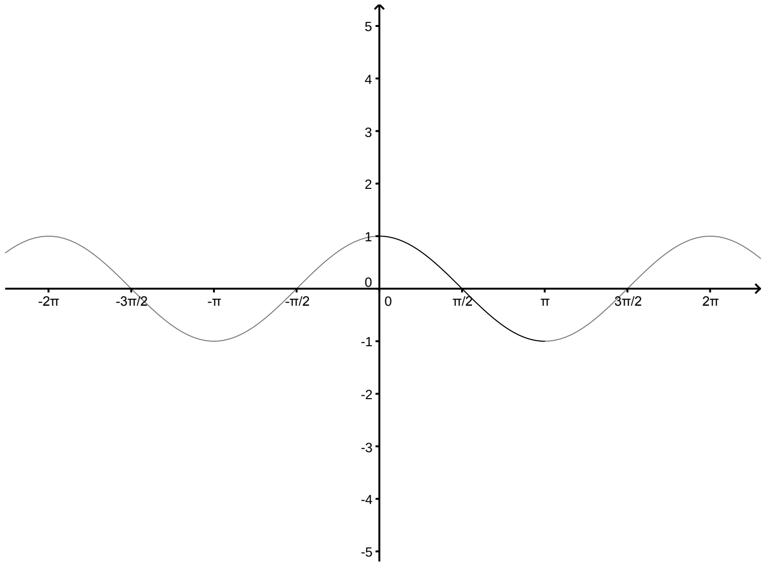
eg. $\int_0^{\sqrt{3}}\dfrac{dx}{\sqrt{4-x^2}}=$

$=\sin^{-1}(\frac{x}{2})\Big|_0^{\sqrt{3}}=\sin^{-1}\dfrac{\sqrt{3}}{2}-\sin^{-1}0=\dfrac{\pi}{3}-0=\dfrac{\pi}{3}$

哪一個角度取值爲√3/2，π/31)；哪一個角度取值爲 0，0

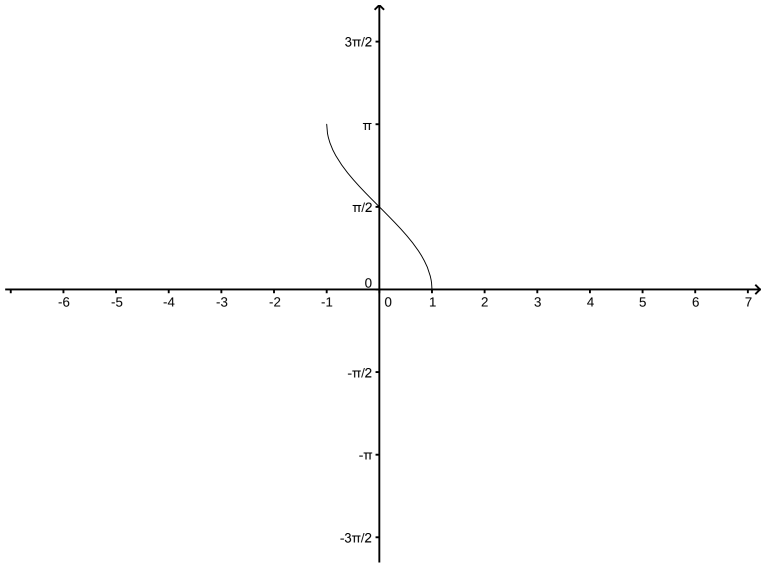
L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

(ii) $\cos^{-1}x$



∵ $\cos:[0,\pi]\to[-1,1]$ is one-to-one

∴ $\cos^{-1}:[-1,-1]\to[0,\pi]$ exists



其圖形為

Question:$(\cos^{-1}x)'=?$

Answer:Let $y=\cos^{-1}x$, then $\cos y=x$, $y\in[0,\pi]$

$(\cos^{-1}x)'=1/\cos'y=1/-\sin y=-1/+\sqrt{1-\cos^2y}=-1/\sqrt{1-x^2}$ ∵ $y\in(0,\pi)$ ∴ $\sin y>0$

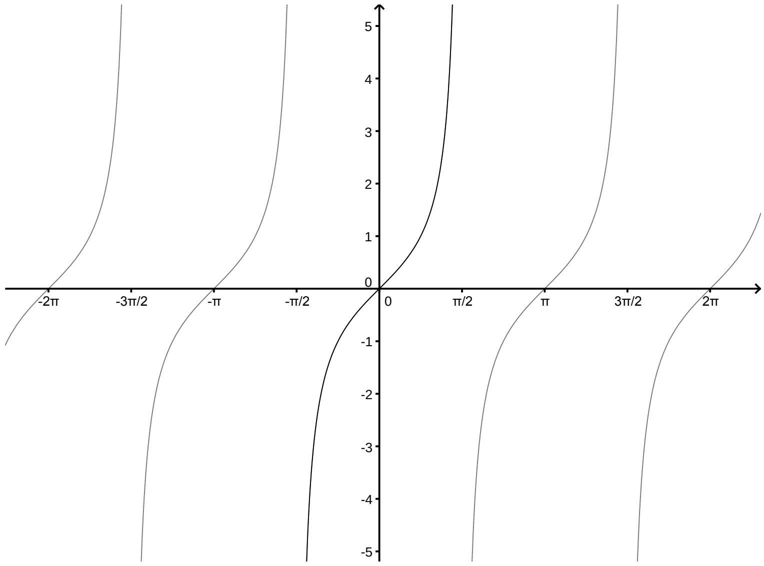
L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

Thm: $(\cos^{-1}x)' = -1/(\sqrt{1-x^2})$, $x \in (-1,1)$

Q:是 0 to π 還是-1 to 1？A: -1 to 1

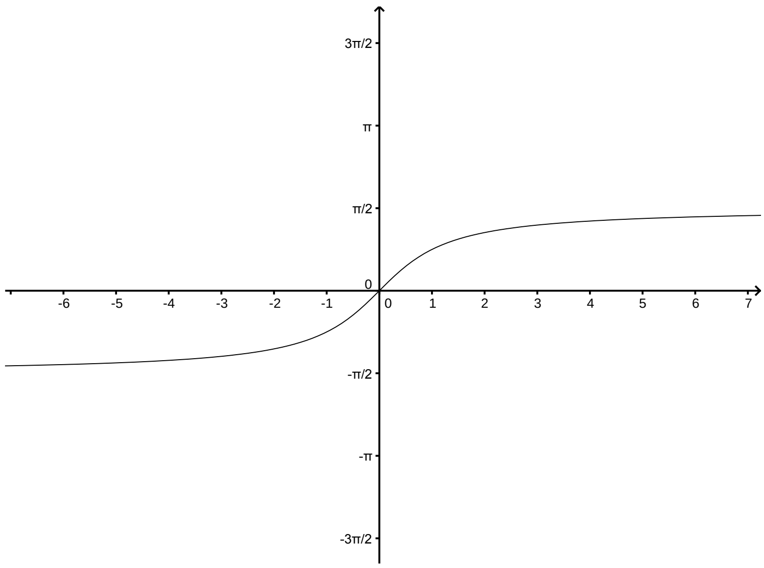
(iii) $\tan^{-1}x$

Q:tan 角度取爲多少？A:-2/π to 2/π ，tan 有漸進線，開區間。
Q:tan 的週期是？A: π



∵ $\tan:(-\pi/2, \pi/2) \to \mathbb{R}$ is one-to-one

∴ $\tan^{-1}:\mathbb{R} \to (-\pi/2, \pi/2)$ exists



其圖形爲

L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

Question: $(\tan^{-1}x)'=?$

Answer:Let $y=\tan^{-1}x$, then $\tan y=x$, $y\in(-\pi/2,\pi/2)$

$(\tan^{-1}x)'=1/\tan'y=1/\sec^2y=1/1+\tan^2=1/1+x^2$

Thm: $(\tan^{-1}x)'=1/1+x^2$ on $\mathbb{R}$

可積是因爲它是一個特殊函數，不是因爲 substitution

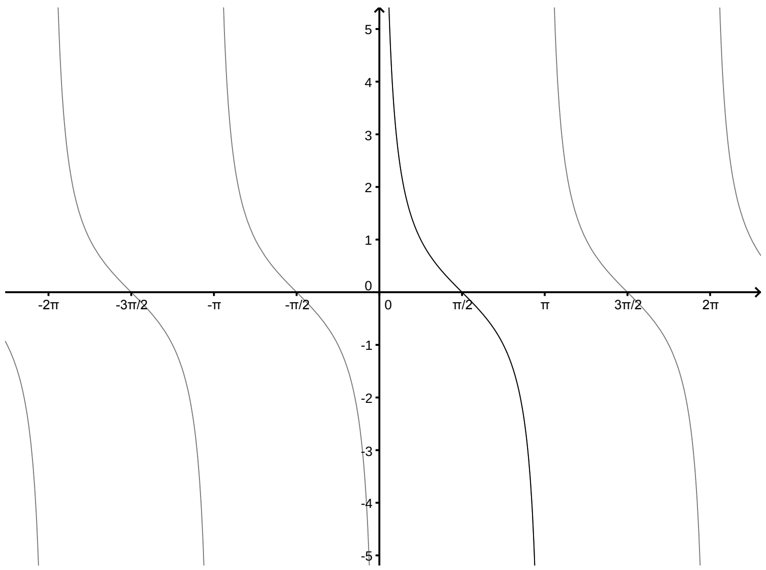
Thm: $\int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

pf: $\int\frac{dx}{a^2+x^2}=\frac{1}{a^2}\int\frac{dx}{1+(\frac{x}{a})^2}=\frac{1}{a}\int\frac{\frac{1}{a}dx}{1+(\frac{x}{a})^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

eg. $\int_0^2\frac{dx}{4+x^2}=\frac{1}{2}\tan^{-1}\frac{x}{2}\Big|_0^2=\frac{1}{2}\tan^{-1}1-\frac{1}{2}\tan^{-1}0=\frac{1}{2}\cdot\frac{\pi}{4}-\frac{1}{2}\cdot 0=\frac{\pi}{8}$

哪一個角度取値爲 1，$\pi/2$；哪一個角度取値爲 0，0

(VI) $\cot^{-1}x$



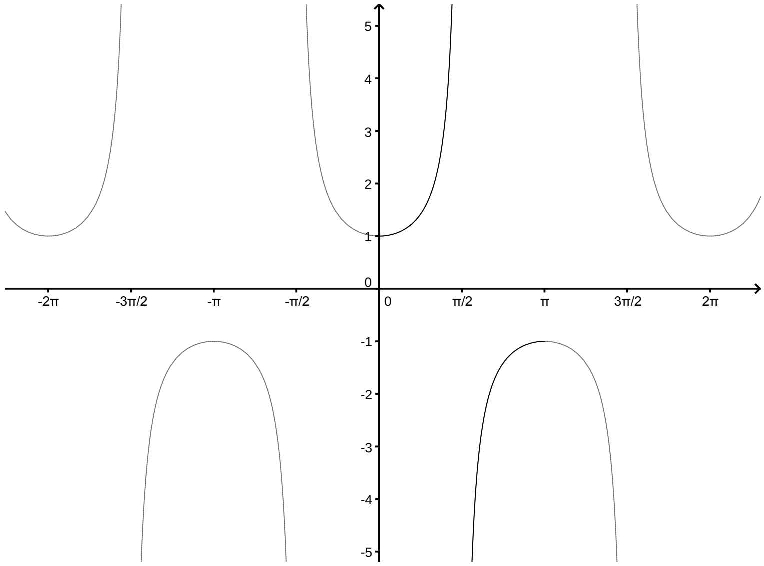
∵ $\cot:(0,\pi)\to\mathbb{R}$ is one-to-one

∴ $\cot^{-1}:\mathbb{R}\to(0,\pi)$ exists

$(\cot^{-1}x)'=-1/1+x^2$ on $\mathbb{R}$

L27 The exponential function (Conti.) (續.指數函數)
7.7 The inverse Triogonometric functions (反三角函數)

(V) $\sec^{-1}$



∵ $\sec:[0,\pi/2)\cup(\pi/2,\pi]\to(-\infty,-1]\cup[1,\infty]$ is one-to-one

∴ $\sec^{-1}:(-\infty,-1]\cup[1,\infty]\to[0,\pi/2)\cup(\pi/2,\pi]$ exists

其圖形爲