L17 critical point(臨界點)
First derivative test(一階導函數測試) Second derivative test(二階導函數測試)
4.6 Concavity and points of inflection(圖形凹性和反曲點) Concave up and concave down

Thm:

If f has a local extreme value at c, then f'(c)=0 or f'(c) doesn't exist.

如果函數在 c 點有局部極值，則若可微必爲 0 或不可微。

局部極值跟連續不連續和可不可微沒有關係。

pf:證微分等於 0，不可微不用證。

assume that f is diff. at c.　沒有函數沒辦法算，要用反證法。

assume f'(c)≠0, say f'(c)>0.

By thm. A, ∃ δ>0 s.t. $\begin{cases} f(x) > f(c), \forall\, x \in (c, c+\delta). \\ f(x) < f(c), \forall\, x \in (c, c+\delta). \end{cases}$

→←(f(c) is local extreme value.) Q:爲什麼會產生矛盾？A:假設不對

Therefore f'(c)=0.

Q:從哪裡找局部極值？A:不可微或微分等於 0
Q:局部極值有沒有可能發生在端點？A:不可能，根據定義。

Def:Let: f:I→ℝ be a function.

Let c is an interior point.

We say that c is critical point of f, if f'(c)=0 or f'(c) doesn't exist.

我們說 c 是函數臨界數，如果微分爲 0 或不存在。

Q:extreme value 多還 critical point?A: critical point

eg.1 Let f(x)=1/(x+1). Find all critical point of f.

pf: Q:它沒有微分等於零的地方？A:沒有。-1 這點沒有定義。

∵ f'(x)=-1/(x+1)^2≠0 ∴ f has on critical.

eg.2 $f(x)=|x+1|$. Find all critical points of f.

pf: $f(x)=x+1$,if $x\geq-1$;$-x-1$,if $x\leq-1$

$f'(x)=1$,if $x>-1$;-1,if $x<-1$

$\Rightarrow f'(-1)$ doesn't exist.

Thus -1 is a critical point of f.

Question:如何從 critical points 中挑出 local extreme values?

Answer:假設 f is cont.從遞增遞減判斷

Thm:(First derivative test)

Let f is a cont. function and c is a critical point.

① If f $f'(x)>0$ on $(c-\delta,c)$ and $'f(x)<0$ on $(c,c-\delta)$,

then f(c) is local maximum value of f.

如果函數在$(c-\delta,c)$一階微分大於0和函數在$(c,c+\delta)$一階微分小於0，

則 f(c)是函數的局部極大值。

① If f $f'(x)<0$ on $(c-\delta,c)$ and $'f(x)>0$ on $(c,c-\delta)$,

then f(c) is local minimum value of f.

如果函數在$(c-\delta,c)$一階微分小於0和函數在$(c,c+\delta)$一階微分大於0，

則 f(c)是函數的局部極小值。

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pf:①

∵ $f'(x)>0$ on $(c-\delta,c)$ and $f'(x)<0$ on $(c,c+\delta)$ ∴ f $\begin{cases} \nearrow & on\ (c-\delta,c) \\ \searrow & on\ (c,c+\delta) \end{cases}$

∵ f is cont. at c. ∴ f $\begin{cases} \nearrow & on\ (c-\delta,c] \\ \searrow & on\ [c,c+\delta) \end{cases}$

⇒f is a local max value at c.

eg. Let f(x)=x^4-2x^3. Find all the local extreme values of f.

f'(x)=4x^3-6x^2=2x^2(2x-3)=0⇒x=0 or 3/2

- 0 - 3/2 +

By 1st derivative test, f(3/2)= is a local min valre of f.

Let f is cont. at c and c is critical point.f'(c)=0 or f'(c)不存在。

f'(c)>0 on (c- δ ,c) and f'(c)<0 on (c,c+ δ). f'(c)=0 進階成

Thm:(Second derivative test)

Let f be cont. and f'(c)=0.

If f''<0, then f(c) is a local max. value of f.

如果函數二階微分小於 0，則 f(c)是函數的局部極大值。

If f''>0, then f(c) is a local min. value of f.

如果函數二階微分大於 0，則 f(c)是函數的局部極小值。

Q:什麼時候用 First、Second？A:取決於微分的微度。例如只能微一次，用 First。

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Eg. f(x)=2x^3-3x^2-12x+5.Detemine all the local extreme values.

pf:

$f'(x)=6x^2-6x-12=6(x^2-x-2)=6(x-2)(x+2)=0\Rightarrow x=2,-1$

$f''(x)=12x-6$

$\because$ $f''(2)>$ and $f''(-1)<0$

$\therefore$ By Second derivative test, $f(2)=16-12-24+5=-15$ is local min value of f

and $f(1)=-2-3+13+5=13$ is local min value of f.

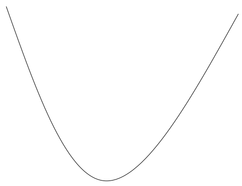
Ex:P173(17.20.28.32.35.39.42)

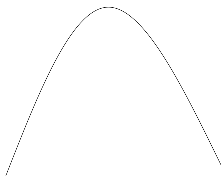
§ 4.6 Concavity and points of infection

Q:遞增(減)有幾種畫法？A:三種(開口朝上、朝下、直線)~直線不討論

① 開口朝上(concave up) f'↗

② 開口朝下(concave down) f'↘





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Def:

① We say that f is concave up on I, if f' increases on I.

我們說函數開口朝上，如果一階微分在 I 上遞增。

② We say that f is concave down I, if f' decreases on I.

我們說函數開口朝下，如果一階微分在 I 上遞減。

Q:什麼樣的函數一階微分遞增(遞減)？A:二階微分大於(小於)0

Thm:

① If f''>0 on I, then f is concave up on I.

如果二階微分大於 0，則函數在 I 上開口朝上。

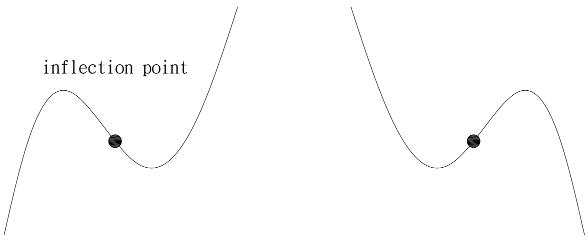
② If f''<0 on I, then f is concave down on I.

如果二階微分小於 0，則函數在 I 上開口朝下。

竟然如此，所有圖形都是遞增遞減。
討論到這裡了，有一個點不得不討論。開口朝上(下)接開口朝下(上)。
圖形上的點叫反曲點，數學上叫 inflection point or point of inflection.

Question:如何找反曲點？

Answer:一階微分遞增遞減(局部極大)



Thm:

If f has an inflection at c, then f''(c)=0 or f''(c) doesn't exist.

如果函數在 c 有反曲點，則二階微分為 0 或二階微分不存在。