L13

3.4 The derivative as a rate of change (視微分爲變化率)
3.5 Chain rule (連鎖律)
3.6 Differentiaing the trigonometric functions (三角函數的微分)
3.7 Implicit differentiation, rational powers (隱函數的微分方式，有理數的次方)

§ 3.4 The derivative as a rate of change

f'(x)=lim(h→0)[f(x+h)-f(x)]/h $\frac{\Delta y}{\Delta x}$ 平均變化率

=lim(Δ→0)Δy/Δx=瞬間變化率 of f at x

≙變化率 of f at x=rate of change of f at x.

rate of change=change rate 在工程裡頭變化率，就是對它做微分。

eg. 若 f(x)=點 x 的位置，則 f'(x)=點 x 的速度，f'' (x)=點 x 的加速度

Ex:P132(6.7)

§ Thm:(The Chain rule)

If g is diff. at x, and f is diff. at g(x), then f∘g is diff. at x and

(f∘g)'(x)=[f(g(x))]'=f'(g(x))·g'(x) (f∘g)'(x)=[f(g(x))]'≠f'(g(x))

口語：f 合成 g 的微分等於 f 的微分帶入 g(x)乘上 g(x)的微分

pf:證明留到高微

cor:If y=f(u) and u=g(x). y 是 u 的變數、u 是 x 的變數

Then dy/dx=dy/du·du/dx.

eg.

① y=(u-1)/(u+1), u=x^2. Find dy/dx.

pf:dy/dx=dy/du·du/dx=[(u+1)-(u-1)]/(u+1)^2·2x=4x/(u+1)^2=4x/(x^2+1)^2

② d/dx[(x^2-1)^100] x^100 的合成函數，代入x^2-1

pf:100(x^2-1)^99·2x=200x(x^2-1)^99

③ [1/(x^4+2x+1)^2]'

pf:-2/(x^4+2x+1)^3·(4x^3+2)　x^-2 的合成函數，代入 x^4+2x+1

④ y=2u/(1-4u), u=(5x^2+1)^4. Find dy/dx.

pf:dy/dx=dy/du·du/dx=[2(1-4u)+8u]/(1-4u)^2·4(5x^2+1)^3·10x=…

⑤ d/dx[f(x^2+1)]=　x的合成函數，代入x^2+1

pf:f'(x^2+1)·2x

⑥ d/dx[f^3(x^2+1)]=　x^3 的合成函數，代入f(x^2+1)

pf:3f^2(x^2+1)·f'(x^2+1)·2x

Ex:P138(5.7.16.24.27.44.45.60)

§ 3.6 Differentiating the trigonometric functions

Thm:(sinx)'=cosx, (cosx)'=-sinx.

pf:Let x∈R lim(h→0)sinx= lim(h→0)[sin(x+h)-sin(x)]/h

=lim(h→0)(sinxcosh+cosxsinh-sinx)/h

=lim(h→0)[sinx(cosh-1)+cosxsinh]/h

=lim(h→0)[sinx·(cosh-1)/h+cosx·sinh/h]

sinx→sinx、(cosh-1)/h→0、cosx→cosx、sinh/h→1

L13
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$=\sin x\cdot 0+\cos x\cdot 1=\cos x$

Thm: $(\tan x)'=\sec^2 x$、$(\sec x)'=\sec x\tan x$，$(\cot x)'=-\csc^2 x$、$(\csc x)'=-\csc x\cot x$

pf: $(\tan x)'=(\sin x/\cos x)'$ tanx 本來就定義在 cosx 不為零的地方

$=(\cos^2 x+\sin^2 x)/\cos^2 x=\sec^2 x$

eg.

① $[(1-\sec x)/\tan x]'=[(-\sec x\tan^2 x)-(1-\sec x)\sec^2 x]/\tan^2 x$

② $d/dx[\sec(x^2+1)]=\sec(x^2+1)\tan(x^2+1)\cdot 2x$

③ $[x^3x-\sin(2x^2)]'=3x^2-\cos(2x^2)\cdot 4x$

④ $d/dx[\csc(f(3\cos x))]=-\csc(f(3\cos x))\cot(f(3\cos x))\cdot f'(3\cos x)\cdot(-3\sin x)$

Ex145(12.24.27.55.56.67)

§ 3.7 Implicit differentiation, rational powers.

eg.

① $3x^3y-4y-2x+\sin x=0$. Find $y'=?$ $y=f(x)$

$9x^2y+3x^3y'-4y'-2+\cos x=0$

$(3x^3-4)y'=-9x^2y+2-\cos x$

$\Rightarrow y'=(-9x^2y+2-\cos x)/(3x^3-4)$

② $\cos(x-y)=(2x+1)^2y^2$. Find y'.

pf: $-\sin(x-y)\cdot(1-y')=2(2x+1)\cdot 2\cdot y^2+(2x+1)^2\cdot 2y\cdot y'$...化簡

L13

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Thm:Let p,q ∈ℤ q≠0,(x^(p/q))'=(p/g)x^(p/q-1)

pf: Let y=x^(p/q).Then y^q=x^p.

qy^(q-1)·y'=px^(p-1)

q(x^(p/q))^(q-1)·y'= px^(p-1)

y'=(p/q)x^(p-1)(x^(p/q))^(1-q)=(p/q)x^(p-1+p/q-p)=(p/q)x^(p/q-1)

eg.

$x^{\frac{p}{q}} = \sqrt[q]{x^p}$

① (x^(2/3))'=(2/3)x^(-1/3)=$\frac{2}{3\sqrt[3]{x}}$

② {√[(secx/(1+x^2)]}'=$\frac{1}{2}\frac{1}{\sqrt{\frac{\sec x}{1+x^2}}}\cdot\frac{\sec x\tan x(1+x^2)-2x\sec x}{(1+x^2)^2}$

Ex:P150(10.18.32.34.42.48)