Chapter 6 Magnetic Fields in Matter

6.1 Magnetization

6.1.1 Diamagnets, Paramagnets, Ferromagnets

All the magnetic phenomena are due to electric charges in motion:

Electrons orbiting around nuclei
Electrons spinning about their axes } magnetic dipoles

When a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or magnetized.

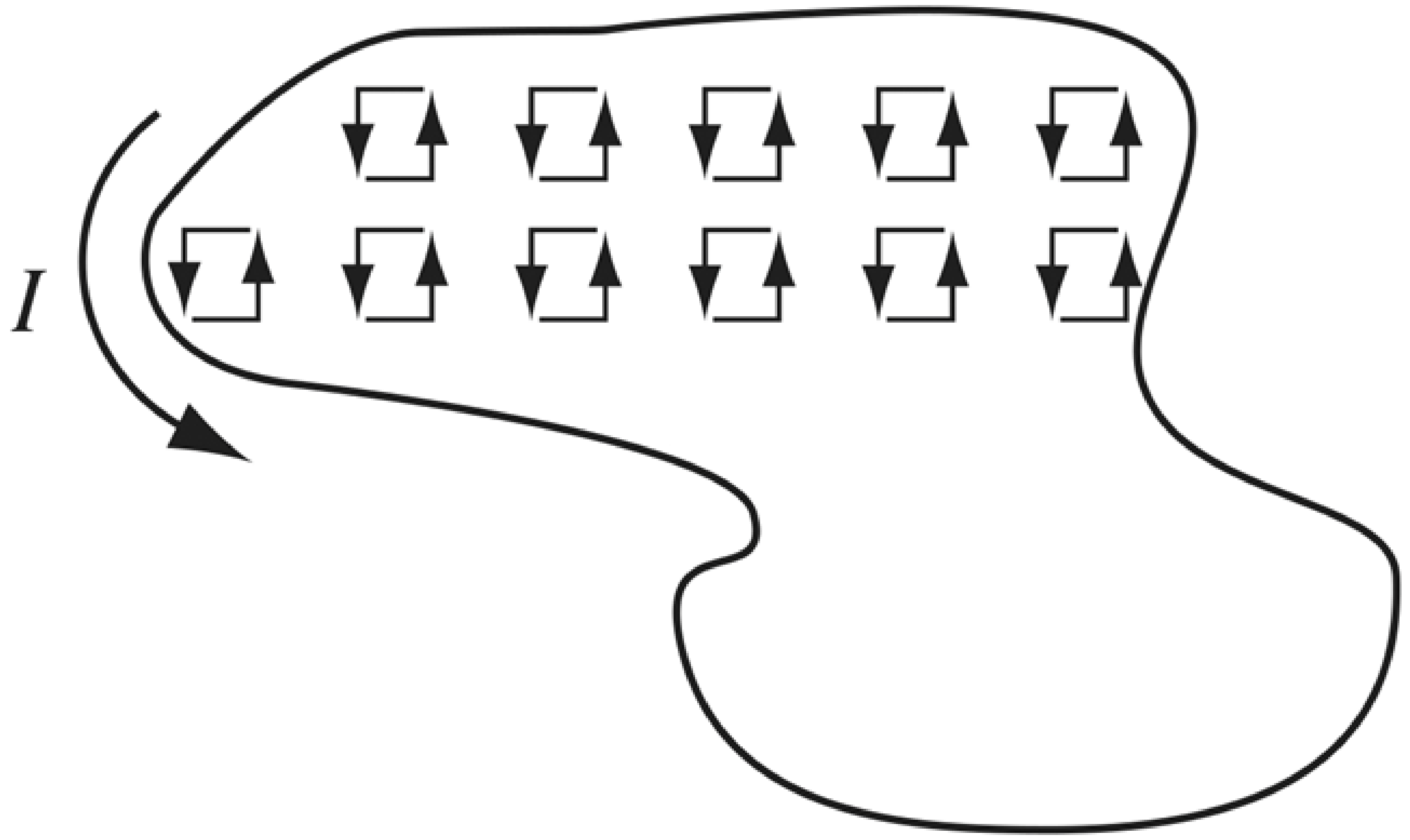
The magnetic polarization $\mathbf{M}$, unlike electrical polarization $\mathbf{P}$, might be parallel to $\mathbf{B}$ (*paramagnets*) or opposite to $\mathbf{B}$ (*diamagnets*).

A few substances (*ferromagnets and ferrimagnets*) retain their magnetization even after the external field has been removed.

6.1.2. Torques and Forces on Magnetic Dipoles

A magnetic dipole experiences a torque in a magnetic field, just as an electric dipole does in an electric field.

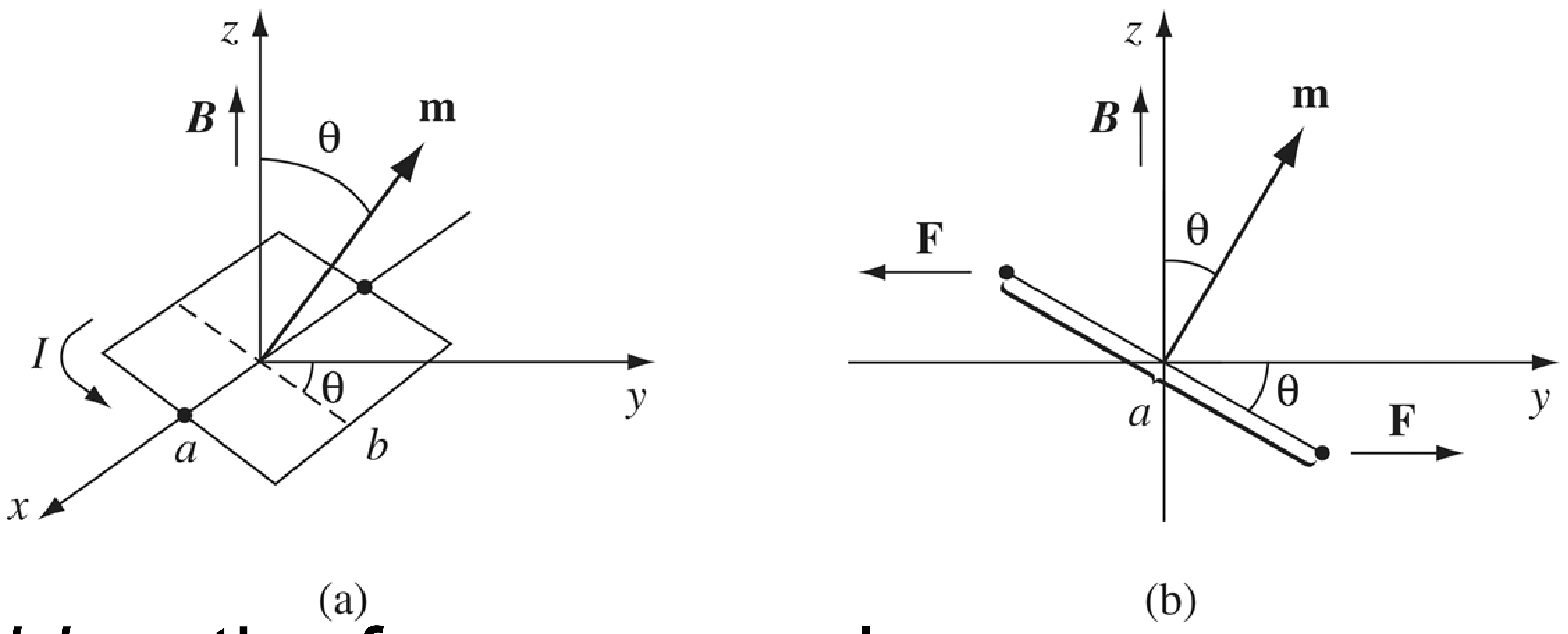
Any current loop could be built up from infinitesimal *rectangles*, with all the “internal” side canceling. There is no actual loss of generality in using the shape.



Let’s calculate the torque on a rectangular current loop in a uniform magnetic field.

Torques and Forces on Magnetic Dipoles

Center the loop at the origin, and tilt it an angle θ from the z axis towards the y axis. Let $\mathbf{B}$ point in the z direction.



Sloping sides: the forces cancel.
Horizontal sides: the forces cancel but they generate a torque.

$$\mathbf{N} = \mathbf{L} \times \mathbf{F} = aF\sin\theta\ \hat{\mathbf{x}}$$

The magnitude of the force on each of these segments is:

$$F = IbB$$

Torques and Dipole Moment

$$\mathbf{N} = IabB\sin\theta\ \hat{\mathbf{x}} = mB\sin\theta\ \hat{\mathbf{x}} = \mathbf{m}\times\mathbf{B}$$

where $m = Iab$ is the magnetic dipole moment of the loop.

This equation is identical in form to the electrical analogy.

$$\mathbf{N} = \mathbf{p}\times\mathbf{E}$$

The torque is again in such a direction as to line the dipole up parallel to the field (***paramagnetism***).

QM: The Pauli exclusion principle dictates that the electrons within a given atom lock together in pairs with opposite spins, and this effectively neutralizes the torque on the combination.

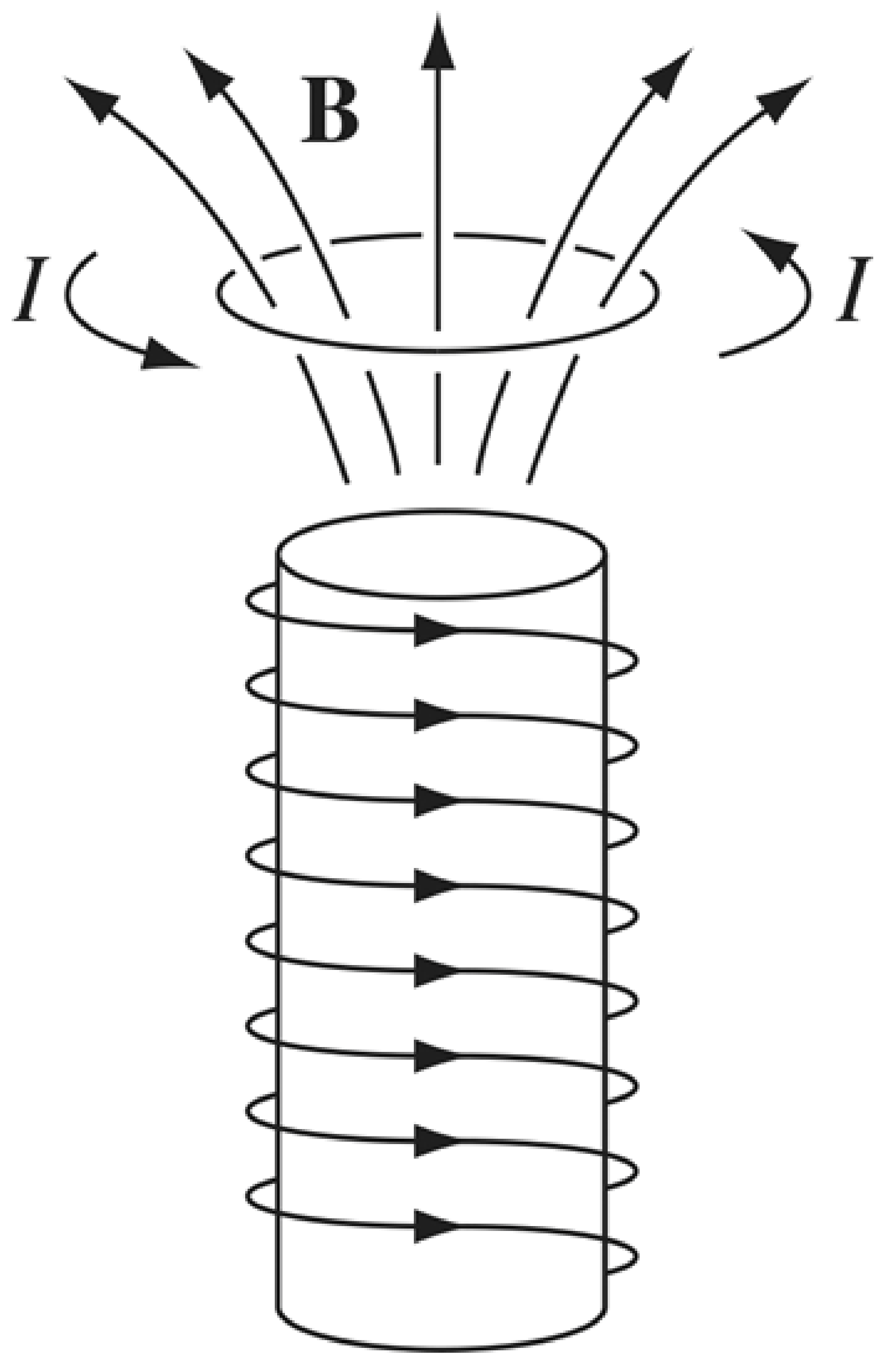
Forces in Nonuniform Magnetic Field

In a uniform field, the net force on a current loop is zero:

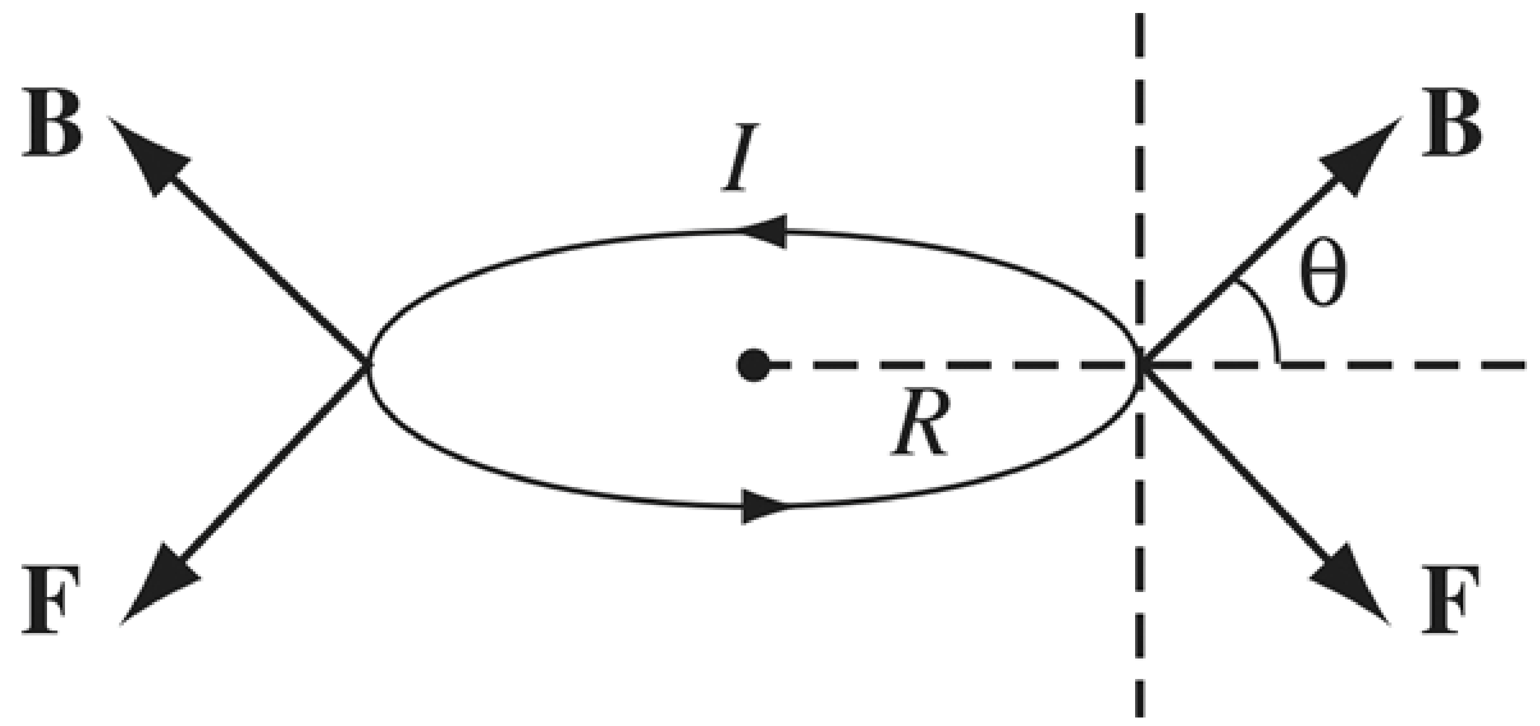
$$F = I\oint(d\mathbf{l}\times\mathbf{B}) = I\oint(d\mathbf{l})\times\mathbf{B} = 0$$

=0

In a nonuniform field this is no longer the case, because the magnetic field B could not come outside the integral.



Fringing field effect: $F = 2\pi IRB\cos\theta$

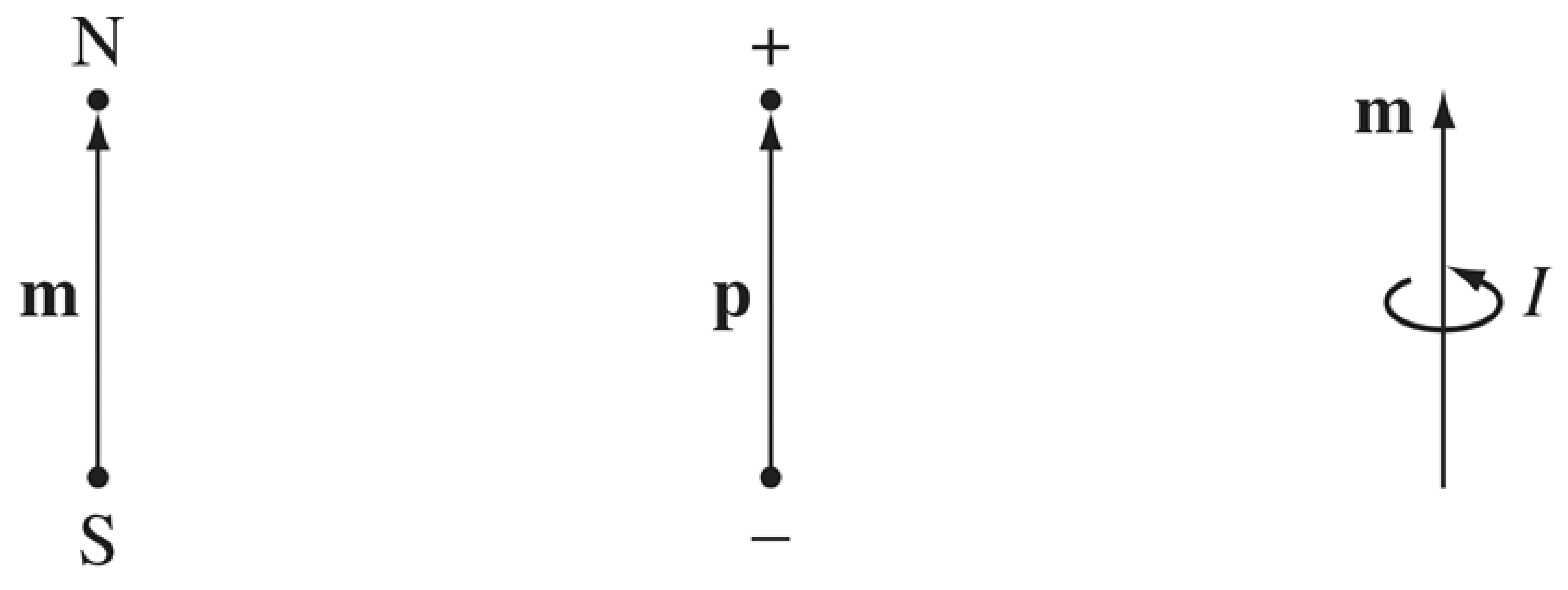


Forces on an Infinitesimal Current Loop and Model

$$\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B}) \qquad (= -\nabla U, \text{ where } U = -(\mathbf{m}\cdot\mathbf{B}))$$

Identical to the electrical formula $\mathbf{F} = \nabla(\mathbf{p}\cdot\mathbf{E})$

Does the magnetic dipole consist of a pair of opposite magnetic monopoles just like an electric dipole?



(a) Magnetic dipole (Gilbert model) (b) Electric dipole (c) Magnetic dipole (Ampère model)

6.1.3. Effect of a Magnetic Field on Atomic Orbits

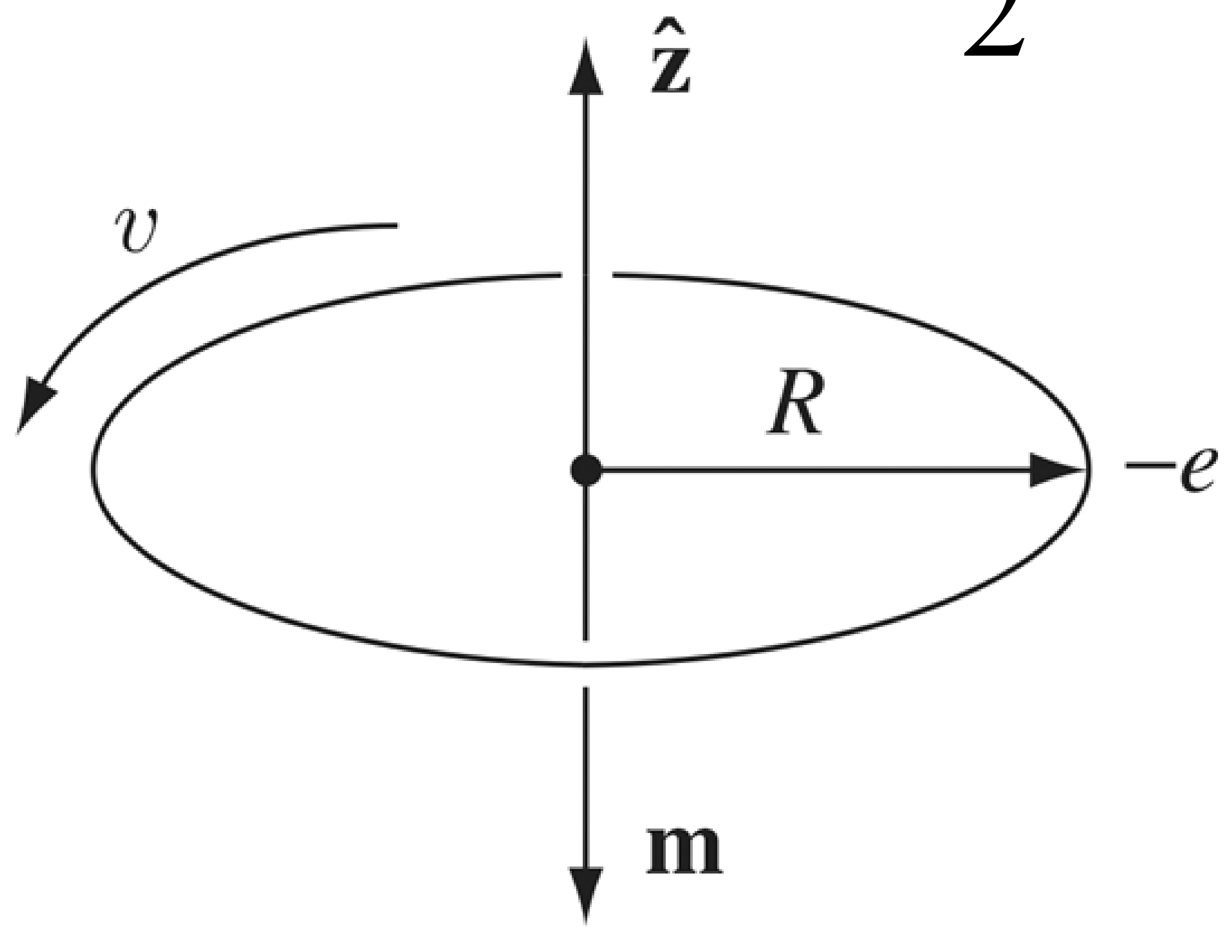
Electrons not only spin; they also revolve around the nucleus.

Let's assume the orbit is a circle of radius R. The current looks like steady (really?)

$$\text{Current } I = \frac{-e}{T} = \frac{-e}{\frac{2\pi R}{v}} = \frac{-ev}{2\pi R}$$

The negative charge of the electron

$$\text{Orbital dipole moment } \mathbf{m} = -\frac{evR}{2}\hat{\mathbf{z}} \qquad (m = I\pi R^2)$$

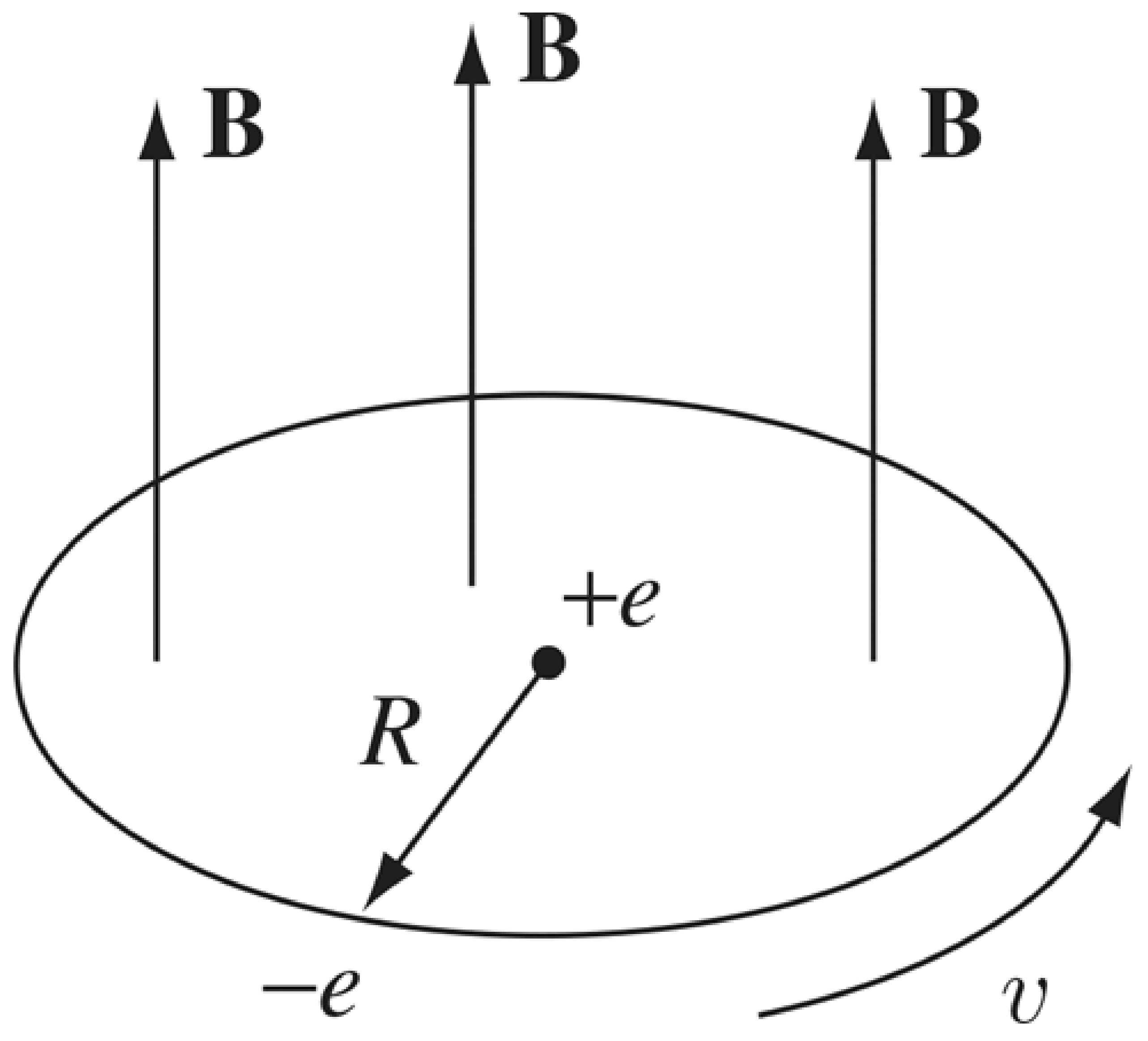


Electron Speeds Up or Slows Down

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}$$ without the magnetic field.

The centripetal force comes from two sources: *the electric force* and *the magnetic force*.

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R'^2} - (-e)v'B = m_e\frac{v'^2}{R'}$$



Assume $R' \cong R$

$$ev'B = \frac{m_e(v'^2 - v^2)}{R} = \frac{m_e(v'+v)(v'-v)}{R}$$

$$\therefore \Delta v = (v' - v) \approx \frac{eRB}{2m_e}$$ When **B** is turned on, the electron speeds up.

The Dipole Moment and The Diamagnetism

A change in the orbital speed means a change in the dipole moment

$$\Delta \mathbf{m} = -\frac{1}{2} e \Delta v R \hat{\mathbf{z}} = -\frac{e^2 R^2}{4 m_e} \mathbf{B}$$

The change in $\mathbf{m}$ is opposite to the direction of $\mathbf{B}$.

In the presence of a magnetic field, each atom picks up a little “extra” dipole moment, and the increments are all antiparallel to the field. This is the mechanism responsible for **diamagnetism**.

This is a universal phenomenon, affecting all atoms, but it is typically much *weaker* than **paramagnetism**.

6.1.4 Magnetization

In the present of a magnetic field, matter becomes magnetized. Upon microscopic examination, it contains many tiny dipoles, with a net alignment along some direction.

Two mechanisms account for this magnetic polarization:

1. Paramagnetism: the dipoles associated with the spins of unpaired electrons experience a torque tending to line them up parallel to the field.

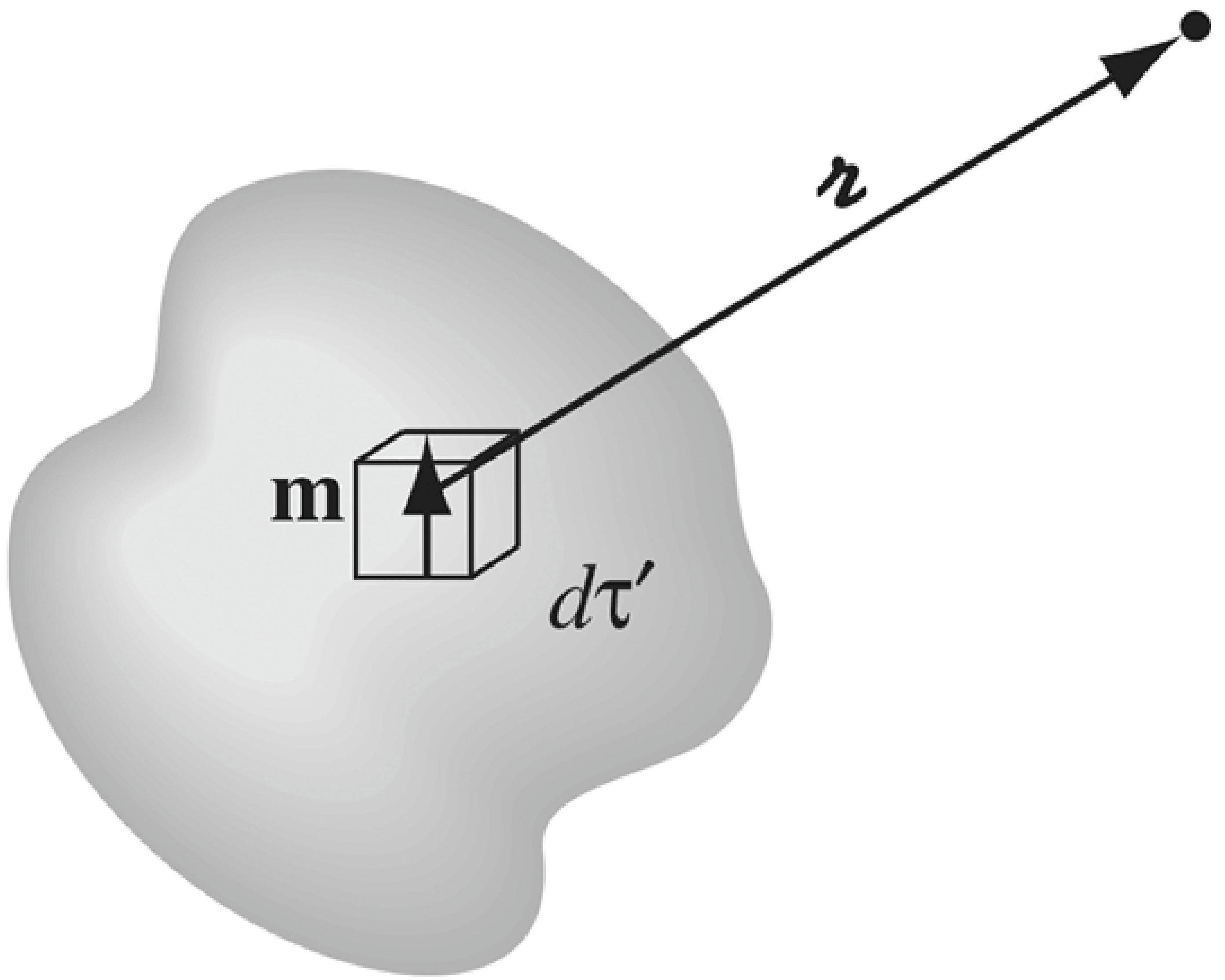
2. Diamagnetism: the orbital speed of the electrons is altered in such a way as to change the orbital dipole moment in the direction opposite to the field.

We describe the state of magnetic polarization by the vector quantity:

$$\mathbf{M} \equiv \text{magnetic dipole moment per unit volume.}$$

6.2 The Field of a Magnetized Object
6.2.1 Bound Currents

Suppose we have a piece of magnetized material (i.e., $\mathbf{M}$ is given). What field does this object produce?



The vector potential of a single dipole $\mathbf{m}$ is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

In the magnetized object, each volume element carries a dipole moment $\mathbf{M}d\tau'$, so the total vector potential is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{M}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}d\tau'$$

Vector Potential and Bound Currents

Can the equation be expressed in a more illuminating form, as in the electrical case? Yes!

By exploiting the identity,

$$\nabla' \frac{1}{\mathscr{r}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$(\hat{\mathbf{x}}'\frac{\partial}{\partial x'} + \hat{\mathbf{y}}'\frac{\partial}{\partial y'} + \hat{\mathbf{z}}'\frac{\partial}{\partial z'})\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

$$= \frac{\hat{\mathbf{x}}'(x-x') + \hat{\mathbf{y}}'(y-y') + \hat{\mathbf{z}}'(z-z')}{((x-x')^2+(y-y')^2+(z-z')^2)^{3/2}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

The vector potential is $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \mathbf{M}(\mathbf{r}') \times (\nabla' \frac{1}{\mathscr{r}}) d\tau'$

Using the product rule $\nabla \times (\frac{1}{\mathscr{r}}\mathbf{M}) = \nabla \frac{1}{\mathscr{r}} \times \mathbf{M} + \frac{1}{\mathscr{r}}(\nabla \times \mathbf{M})$

and integrating by part, we have

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]d\tau' - \int \nabla' \times [\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}]d\tau'\right\}$$

↓ how? Prob. 1.60

$$= \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]d\tau'\right\} + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times \hat{\mathbf{n}}']da'$$

Problem 1.60 Although the gradient, divergence, and curl theorems are the fundamental integral theorems of vector calculus, it is possible to derive a number of corollaries from them. Show that:

(a) $\int_{\mathcal{V}} (\nabla T)\, d\tau = \oint_{\mathcal{S}} T\, d\mathbf{a}$. [*Hint:* Let $\mathbf{v} = \mathbf{c}T$, where $\mathbf{c}$ is a constant, in the divergence theorem; use the product rules.]

(b) $\int_{\mathcal{V}} (\nabla \times \mathbf{v})\, d\tau = -\oint_{\mathcal{S}} \mathbf{v} \times d\mathbf{a}$. [*Hint:* Replace $\mathbf{v}$ by $(\mathbf{v} \times \mathbf{c})$ in the divergence theorem.]

(c) $\int_{\mathcal{V}} [T\nabla^2 U + (\nabla T)\cdot(\nabla U)]\, d\tau = \oint_{\mathcal{S}} (T\nabla U)\cdot d\mathbf{a}$. [*Hint:* Let $\mathbf{v} = T\nabla U$ in the divergence theorem.]

(b) Gauss's law $\int_v (\nabla \cdot \mathbf{E}) d\tau = \oint_S \mathbf{E} \cdot d\mathbf{a}$

Let $\mathbf{E} = \mathbf{v} \times \mathbf{c}$, where $\mathbf{c}$ is a constant vector. We have

$$\begin{cases} \int_v (\nabla \cdot (\mathbf{v} \times \mathbf{c})) d\tau = \mathbf{c} \cdot \int_v (\nabla \times \mathbf{v}) d\tau \\ \oint_S (\mathbf{v} \times \mathbf{c}) \cdot d\mathbf{a} = -\mathbf{c} \cdot \oint_S \mathbf{v} \times d\mathbf{a} \end{cases} \Rightarrow \int_v (\nabla \times \mathbf{v}) d\tau = -\oint_S \mathbf{v} \times d\mathbf{a}$$

Vector Potential and Bound Currents

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}')\times \hat{\mathbf{n}}']da'$$

$$\mathbf{J}_b = \nabla' \times \mathbf{M}(\mathbf{r}')$$

volume current

$$\mathbf{K}_b = \mathbf{M}(\mathbf{r}')\times \hat{\mathbf{n}}'$$

surface current

bound currents

With these definitions,

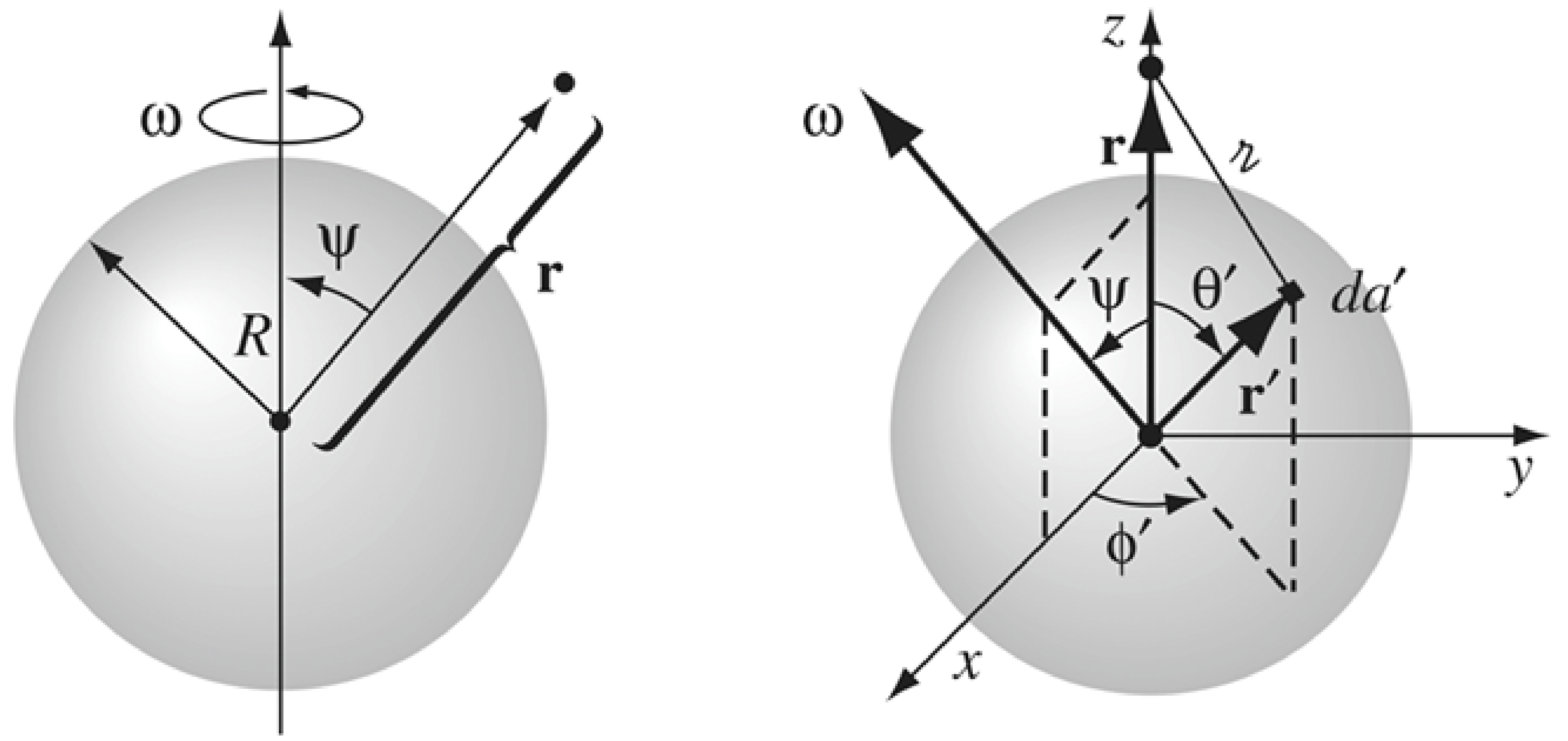
$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_v \frac{\mathbf{J}_b}{\mathscr{r}}d\tau' + \frac{\mu_0}{4\pi}\oint_S \frac{\mathbf{K}_b}{\mathscr{r}}da'$$

The electrical analogy

volume charge density $\rho_b = -\nabla \cdot \mathbf{P}$

surface charge density $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$

Example 5.11 A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. Find the vector potential it produce at point **r**.



Sol: First, let the observer is in the z axis and ω is tilted at an angle ψ

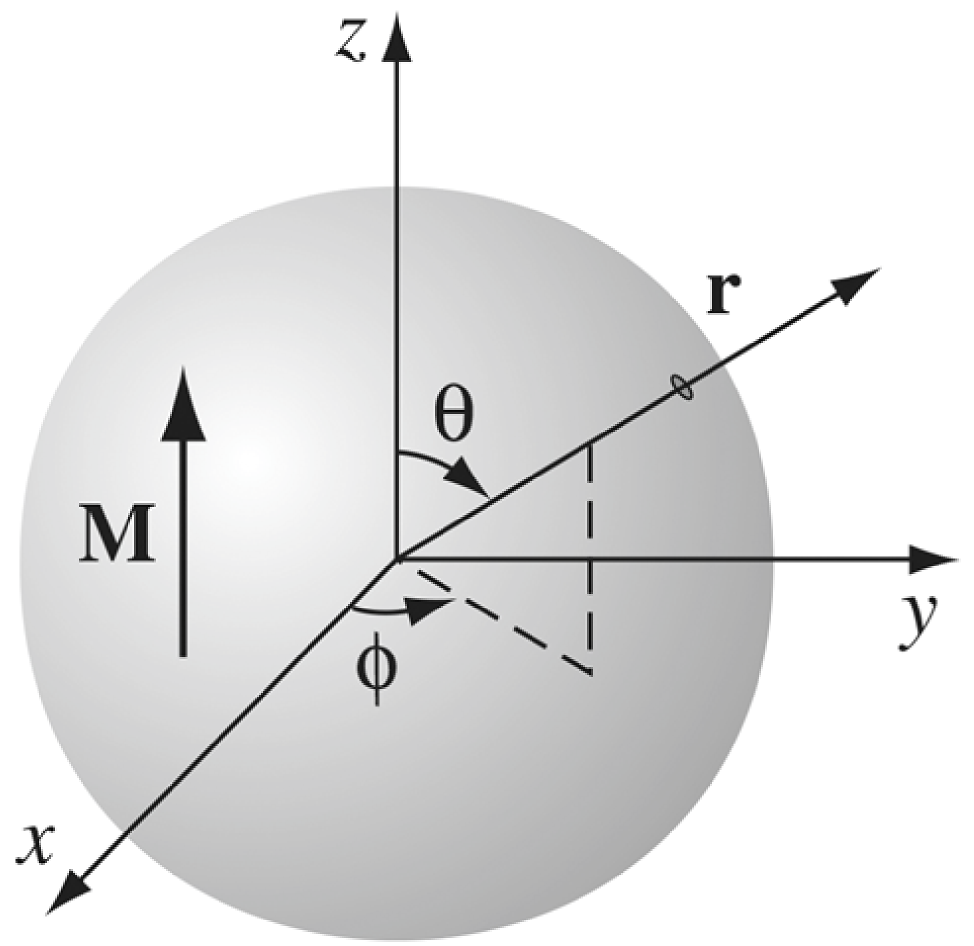
Vector potential is $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \dfrac{\mathbf{K}(\mathbf{r}')}{\mathscr{r}} da'$

The surface current density $\mathbf{K}(\mathbf{r}') = \sigma \mathbf{v}'$

$$\mathbf{A}(r,\theta,\phi) = \begin{cases} \dfrac{\mu_0 R\omega\sigma}{3} r\sin\theta\hat{\boldsymbol{\phi}} & r \le R \\ \dfrac{\mu_0 R^4\omega\sigma}{3}\dfrac{\sin\theta}{r^2}\hat{\boldsymbol{\phi}} & r \ge R \end{cases}$$

$$\mathbf{B} = \nabla\times\mathbf{A} = \frac{2}{3}\mu_0\sigma R\omega\hat{\mathbf{z}}$$

Example 6.1 Find the magnetic field of a uniformly magnetized sphere of radius R.



Sol: Choosing the z axis along the direction of M,

$$\text{we have } \begin{cases} \mathbf{J}_b' = \nabla \times \mathbf{M} = 0 \\ \mathbf{K}_b' = \mathbf{M} \times \hat{\mathbf{n}}' = M \sin\theta \ \hat{\boldsymbol{\phi}} \end{cases}$$

The surface current density is analogous to that of a spinning spherical shell with uniform surface current density.

$$\mathbf{K}_b' = \mathbf{M} \times \hat{\mathbf{n}}' = M \sin\theta \ \hat{\boldsymbol{\phi}} \quad \Leftrightarrow \quad \mathbf{K}' = \sigma \mathbf{v}' = \sigma R \omega \sin\theta \ \hat{\boldsymbol{\phi}}$$

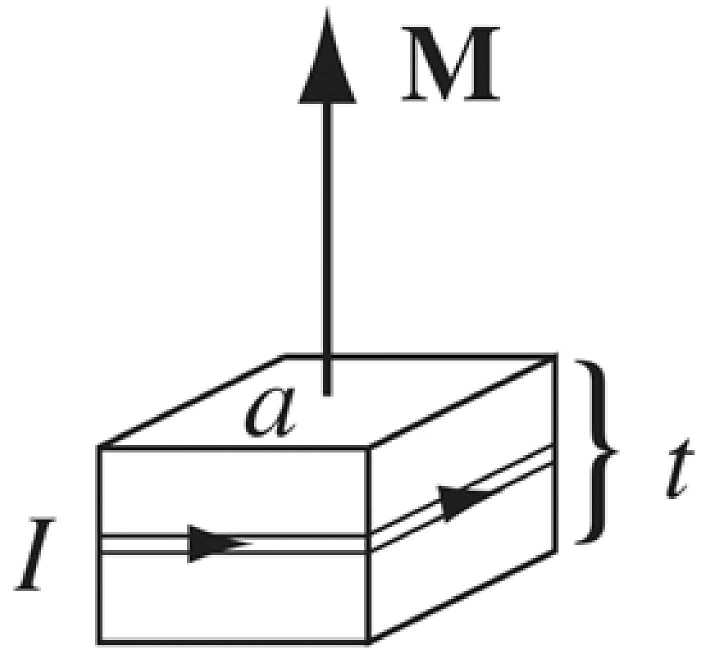
$$\sigma R \omega \to M$$

$$\mathbf{B} = \frac{2}{3} \mu_0 \mathbf{M} \ \text{(inside)}$$

Can you find a more direct method?

6.2.2 Physical Interpretation of Bound Currents

Bound surface current $\mathbf{K}_b$:

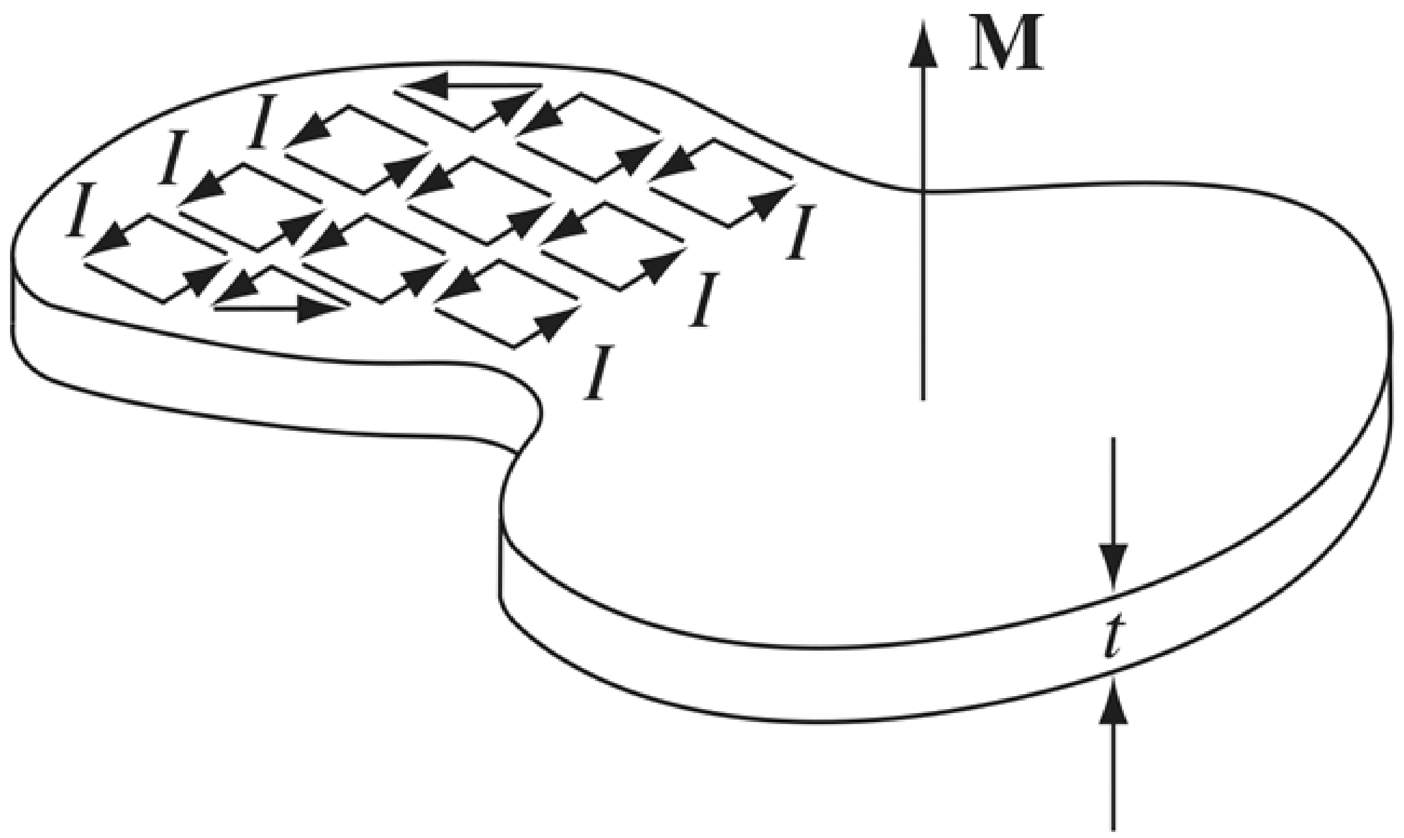


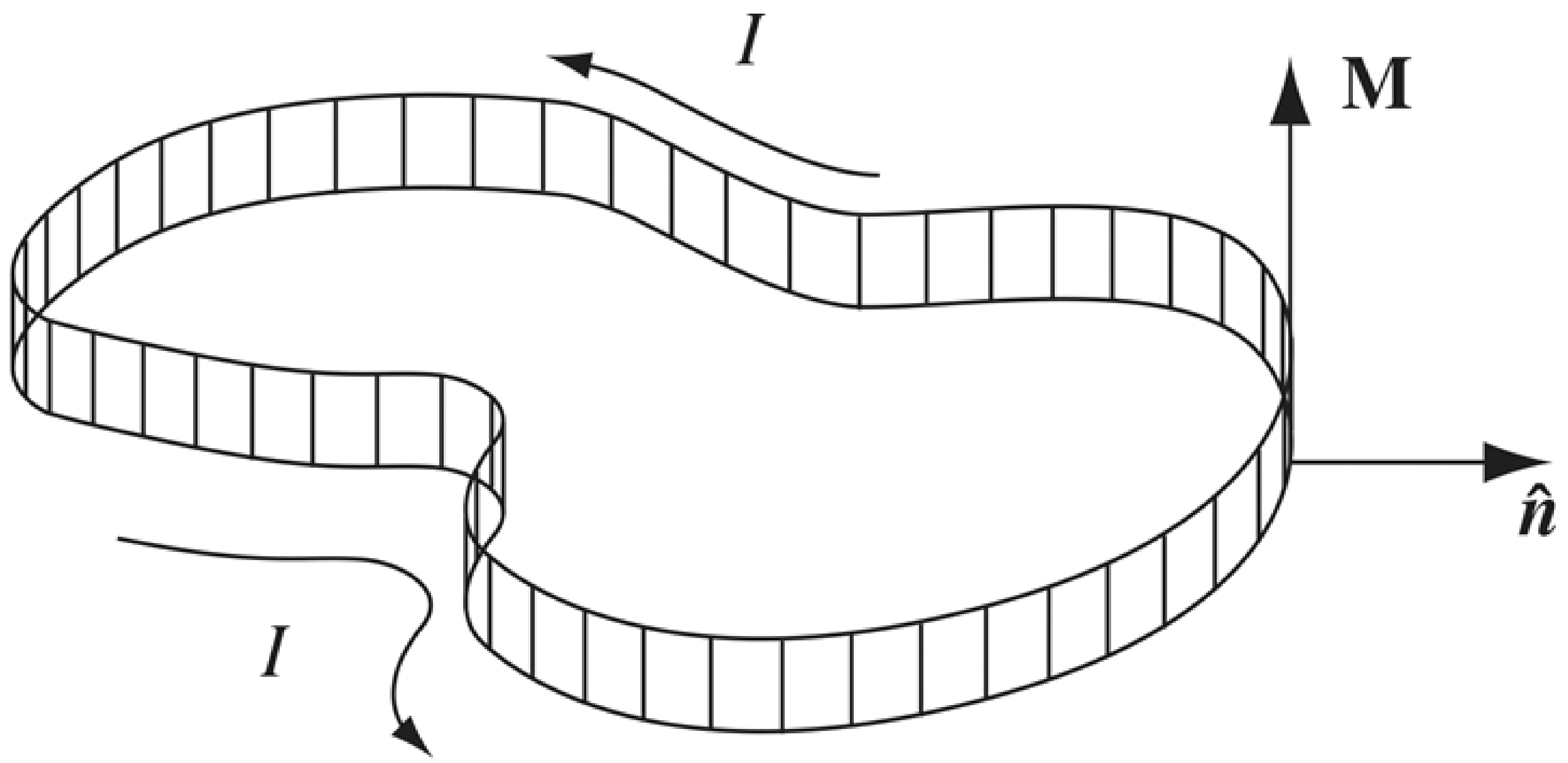
What is the current in terms of $\mathbf{M}$?

In terms of the magnetization $\mathbf{M}$, its dipole moment is $m = Mat = Ia$. So, $M = I/t = K_b$

Consider a thin slab of *uniformly* magnetized material, with the dipoles represented by tiny current loops.

All the "internal" currents cancel. However, at the edge there is no adjacent loop to do the canceling.





Physical Interpretation of Bound Current

Bound current density $\mathbf{J}_b$:

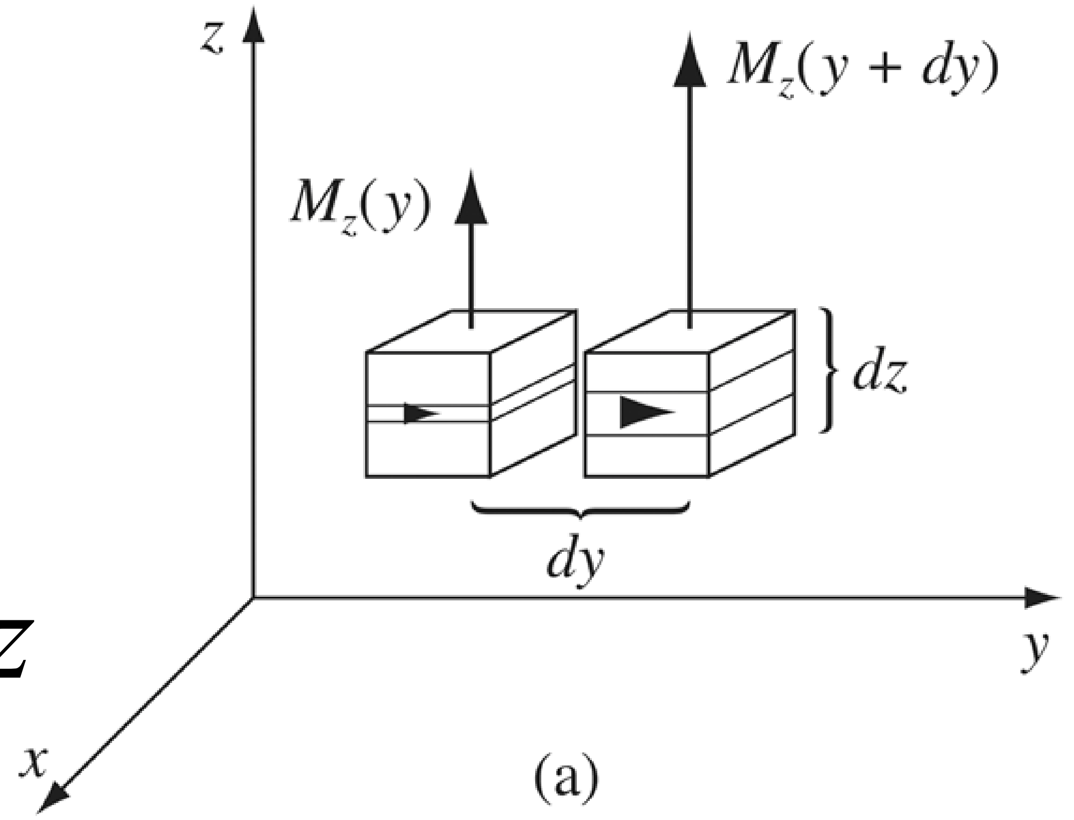
What if the magnetization is not uniform?

The adjacent current loops do not completely cancel out.

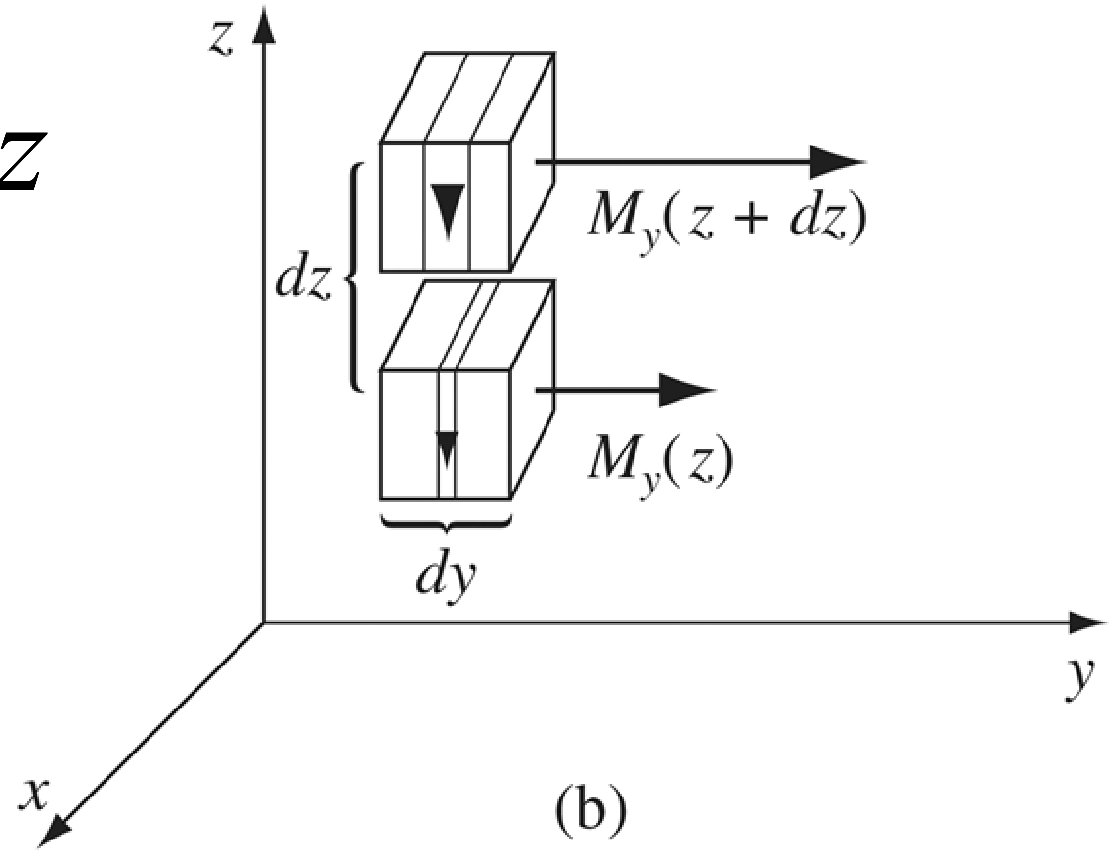
$$\text{Case (a) } I_x = [M_z(y+dy) - M_z(y)]dz = \frac{\partial M_z}{\partial y} dydz$$

$$\text{Case (b) } I_x = [M_y(z+dz) - M_y(z)]dy = \frac{\partial M_y}{\partial z} dydz$$

$$\therefore (J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z} \quad \Rightarrow \quad \mathbf{J}_b = \nabla \times \mathbf{M}$$



(a)



(b)

6.3 The Auxiliary Field **H**

6.3.1 Ampere's Law in Magnetized Materials

What is the difference between bound current and free current?

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$$

Ampere's law can be written:

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}_f + \nabla \times \mathbf{M}$$

$$\Rightarrow \quad \nabla \times \underbrace{\left(\frac{1}{\mu_0}\mathbf{B} - \mathbf{M}\right)}_{\mathbf{H}} = \mathbf{J}_f$$

In terms of **H**, then the Ampere's law reads

$$\nabla \times \mathbf{H} = \mathbf{J}_f \quad \text{(differential form)}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{fenc} \quad \text{(integral form)}$$

The Role of **H** in Magnetostatics

H plays a role in magnetostatics analogous to **D** in the electrostatics.

D allows us to write Gauss's law in terms of free charge alone.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad \nabla \cdot \mathbf{D} = \rho_f$$

H permits us to express Ampere's law in terms of free current alone.

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}, \quad \nabla \times \mathbf{H} = \mathbf{J}_f$$

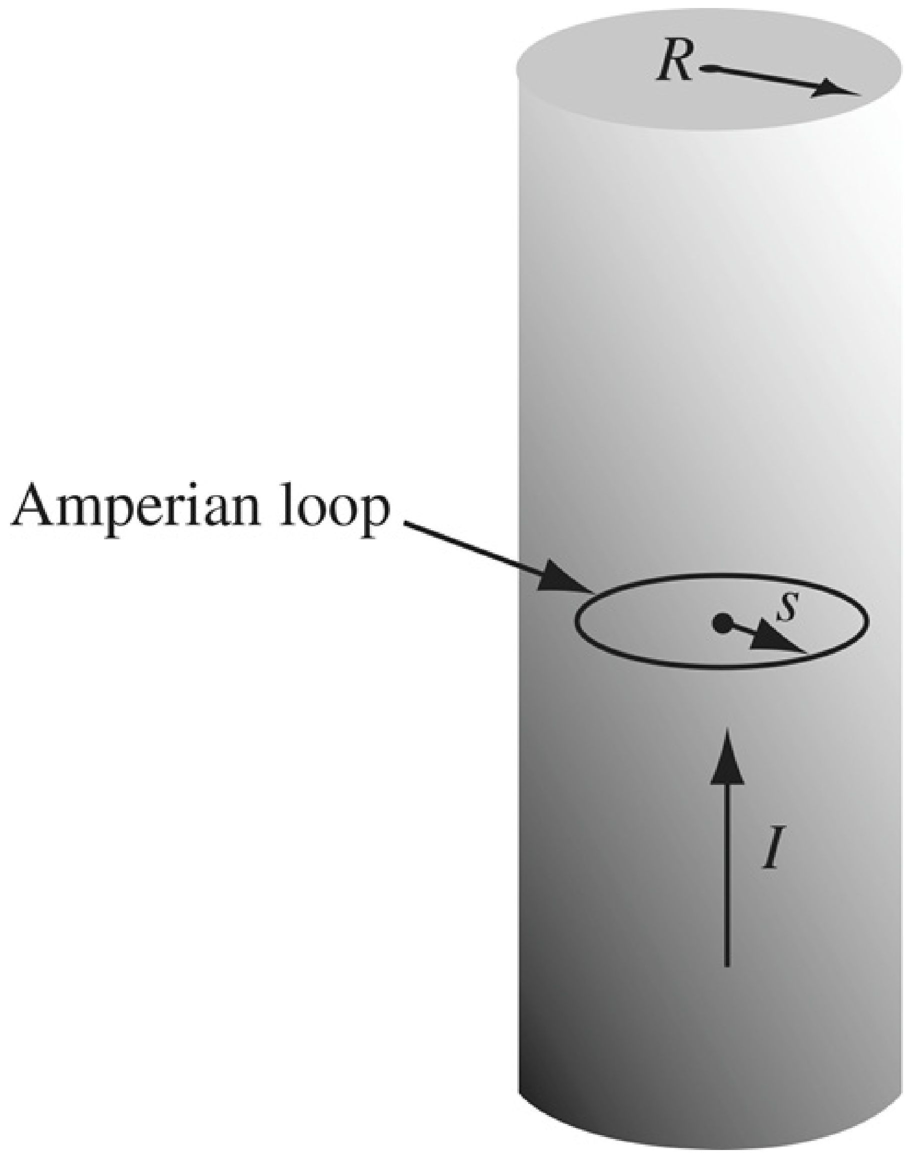
What we can control directly.

Why can't we turn the bound currents on or off independently?

Example 6.2 A long copper rod of radius R carries a uniformly distributed (free) current I. Find $\mathbf{H}$ inside and outside the rod.

Sol :

Use the Ampere's law in the integral form and properly choose a suitable Amperian loop.



$$s \le R: \quad H(2\pi s) = I_{f_{enc}} = I\frac{\pi s^2}{\pi R^2}$$

$$\text{so } \mathbf{H} = \frac{sI}{2\pi R^2}\hat{\phi}$$

$$s > R: \quad H(2\pi s) = I, \quad \text{so } \mathbf{H} = \frac{I}{2\pi s}\hat{\phi}$$

How to choose a suitable Amperian loop? Symmetry.

How to determine the magnetic field $\mathbf{B}$?

H and **B**, **D** and **E**

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

$$\nabla \cdot \mathbf{D} = \rho_f$$

Which equation is more useful?

We can easily control the free current **I**, but not the free charge. So **H** can be determined accordingly.

On the other hand, the potential difference **V** can be read from the voltmeter, which can be used to determine **E**.

The name of **H**: Some author call **H**, not **B**, the "magnetic field", but it is not a good choice. Let's just call it "**H**".

6.3.2 A Deceptive Parallel

In free space

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$
$$\nabla \cdot \mathbf{B} = 0$$

In matters

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$
$$\nabla \cdot \mathbf{H} = \nabla \cdot \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) = -\nabla \cdot \mathbf{M} \neq 0$$

At what condition the divergence of **H** is equal to zero?

$\mathbf{M} \mathbin{//} \mathbf{B}$ i.e., $\mathbf{M} \mathbin{//} \mathbf{B} \mathbin{//} \mathbf{H}$ for uniform material only.

6.3.3 Boundary Conditions

The magnetostatic boundary conditions can be rewritten in terms of $\mathbf{H}$ and the free surface current $\mathbf{K}_f$.

$$\nabla \times \mathbf{H} = \mathbf{J}_f \quad \Rightarrow \quad \mathbf{H}^{\|}_{above} - \mathbf{H}^{\|}_{below} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \quad \Rightarrow \quad H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$$

The corresponding boundary conditions in terms of $\mathbf{B}$ and total surface current $\mathbf{K}$.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \Rightarrow \quad \mathbf{B}^{\|}_{above} - \mathbf{B}^{\|}_{below} = \mu_0 (\mathbf{K} \times \hat{\mathbf{n}})$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad B^{\perp}_{above} - B^{\perp}_{below} = 0$$

How to express the boundary conditions at metal or dielectric interface?

Homework of Chap. 6 (part I)

Problem 6.4 Derive Eq. 6.3. [Here's one way to do it: Assume the dipole is an infinitesimal square, of side ε (if it's not, chop it up into squares, and apply the argument to each one). Choose axes as shown in Fig. 6.8, and calculate $\mathbf{F} = I\int (d\mathbf{I} \times \mathbf{B})$ along each of the four sides. Expand $\mathbf{B}$ in a Taylor series – on the right side, for instance,

$$\mathbf{B} = \mathbf{B}(0,\varepsilon,z) \cong \mathbf{B}(0,0,z) + \varepsilon \left.\frac{\partial \mathbf{B}}{\partial y}\right|_{(0,0,z)} .$$

For a more sophisticated method, see Prob. 6.22.]

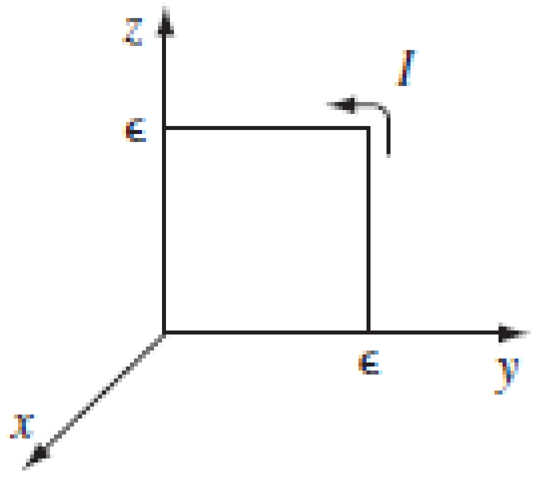


FIGURE 6.8

Problem 6.10 An iron rod of length L and square cross section (side a) is given a uniform longitudinal magnetization **M**, and then bent around into a circle with a narrow gap (width ω), as shown in Fig. 6.14. Find the magnetic field at the center of the gap, assuming $\omega \ll a \ll L$. [*Hint*: treat it as the superposition of a *complete* torus plus a square loop with reversed current.]

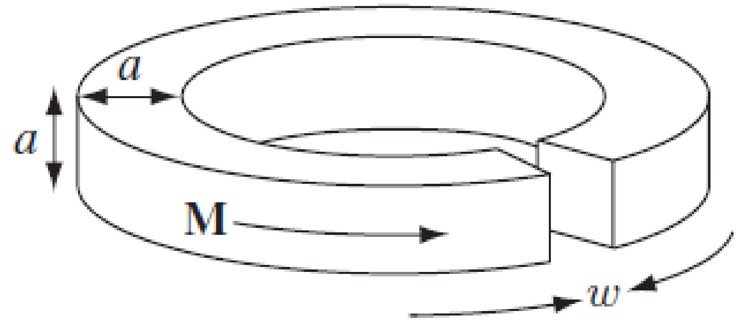


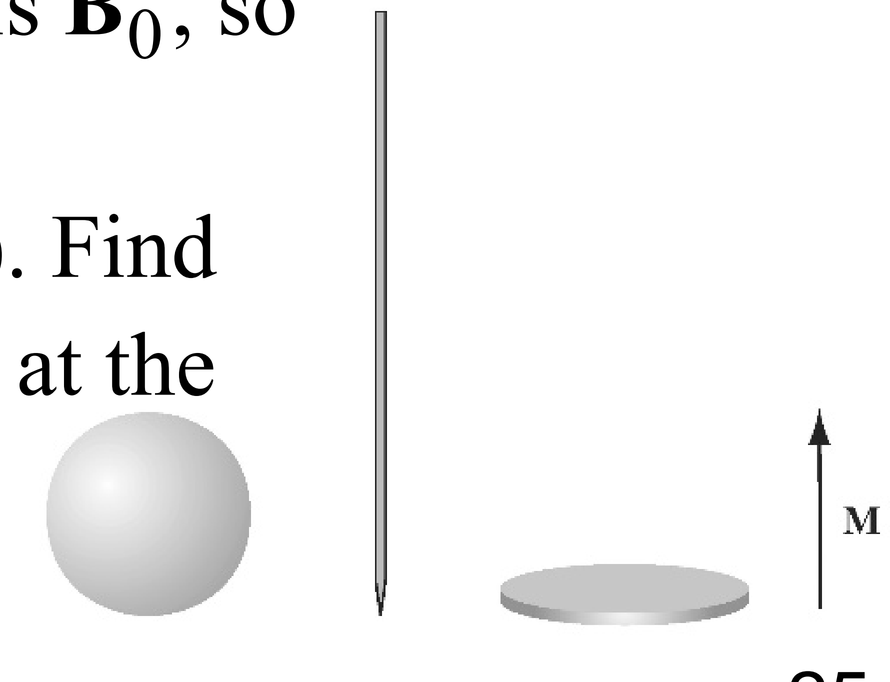
FIGURE 6.14

Problem 6.13 Suppose the field inside a large piece of magnetic material is $\mathbf{B}_0$, so that $\mathbf{H}_0 = (1/\mu_0)\mathbf{B}_0 - \mathbf{M}$, where **M** is a "frozen-in" magnetization.

(a) Now a small spherical cavity is hollowed out of the material (Fig. 6.21). Find the field at the center of the cavity, in terms of $\mathbf{B}_0$ and **M**. Also find **H** at the center of the cavity, in terms of $\mathbf{H}_0$ and **M**.

(b) Do the same for a long needle-shaped cavity running parallel to **M**.

(c) Do the same for a thin wafer-shaped cavity perpendicular to **M**.



(a) Sphere (b) Needle (c) Wafer

FIGURE 6.21

Homework of Chap. 6 (part I)

Problem 6.15 If $\mathbf{J}_f = \mathbf{0}$ everywhere, the curl of $\mathbf{H}$ vanishes (Eq. 6.19), and we can express $\mathbf{H}$ as the gradient of a scalar potential W:

$$\mathbf{H} = -\nabla W.$$

According to Eq. 6.23, then,

$$\nabla^2 W = (\nabla \cdot \mathbf{M}),$$

so W obeys Poisson's equation, with $\nabla \cdot \mathbf{M}$ as the "source." This opens up all the machinery of Chapter 3. As an example, find the field inside a uniformly magnetized sphere (Ex. 6.1) by separation of variables. [*Hint*: $\nabla \cdot \mathbf{M} = 0$ everywhere except at the surface ($r = R$), so W satisfies Laplace's equation in the regions $r < R$ and $r > R$; use Eq. 3.65, and from Eq. 6.24 figure out the appropriate boundary condition on W.]

Problem 6.17 A current I flows down a long straight wire of radius a. If the wire is made of linear material (copper, say, or aluminum) with susceptibility χ_m, and the current is distributed uniformly, what is the magnetic field a distance s from the axis? Find all the bound currents. What is the *net* bound current flowing down the wire?

6.4 Linear and Nonlinear Media
6.4.1 Magnetic susceptibility and Permeability

The magnetization of paramagnetic and diamagnetic materials is sustained by the field, i.e., when **B** is removed, **M** disappears.

$$\mathbf{M} = \chi_m \mathbf{H},$$

where the proportionality constant χ_m is called the magnetic susceptibility.

Why not use $\mathbf{M} = \frac{\chi_m}{\mu_0}\mathbf{B}$? Because $\mathbf{M} = \chi_m \mathbf{H} \propto I_f$

Materials that obey $\mathbf{M} = \chi_m \mathbf{H}$ are called linear media.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m)\mathbf{H} = \mu \mathbf{H},$$

where $\mu = \mu_0(1 + \chi_m)$ is call the **permeability** of the material.

Material Susceptibility

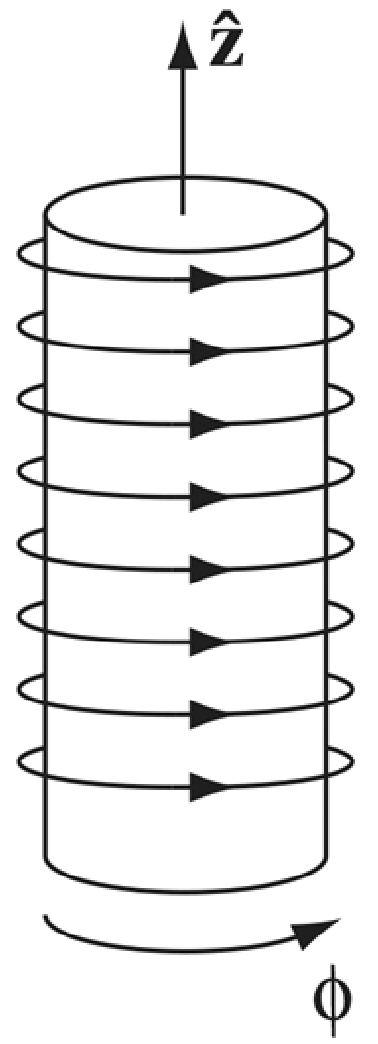
Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.7×10^{-4}	Oxygen (O_2)	1.7×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.2×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.0×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.7×10^{-4}
Carbon Dioxide	-1.1×10^{-8}	Liquid Oxygen (−200° C)	3.9×10^{-3}
Hydrogen (H_2)	-2.1×10^{-9}	Gadolinium	4.8×10^{-1}

釓ㄍㄚˊ (Gd)

Table 6.1 Magnetic Susceptibilities (unless otherwise specified, values are for 1 atm, 20° C). *Source: Handbook of Chemistry and Physics,* 67th ed. (Boca Raton: CRC Press, Inc., 1986).

Magnetic permeability & susceptibility for selected materials			
Medium	Susceptibility	Permeability $\times 10^{-6}$	
Mu-metal	20,000 [1]	25,000 N/A^2	at 0.002 T
Permalloy	8000 [1]	10,000 N/A^2	at 0.002 T
Transformer iron with ρ=0.01 μΩ·m	4000 [1]	5000 N/A^2	at 0.002 T
Steel	700 [1]	875 N/A^2	at 0.002 T
Nickel	100 [1]	125 N/A^2	at 0.002 T
soft ferrite with ρ=0.1 Ωm	source, ferroxcube	5000 N/A^2	< 0.1 mT
soft ferrite with ρ=10 Ωm	source, ferroxcube	2500 N/A^2	< 0.1 mT
Platinum	2.65×10^{-4}	1.2569701 N/A^2	
Aluminum	2.22×10^{-5} [2]	1.2566650 N/A^2	
Hydrogen	8×10^{-9} or 2.2×10^{-9} [2]	1.2566371 N/A^2	
Vacuum	0	1.2566371 N/A^2	
Sapphire	-2.1×10^{-7}	1.2566368 N/A^2	
Copper	-6.4×10^{-6} or -9.2×10^{-6} [2]	1.2566290 N/A^2	

Example 6.3 An infinite solenoid (n turns per unit length, current I) is filled with linear material of susceptibility χ_m: Find the magnetic field inside the solenoid.



Sol: The problem exhibits solenoidal symmetry. Thus, we can employ the Ampere's law.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{fenc} \quad \text{(integral form)}$$

$$H\ell = n\ell I \quad \therefore \quad \mathbf{H} = nI\hat{\mathbf{z}}$$

$$\mathbf{B} = \mu_0(1+\chi_m)nI\hat{\mathbf{z}}$$

The enhancement of the magnetic field strength depends on the susceptibility of the material.

Is there a material that the field is significantly enhanced?

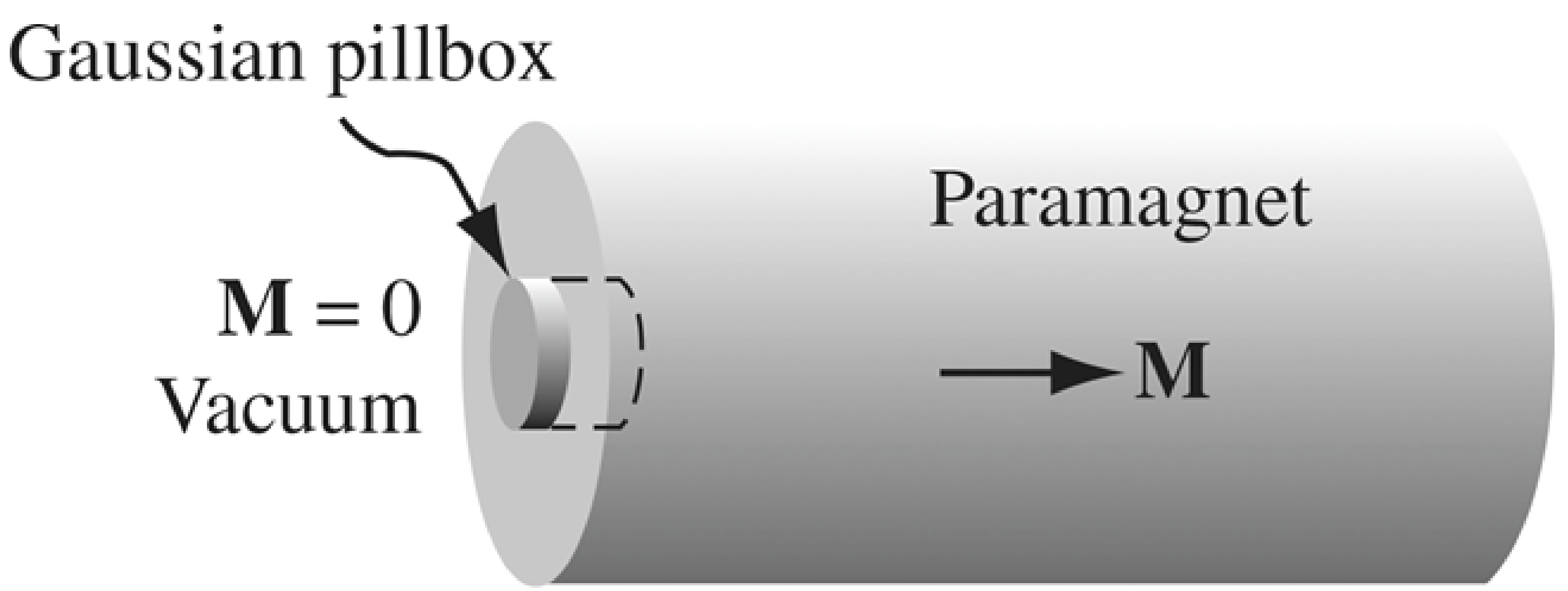
Divergence of the Magnetization

Does the linear media avoid the defect that the divergence of $\mathbf{M}$ is zero? **No!**

Even though $\mathbf{M}$, $\mathbf{H}$, and $\mathbf{B}$ are parallel, the divergence of $\mathbf{M}$ is not zero at the boundary. Consider the following example.

$$\oint_{\text{Gaussian pillbox}} \mathbf{M} \cdot d\mathbf{a} \neq 0$$

$$\Rightarrow \quad \nabla \cdot \mathbf{M} \neq 0$$

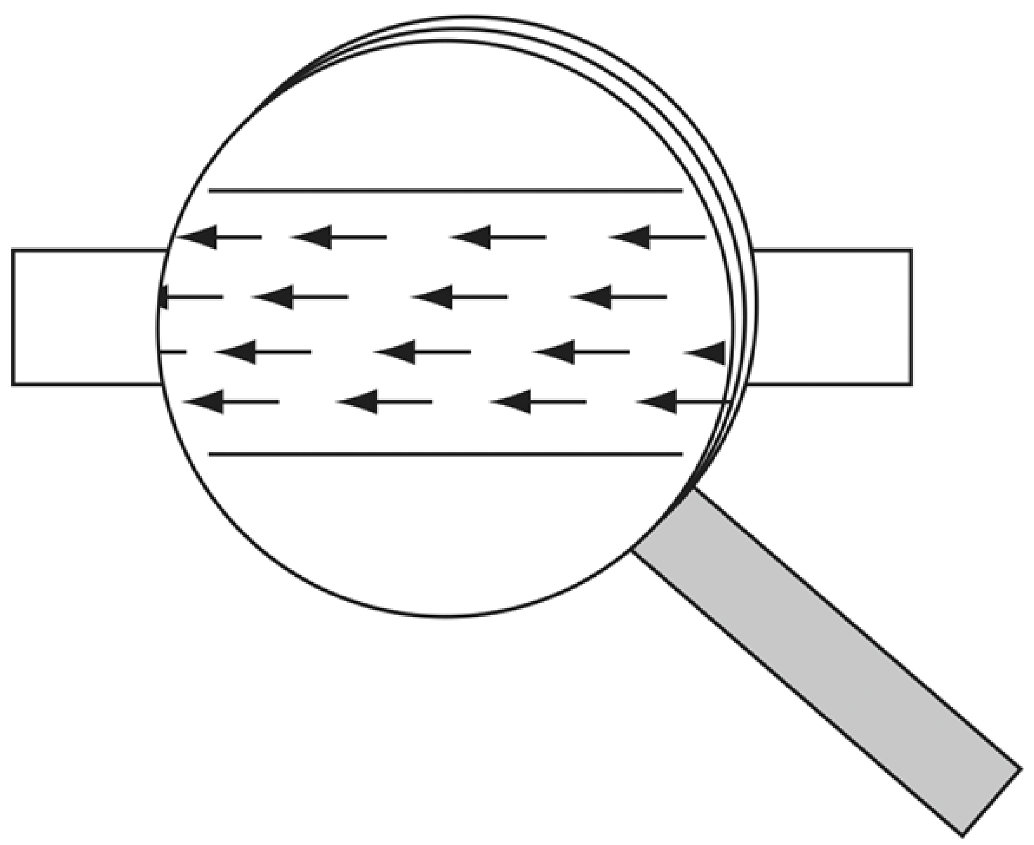


$$\text{and } \mathbf{J}_b = \nabla \times \mathbf{M} = \nabla \times \chi_m \mathbf{H} = \chi_m \mathbf{J}_f$$

6.4.2 Ferromagnetism

Ferromagnets---which are not linear---require no external fields to sustain the magnetization unlike paramagnets and diamagnets.

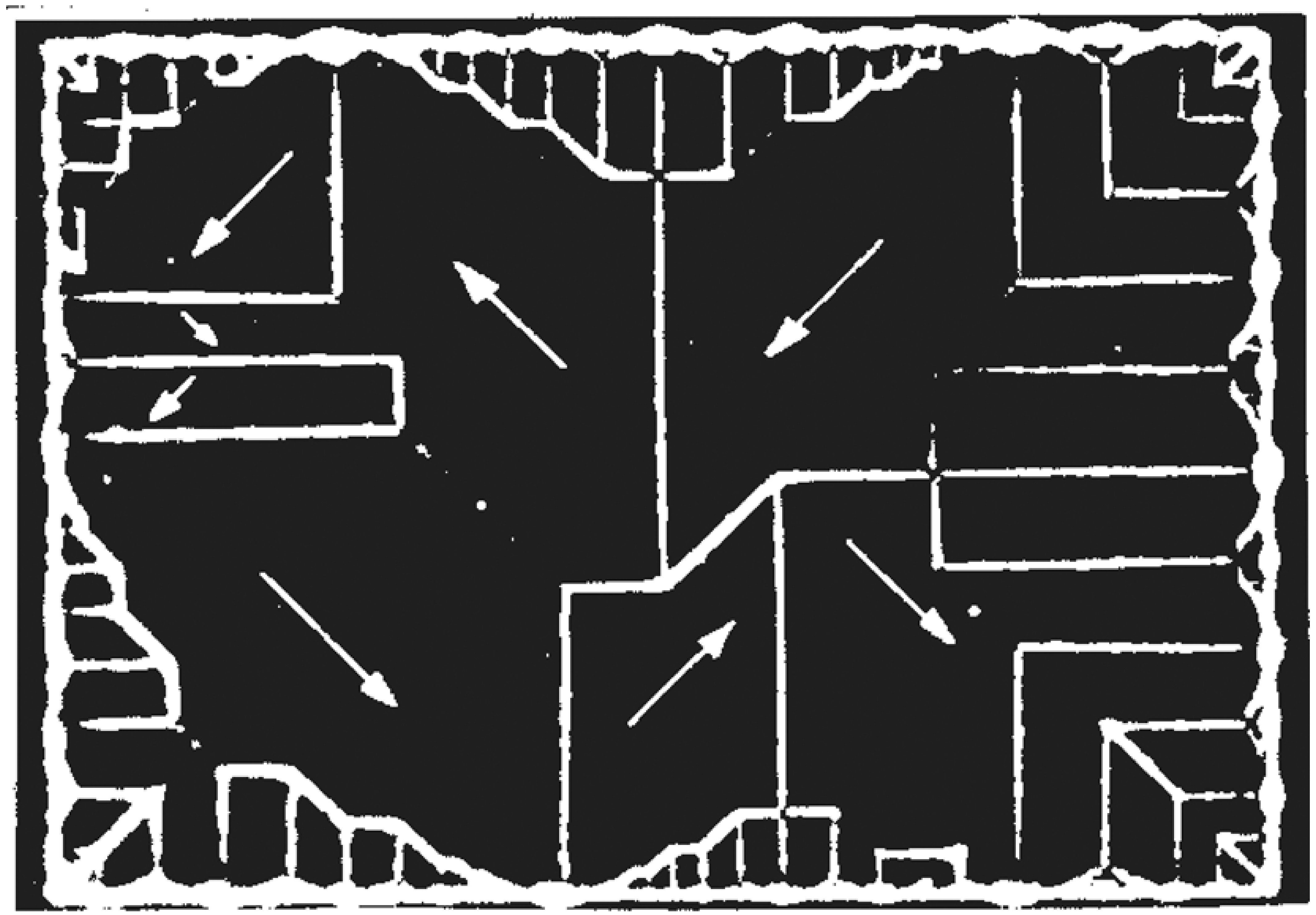
In a ferromagnet, each dipole “like” to point in the same direction as its neighbors. All the spins point the same way.



Why isn’t every wrench and nail a powerful magnet?

Domains.

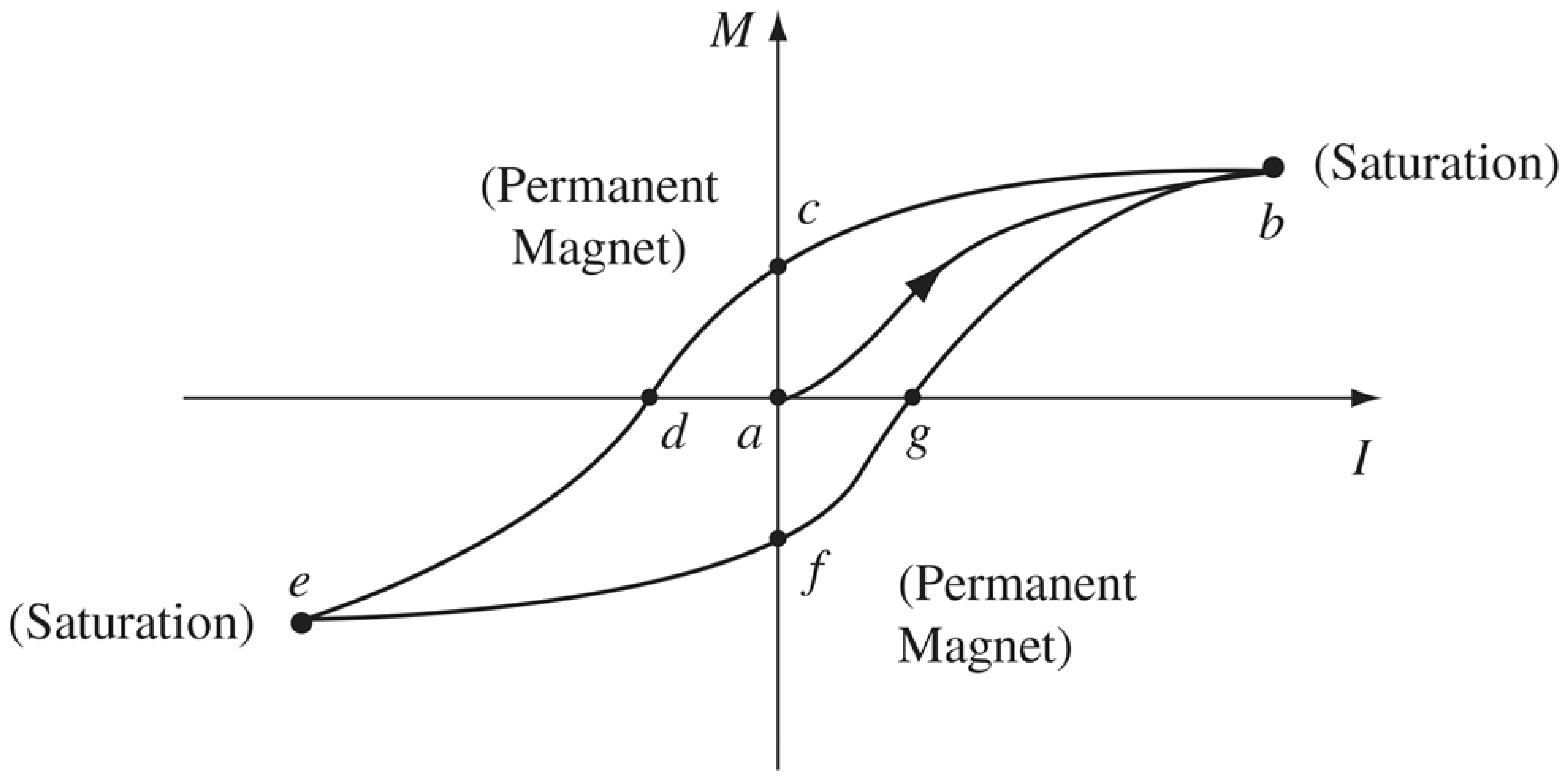
Ferromagnetic Domains



Domain boundaries: Domains parallel to the field grow, and the others shrink.

If the field is strong enough, one domain takes over entirely, and the iron is said to be "**saturated**".

Hysteresis Loop



Hysteresis: The path we have traced out.

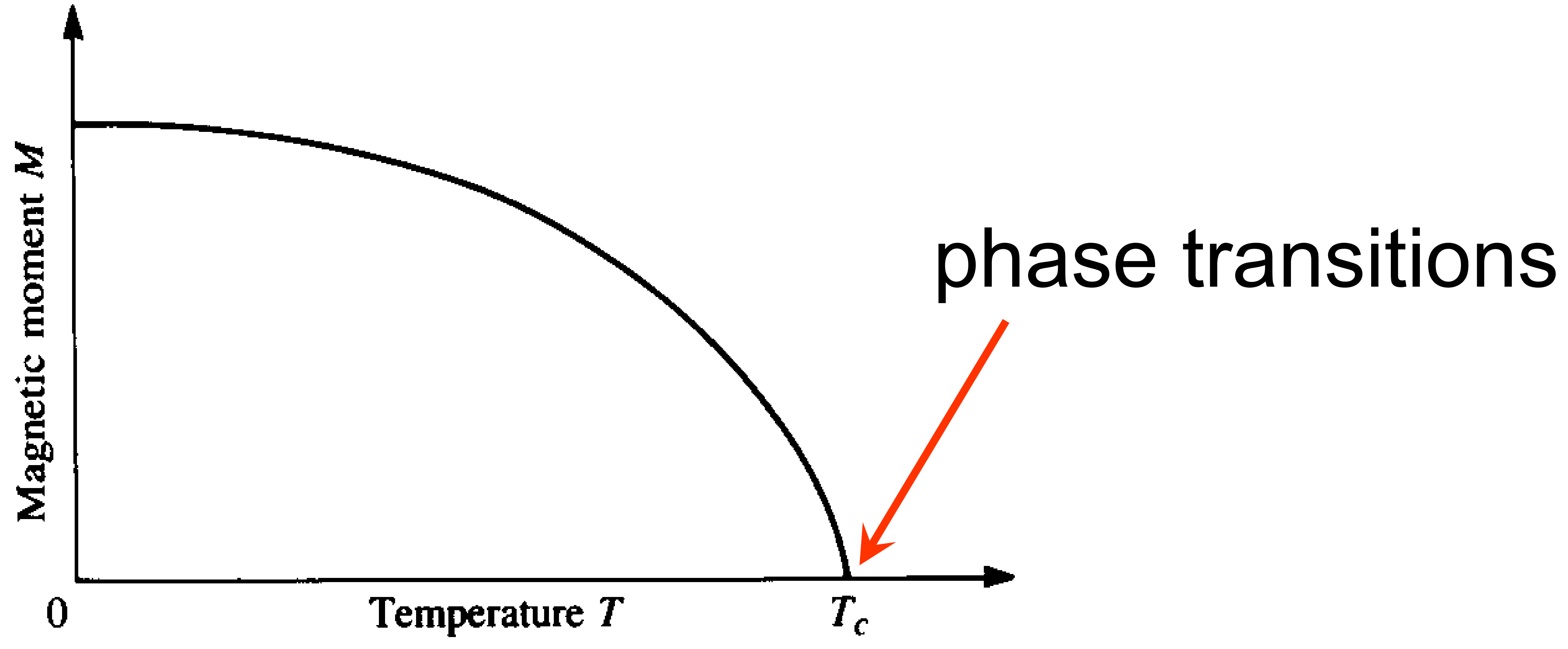
In the experiment, we adjust the current I, i.e., control $\mathbf{H}$.

In practice M is huge compared to H.

Curie Temperature and Phase Transitions

Temperature effect: The dipoles within a given domain line up parallel to one another. However, the random thermal motions complete with this ordering.

Curie temperature: As the temperature increases, the alignment is gradually destroyed. At certain temperature the iron completely turns into paramagnet. This temperature is called the curie temperature.



Homework of Chap. 6 (part II)

Problem 6.21

(a) Show that the energy of a magnetic dipole in a magnetic field **B** is

$$\boxed{U = -\mathbf{m} \cdot \mathbf{B}.} \tag{6.34}$$

[Assume that the *magnitude* of the dipole moment is fixed, and all you have to do is move it into place and rotate it into its final orientation. The energy required to keep the current flowing is a different problem, which we will confront in Chapter 7.] Compare Eq. 4.6.

(b) Show that the interaction energy of two magnetic dipoles separated by a displacement **r** is given by

$$U = \frac{\mu_0}{4\pi} \frac{1}{r^3} [\mathbf{m}_1 \cdot \mathbf{m}_2 - 3(\mathbf{m}_1 \cdot \hat{\mathbf{r}})(\mathbf{m}_2 \cdot \hat{\mathbf{r}})] \tag{6.35}$$

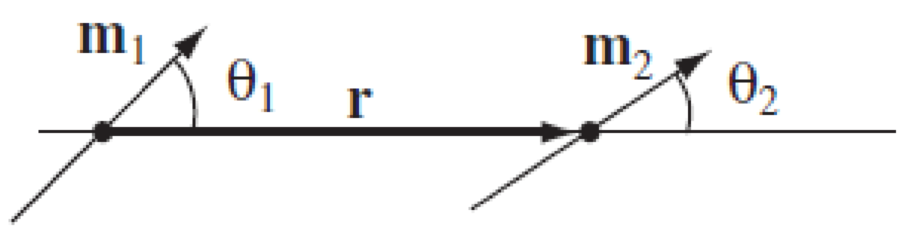


FIGURE 6.30

Compare Eq. 4.7.

(c) Express your answer to (b) in terms of the angles θ_1 and θ_2 in Fig. 6.30, and use the result to find the stable configuration two dipoles would adopt if held a fixed distance apart, but left free to rotate.

(d) Suppose you had a large collection of compass needles, mounted on pins at regular intervals along a straight line. How would they point (assuming the earth's magnetic field can be neglected)? [A rectangular array of compass needles aligns itself spontaneously, and this is sometimes used as a demonstration of "ferromagnetic" behavior on a large scale. It's a bit of a fraud, however, since the mechanism here is purely classical, and much weaker than the quantum mechanical **exchange forces** that are actually responsible for ferromagnetism.[13]]

Homework of Chap. 6 (part II)

Problem 6.25 Notice the following parallel:

$$\begin{cases} \nabla\cdot\mathbf{D}=0, \quad \nabla\times\mathbf{E}=0, \quad \varepsilon_0\mathbf{E}=\mathbf{D}-\mathbf{P}, & \text{(no free charge)} \\ \nabla\cdot\mathbf{B}=0, \quad \nabla\times\mathbf{H}=0, \quad \mu_0\mathbf{H}=\mathbf{B}-\mu_0\mathbf{M}, & \text{(no free current)} \end{cases}$$

Thus, the transcription $\mathbf{D}\to\mathbf{B}$, $\mathbf{E}\to\mathbf{H}$, $\mathbf{P}\to\mu_0\mathbf{M}$, $\varepsilon_0\to\mu_0$ turns an electrostatic problem into an analogous magnetostatic one. Use this, together with your knowledge of the electrostatic results, to rederive

(a) the magnetic field inside a uniformly magnetized sphere (Eq. 6.16);

(b) the magnetic field inside a sphere of linear magnetic material in an otherwise uniform magnetic field (Prob. 6.18);

(c) the average magnetic field over a sphere, due to steady currents within the sphere (Eq. 5.93).

Problem 6.27 At the interface between one linear magnetic material and another, the magnetic field lines bend (Fig. 6.32). Show that $\tan\theta_2/\tan\theta_1 = \mu_2/\mu_1$, assuming there is no free current at the boundary. Compare Eq. 4.68.

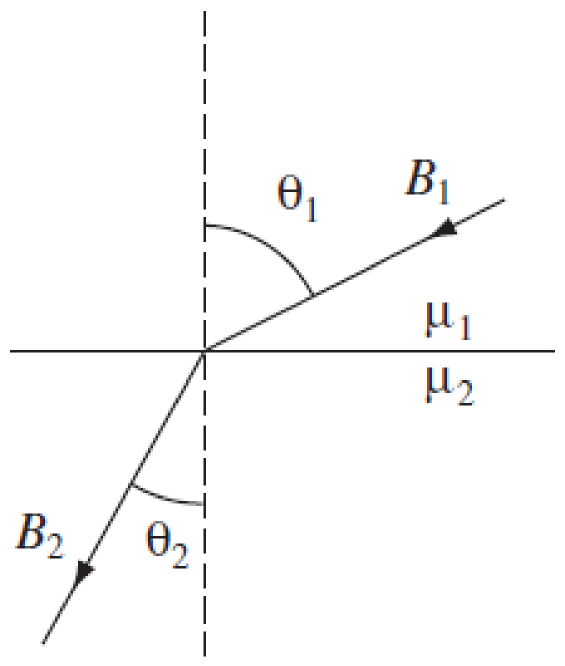


FIGURE 6.32