Chap.5 Supplement

For the EM Course Lectured by Prof. Tsun-Hsu Chang

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Problem 5.6

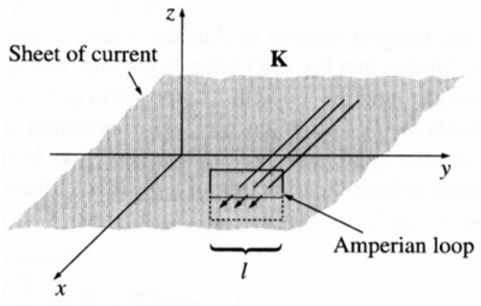
(a) A phonograph record carries a uniform density of "static electricity" σ. If it rotates at angular velocity ω, what is the surface current density K at a distance r from the center?

(b) A uniformly charged solid sphere, of radius R and total charge Q, is centered at the origin and spinning at a constant angular velocity ω about the z axis. Find the current density $\mathbf{J}$ at any point (r, θ, ϕ) within the sphere.

$$(a)\ \ \mathbf{K}(\mathbf{r}) = \sigma\mathbf{v} = \sigma(\boldsymbol{\omega}\times\mathbf{r}) = \sigma r\omega\hat{\boldsymbol{\phi}} \quad \text{(b)}\ \mathbf{J} = \rho\mathbf{v} = \rho\omega r\sin\theta\hat{\boldsymbol{\phi}} = \frac{3Qr\omega\sin\theta}{4\pi R^3}\hat{\boldsymbol{\phi}}$$

Infinite Planes

Example 5.8 Find the magnetic field of an infinite uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$, flowing over the xy plane.



Solution:

$$\oint \mathbf{B}\cdot d\mathbf{l} = B2l = \mu_0 K l$$

$$B = \frac{\mu_0 K}{2}$$

$$\mathbf{B} = \begin{cases} \mu_0 K/2\,\hat{\mathbf{y}} & \text{for } z<0 \\ -\mu_0 K/2\,\hat{\mathbf{y}} & \text{for } z>0 \end{cases}$$

Why no z-component? → Rotational symmetry
Why no x-component? → Biot-Savart Law

Solenoid

Example 5.9 An ideal infinite solenoid has n turns per unit length and carries a current I. Find its magnetic field inside.

Solution:

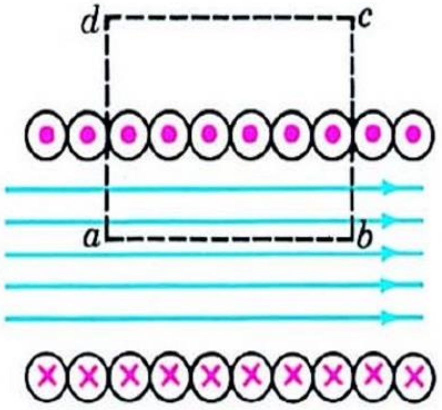
$$\oint \mathbf{B}\cdot d\mathbf{l}$$

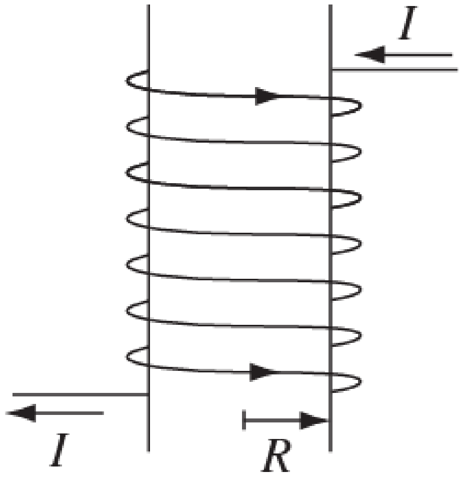
$$= \int_a^b \mathbf{B}\cdot d\mathbf{l} + \int_b^c \mathbf{B}\cdot d\mathbf{l} + \int_c^d \mathbf{B}\cdot d\mathbf{l} + \int_d^a \mathbf{B}\cdot d\mathbf{l}$$

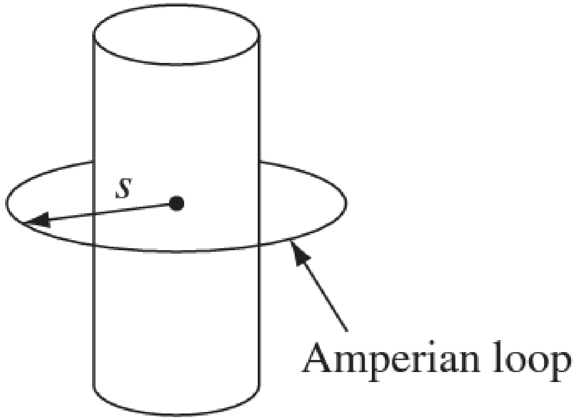
$$= \int_a^b \mathbf{B}\cdot d\mathbf{l}$$

$$BL_{ab} = \mu_0 n L_{ab} I$$

$$B = \mu_0 n I$$







Why no radial part? → Reversing the current direction is not consistent with the upside-down solenoid
Why no circumferential part? →Amperian loop encloses no current

$$\mathbf{B} = \begin{cases} \mu_0 n I\,\hat{\mathbf{z}}, & \text{inside the solenoid,} \\ 0, & \text{outside the solenoid.} \end{cases}$$

Comment: the solenoid is to magnetostatics what the parallel-pater capacitor is to electrostatics: a simple device for producing strong uniform fields.

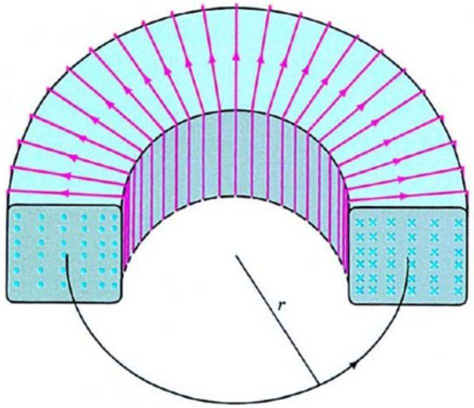
Toroid

Example 5.10 A toroidal coil (shaped like a doughnut) is tightly wound with N turns and carries a current I. We assume that it has a rectangular cross section, as shown below. Find the field strength within the toroid.

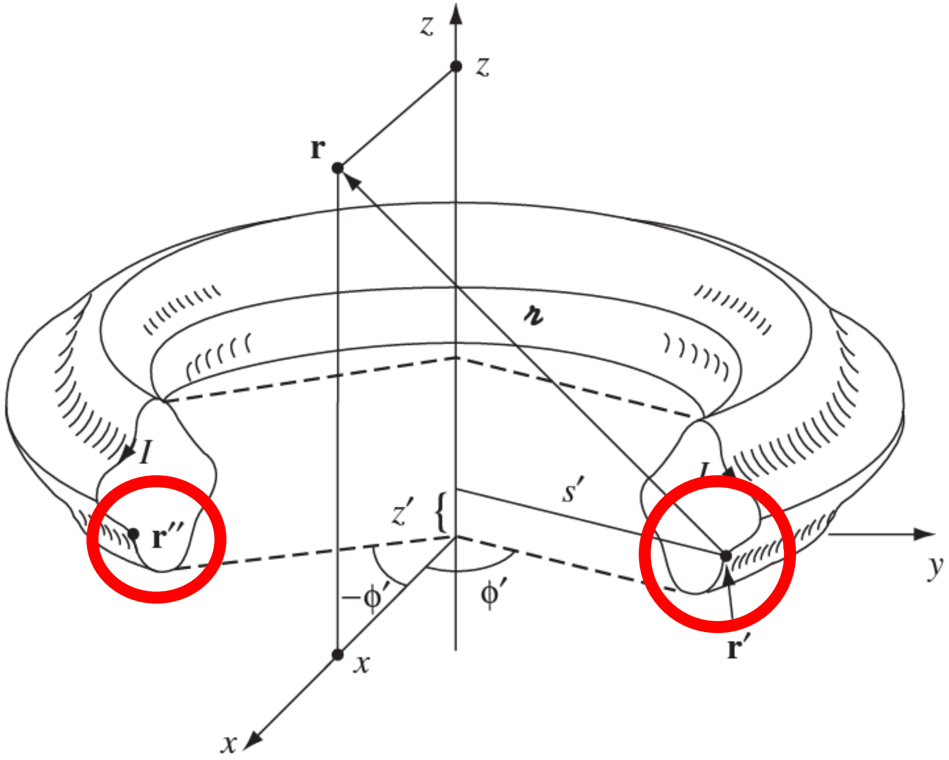
Solution:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$



The field is not uniform; it varies as $1/r$. The toroidal fields are used in research on fusion power.



Why only phi component?
→ Go back to Biot-Savart Law

$$\mathbf{r}' = (s' \cos\phi', s' \sin\phi', z')$$

$$\mathscr{r} = (x - s' \cos\phi', -s' \sin\phi', z - z')$$

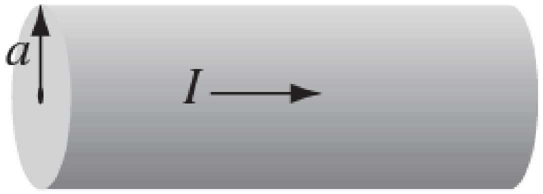
$$\mathbf{I} = (I_s \cos\phi', I_s \sin\phi', I_z)$$

$$\mathbf{I} \times \mathscr{r} = \begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ I_s \cos\phi' & I_s \sin\phi' & I_z \\ (x - s' \cos\phi') & (-s' \sin\phi') & (z - z') \end{bmatrix}$$

$$= \left[\sin\phi' \left(I_s(z - z') + s' I_z\right)\right] \hat{\mathbf{x}} + \left[I_z(x - s' \cos\phi') - I_s \cos\phi'(z - z')\right] \hat{\mathbf{y}}$$

$$+ \left[-I_s x \sin\phi'\right] \hat{\mathbf{z}}.$$

Problem 5.14 A steady current I flows down a long cylindrical wire of radius a (Fig. 5.40). Find the magnetic field, both inside and outside the wire, if



(a) The current is uniformly distributed over the outside surface of the wire.

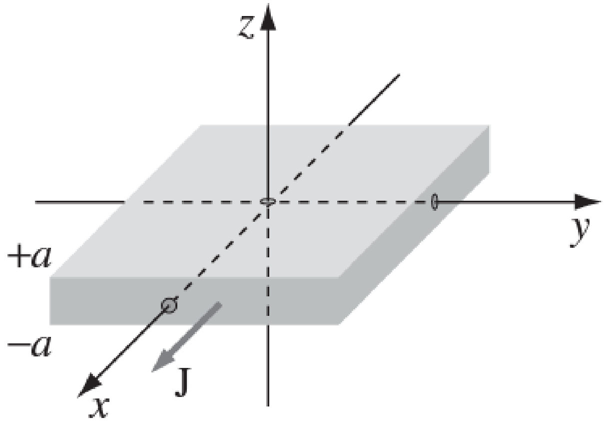
(b) The current is distributed in such a way that J is proportional to s, the distance from the axis.

$$(a)\ \mathbf{B}(s)=\begin{cases}\dfrac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\theta}}, & s>a\\ 0, & s<a\end{cases}$$

$$\text{(b) } \mathbf{B}_{in}=\frac{\mu_0 I_{enc}}{2\pi s}\hat{\boldsymbol{\theta}}=\frac{\mu_0}{2\pi s}\hat{\boldsymbol{\theta}}\int_0^s Cs'2\pi s'ds'=\frac{C\mu_0 s^2}{3}\hat{\boldsymbol{\theta}}\quad \because \int_0^s Cs'2\pi s'ds'=I\Rightarrow C=\frac{3}{2\pi a^3}I\quad \therefore \mathbf{B}_{in}=\frac{\mu_0 I s^2}{2\pi a^3}\hat{\boldsymbol{\theta}}$$

And of course, $\mathbf{B}_{out}=\dfrac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\theta}}$

Problem 5.15 A thick slab extending from $z=-a$ to $z=+a$ (and infinite in the x and y directions) carries a uniform volume current $\mathbf{J}=J\,\hat{\mathbf{x}}$ (Fig. 5.41). Find the magnetic field, as a function of z, both inside and outside the slab.



Convince yourself that the magnetic field only has y-component!

With Amperian's loop, one obtains

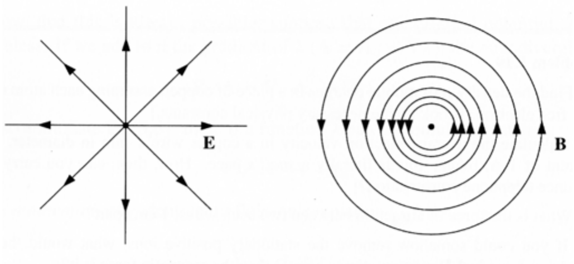
$$\begin{cases}\mathbf{B}=-\mu_0 Jz\hat{\mathbf{y}}, & -a<z<a\\ \mathbf{B}=-\mu_0 Ja\hat{\mathbf{y}}, & z>a\\ \mathbf{B}=\mu_0 Ja\hat{\mathbf{y}}, & z<-a\end{cases}$$

More on Magnetic Vector Potential

5.3.4 Comparison of Magnetostatics and Electrostatics

$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	Gauss's law
$\nabla \times \mathbf{E} = 0 \ (-\frac{\partial \mathbf{B}}{\partial t})$	No name (Faraday's law)
$\nabla \cdot \mathbf{B} = 0$	Gauss's law for magnetic field
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \ (+\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t})$	Ampere's law (Ampere-Maxwell law)

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ Lorentz's force law



(a) Electrostatic field of a point charge

(b) Magnetostatic field of a long wire

5.4 Magnetic Vector Potential

5.4.1 The Vector Potential

$$\nabla \times \mathbf{E} = 0 \Leftrightarrow \mathbf{E} = -\nabla V \quad \text{and} \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0 \Leftrightarrow \mathbf{B} = \nabla \times \mathbf{A} \quad \text{and} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \Rightarrow \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}$$

Is it possible for us to set $\nabla \cdot \mathbf{A} = 0$? Yes.

The Coulomb gauge

Proof: If $\nabla \cdot \mathbf{A}_0 \neq 0$, let $\mathbf{A} = \mathbf{A}_0 + \nabla \lambda \Rightarrow \mathbf{B} = \nabla \times \mathbf{A}_0 = \nabla \times \mathbf{A}$

If $\nabla \cdot \mathbf{A} = 0$, then $\nabla^2 \lambda = -\nabla \cdot \mathbf{A}_0$ ← similiar to Poisson's equation

$$\begin{cases} \nabla^2 V = -\rho / \varepsilon_0 & V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r} d\tau' \\ \nabla^2 \lambda = -\nabla \cdot \mathbf{A}_0 & \lambda = \frac{1}{4\pi} \int \frac{\nabla \cdot \mathbf{A}_0}{r} d\tau' \end{cases}$$

It is always possible to make the vector potential divergenceless.

What's the physical picture of Vector potential?

*For more details, see AJP **64**, 1361 (1996)*

More on Magnetic Vector Potential

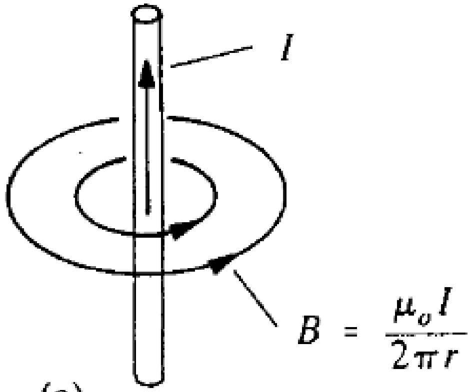
Coulomb Gauge: $\nabla \cdot \mathbf{A} = 0$ ← **Why do we need "Gauge" (規範) here?**

$$\begin{cases} \nabla \cdot \mathbf{B} = 0 \Leftrightarrow \mathbf{B} = \nabla \times \mathbf{A} \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \left(+\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \end{cases}$$

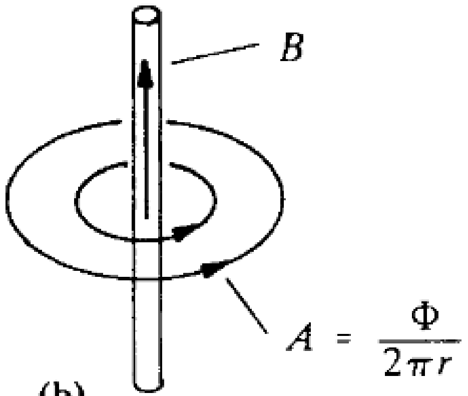
Effect **Cause**

Imply "Ampere's law" for A

The term *gauge* refers to any specific mathematical formalism to regulate redundant degrees of freedom in the Lagrangian of a physical system.



(a)



(b)

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla\left(V_0 + \underline{C}\right)$$

$$\nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B} = \nabla \times \left(\mathbf{A}_0 + \underline{\nabla \lambda}\right)$$

Extra DOFs for potentials (make no harm to the fields)

Maxwell's equations have a gauge symmetry. (Invariant under gauge transformation!)

More on Magnetic Vector Potential

Coulomb Gauge: $\nabla \cdot \mathbf{A} = 0$

$$\begin{cases} \nabla \cdot \mathbf{B} = 0 \Leftrightarrow \mathbf{B} = \nabla \times \mathbf{A} \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \left(+\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \end{cases}$$

Effect **Cause**

If $\nabla \cdot \mathbf{A} = 0$, then $\nabla^2 \lambda = -\nabla \cdot \mathbf{A}_0$

$$\begin{cases} \nabla^2 V = -\rho / \varepsilon_0 & V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{\mathscr{r}} d\tau' \\ \nabla^2 \lambda = -\nabla \cdot \mathbf{A}_0 & \lambda = \frac{1}{4\pi} \int \frac{\nabla \cdot \mathbf{A}_0}{\mathscr{r}} d\tau' \end{cases}$$

A good gauge helps us to solve problems efficiently.

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} = \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{A}) \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\Rightarrow \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \phi \Rightarrow \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

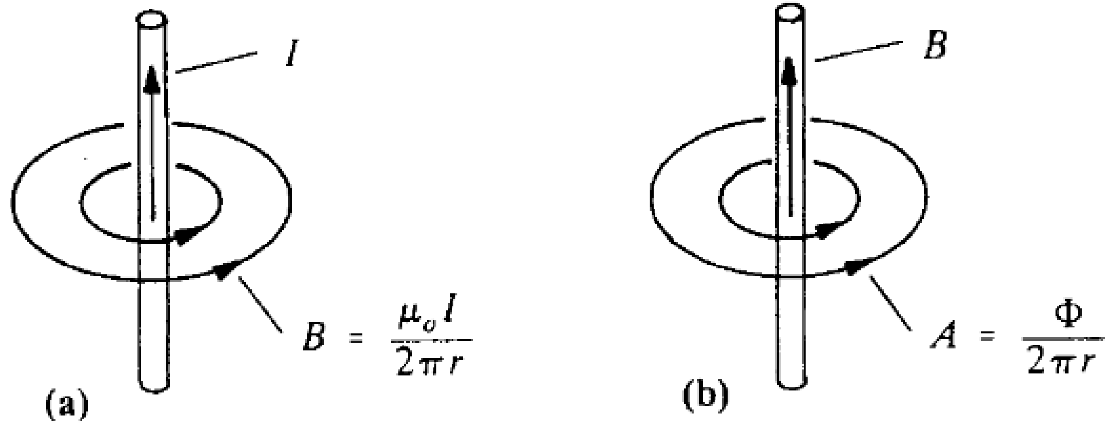
For any scalar field $f \Rightarrow \begin{cases} \mathbf{A}' = \mathbf{A} - \nabla f \\ \phi' = \phi + \frac{\partial}{\partial t} f \end{cases}$

describes the same physical situation.

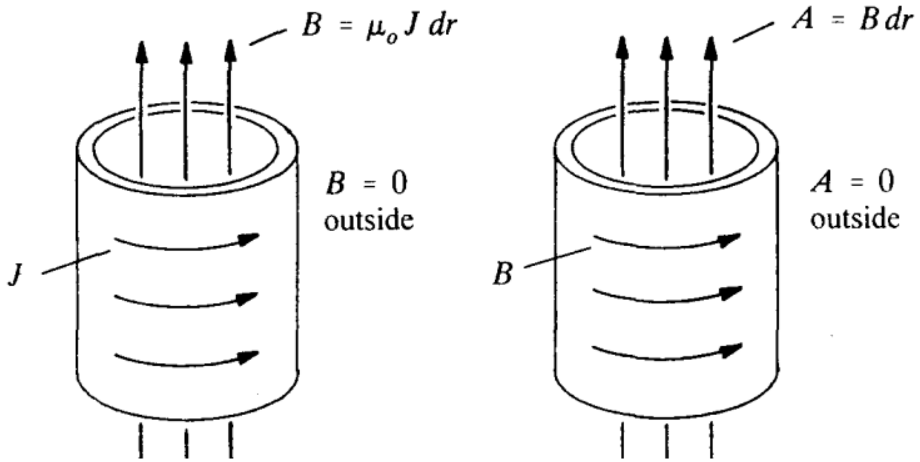
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} = -\nabla^2 \phi - \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) \Rightarrow -\nabla^2 \phi = \frac{\rho}{\varepsilon_0}$$

*For more details, see AJP **64**, 1361 (1996)*

More on Magnetic Vector Potential



$$\oint \mathbf{A}\cdot d\mathbf{l} = \int \mathbf{B}\cdot d\mathbf{a} = \Phi$$



Inside the wire

$$\mathbf{B}\cdot 2\pi r = \mu_0 I\left(\frac{\pi r^2}{\pi a^2}\right) \Rightarrow \mathbf{B} = \frac{\mu_0 I r}{2\pi a^2}\hat{\boldsymbol{\phi}}$$

$$\mathbf{A}\cdot 2\pi r = \Phi\left(\frac{\pi r^2}{\pi a^2}\right) \Rightarrow \mathbf{A} = \frac{\Phi r}{2\pi a^2}\hat{\boldsymbol{\phi}} = \frac{Br}{2}\hat{\boldsymbol{\phi}} = \boxed{\frac{1}{2}\mathbf{B}\times\mathbf{r}}$$

Useful in QM

Known as symmetry gauge (Checked in Prob. 5.25)

Similarly, one can imagine the A-field in other common situations.

More on Magnetic Vector Potential

Is Vector Potential no more than a math trick?

<u>Maxwell-Thomson view</u>

The vector potential can be seen as a "stored momentum" per unit charge in much the same way that electric potential is the "stored energy" per unit charge. One of Maxwell's several names for the vector potential was "electromagnetic momentum."

More on Magnetic Vector Potential

Consider a charged particle in an electromagnetic field.

$$\nabla\times\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \Rightarrow \nabla\times\left(\mathbf{E}+\frac{\partial \mathbf{A}}{\partial t}\right) = 0 \Rightarrow \mathbf{E}+\frac{\partial \mathbf{A}}{\partial t} = -\nabla\phi$$

Generalized potential energy

$$\therefore U = q\phi = q\left(V-\mathbf{A}\cdot\mathbf{v}\right) \Rightarrow L = \frac{1}{2}m\mathbf{v}\cdot\mathbf{v} - q\left(V-\mathbf{A}\cdot\mathbf{v}\right)$$

Generalized momentum

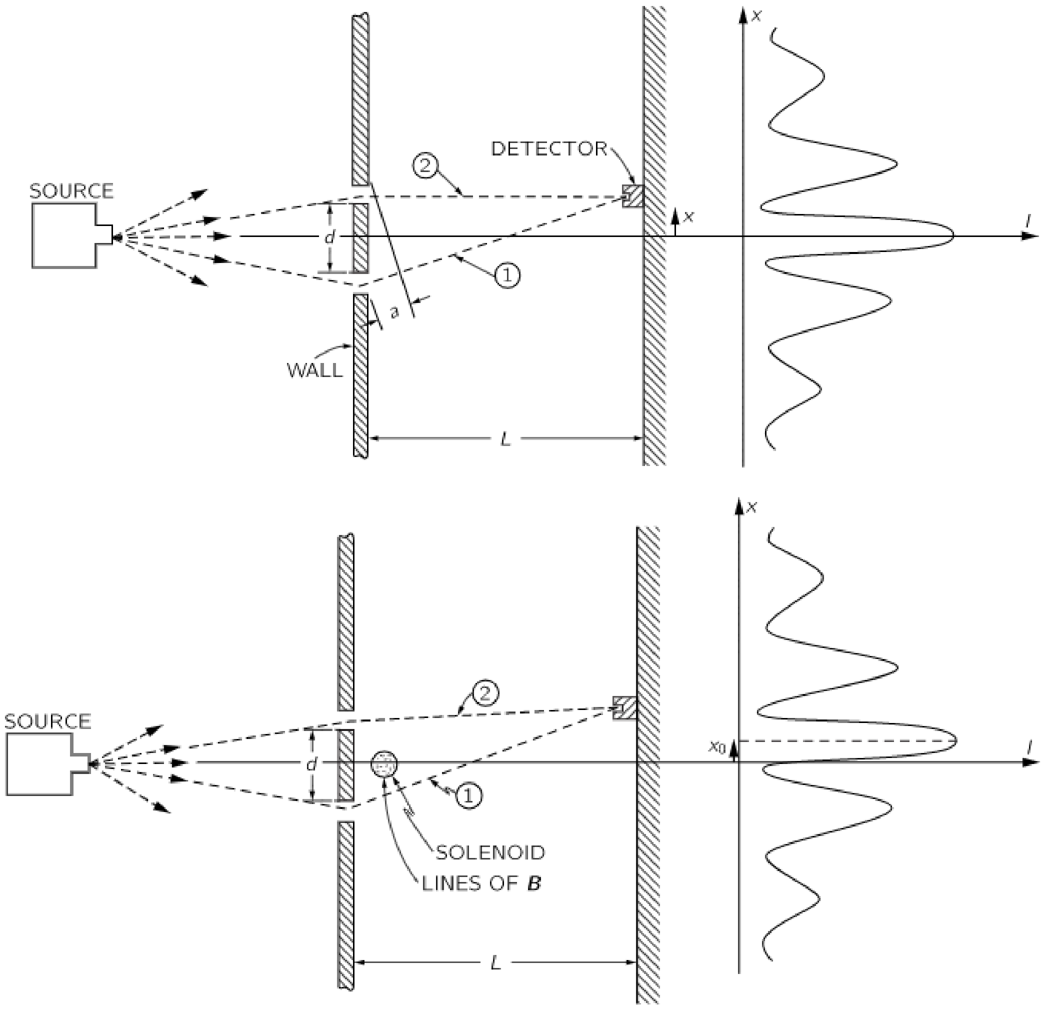
$$p_i \equiv \frac{\partial L}{\partial v_i} = mv_i + qA_i$$

Momentum in the field

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \Rightarrow m\ddot{x} + q\left[\frac{dA_x}{dt} + \frac{\partial V}{\partial x} - \frac{\partial \mathbf{A}}{\partial x}\cdot\mathbf{v}\right] = 0$$

$$\Rightarrow m\ddot{x} = q\left[\left(-\frac{\partial V}{\partial x} - \frac{\partial A_x}{\partial t}\right) + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial y}\right)\dot{y} - \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\dot{z}\right] = q\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}\right]_x$$

More on Magnetic Vector Potential



—Feynman Lectures Vol. 2, Sec.15.5 —

"Because in classical mechanics, *A* did not appear to have any direct importance and because it could be changed by adding a gradient, people repeatedly said that the vector potential had no direct physical significance—that only the magnetic and electric fields are "right" even in quantum mechanics, ……, It is interesting that something like this can be around for thirty years but, because of certain prejudices of what is and is not significant, continues to be ignored."

More on Magnetic Vector Potential

An interpretation for Aharonov-Bohm effect with classical electromagnetic theory

Gaobiao Xiao

The magnetic Aharonov-Bohm effect shows that charged particles may be affected by the vector potential in regions without any electric or magnetic fields [1]. The Aharonov-Bohm effect was experimentally confirmed [2-3] and has been found in many situations [4-6]. A common explanation is based on quantum mechanics, which states that the wavefunctions associated with the charges will accumulate a phase shift due to the vector potential. However, consensus about its nature and interpretation has not been achieved [7-14]. We here propose a simple but reasonable interpretation based on the theory for electromagnetic radiation and couplings [15]. The energy associated with a pulse radiator is divided into a Coulomb-

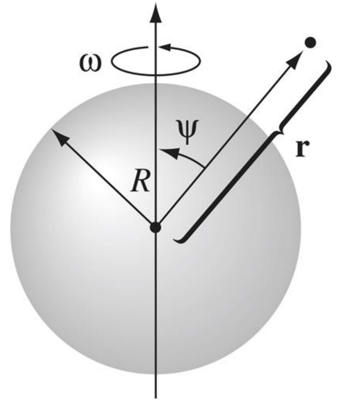
Some are still trying to find evidence of the A-field in classical mechanics.

https://arxiv.org/abs/2201.12292

"A common explanation is based on quantum mechanics, which states that the wavefunctions associated with the charges will accumulate a phase shift due to the vector potential. …… We here propose a simple but reasonable interpretation based on the theory of electromagnetic radiation and couplings. It is derived directly from the Maxwell theory with no modification but only substitution and reorganization."

Spinning Charged Sphere

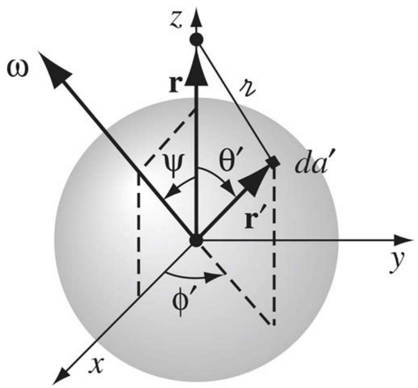
Example 5.11 A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. Find the vector potential it produce at point $\mathbf{r}$.



First, let the observer is in the z axis and ω is tilted at an angle ψ

Vector potential is $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{\mathbf{K}(\mathbf{r}')}{\mathscr{r}} da'$

The surface current density $\mathbf{K}(\mathbf{r}') = \sigma \mathbf{v}'$



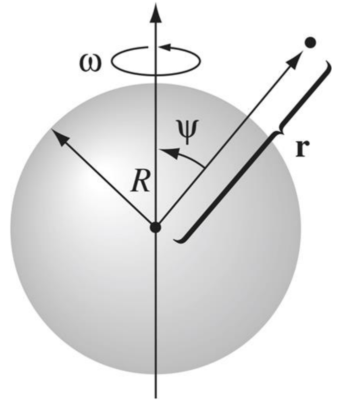
$$\mathbf{v}' = \boldsymbol{\omega} \times \mathbf{r}' = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \omega \sin\psi & 0 & \omega\cos\psi \\ R\sin\theta'\cos\phi' & R\sin\theta'\sin\phi' & R\cos\theta' \end{vmatrix}$$

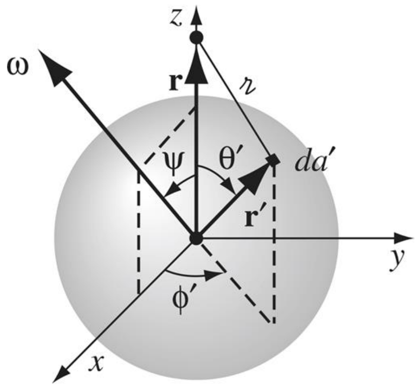
$$= R\omega[-(\cos\psi\sin\theta'\sin\phi')\hat{\mathbf{x}} + (\cos\psi\sin\theta'\cos\phi' - \sin\psi\cos\theta')\hat{\mathbf{y}} + (\sin\psi\sin\theta'\sin\phi')\hat{\mathbf{z}}]$$

Spinning Charged Sphere

Prob. 5.30 deals with the case of a solid sphere

Example 5.11 A spherical shell, of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω. Find the vector potential it produce at point $\mathbf{r}$.





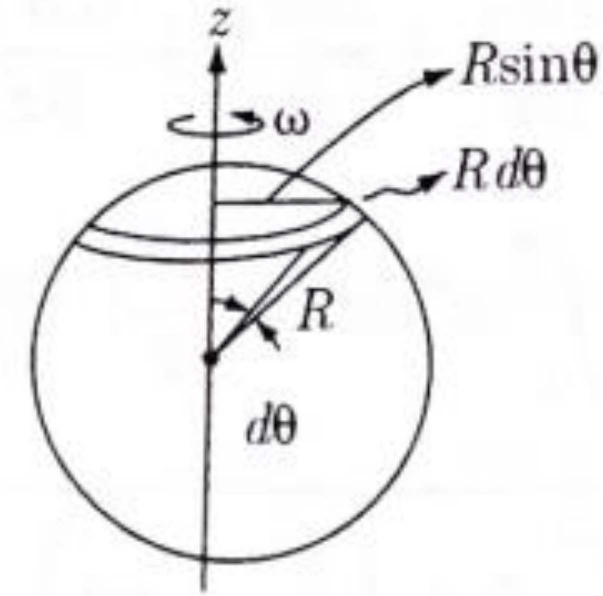
$$\mathbf{A}(\mathbf{r}) = \frac{-R^3\sigma\omega\sin\psi\mu_0\hat{\mathbf{y}}}{4\pi}(2\pi)\int_{-1}^{1}\frac{-\cos\theta'}{\sqrt{r^2+R^2-2rR\cos\theta'}}d(\cos\theta')$$

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0 R\sigma}{3}(\boldsymbol{\omega}\times\mathbf{r}) = \dfrac{\mu_0 R\omega\sigma}{3} r\sin\theta\hat{\boldsymbol{\phi}}, & r \le R \\ \dfrac{\mu_0 R^4\sigma}{3r^3}(\boldsymbol{\omega}\times\mathbf{r}) = \boxed{\dfrac{\mu_0 R^4\omega\sigma}{3}\dfrac{\sin\theta}{r^2}\hat{\boldsymbol{\phi}}, \ r \ge R} \end{cases}$$

Magnetic Dipole Field outside!

$$\mathbf{B} = \nabla\times\mathbf{A} = \frac{2}{3}\mu_0 R\sigma\omega\hat{\mathbf{z}} = \frac{2}{3}\mu_0 R\sigma\boldsymbol{\omega} \leftarrow \text{Uniform B-field inside}$$

Spinning Charged Sphere



$$dq = \sigma\left(2\pi R\sin\theta\right)Rd\theta$$

$$\Rightarrow I = \frac{dq}{dt} = dq\cdot\frac{\omega}{2\pi} = \sigma R^2\omega\sin\theta d\theta$$

$$dm = IdA = \sigma R^2\omega\sin\theta d\theta\left(\pi R^2\sin^2\theta\right)$$

$$\Rightarrow \mathbf{m} = \sigma\omega\pi R^4\hat{\mathbf{z}}\int_0^{\pi}\sin^3\theta d\theta = \frac{4}{3}\sigma\omega\pi R^4\hat{\mathbf{z}}$$

$$\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^2} = \frac{\mu_0}{4\pi r^2}\frac{4}{3}\sigma\omega\pi R^4\sin\theta\hat{\boldsymbol{\phi}} = \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\boldsymbol{\phi}}$$

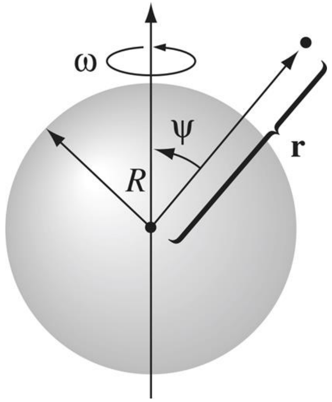
What is the scalar potential of a "pure" magnetic dipole?

Remark: 1. Pure dipole field is the exact solution outside!

2. Same as the uniformly-magnetized sphere.

Spinning Charged Sphere

The analogy with the electrical case: $\mathbf{p}/\varepsilon_0 \leftrightarrow \mu_0 \mathbf{m}$



$$\mathbf{E}_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{p}}{r^3} = -\nabla V,\ \text{with } V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{B}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{m}}{r^3} = -\nabla U,\ \text{with } U = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2}$$

$$\therefore U(\mathbf{r}) = \begin{cases} \dfrac{\mu_0\omega\sigma R^4}{3}\dfrac{\cos\theta}{r^2}, & \text{for } r > R \\ -\dfrac{2}{3}\mu_0\omega\sigma R r\cos\theta, & \text{for } r < R \end{cases}$$

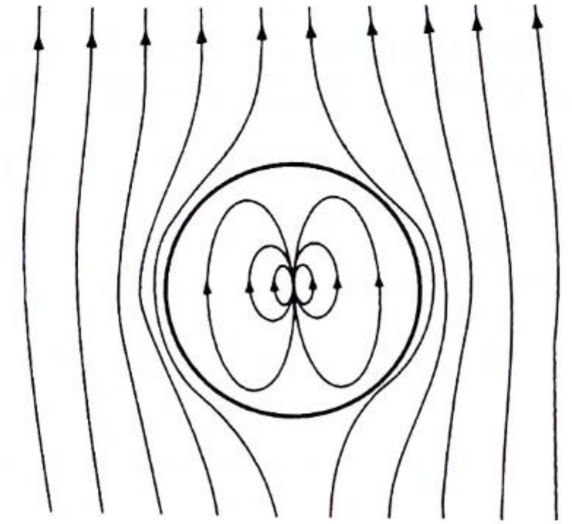
Discontinuity occurs on the surface! WHY?

→ Surface current

See Prob. 5.30, 5.54(c), 5.59, and 5.60 for discussing the rotating solid sphere.

Spinning Charged Sphere

Problem 5.57 A magnetic dipole $\mathbf{m} = -m_0\,\hat{\mathbf{z}}$ is situated at the origin, in an otherwise uniform magnetic field $\mathbf{B} = B_0\,\hat{\mathbf{z}}$. Show that there exists a spherical surface, centered at the origin, through which no magnetic field lines pass. Find the radius of this sphere, and sketch the field lines, inside and out.



$$\mathbf{B}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{m}}{r^3} = \frac{-\mu_0}{4\pi r^3}\left(2\cos\theta\,\hat{\mathbf{r}}+\sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

$$\Rightarrow \mathbf{B}\cdot\hat{\mathbf{r}} = B_0(\hat{\mathbf{z}}\cdot\hat{\mathbf{r}}) - \frac{\mu_0 m_0}{4\pi r^3}2\cos\theta = \left(B_0 - \frac{\mu_0 m_0}{2\pi r^3}\right)\cos\theta = 0\ \forall\theta$$

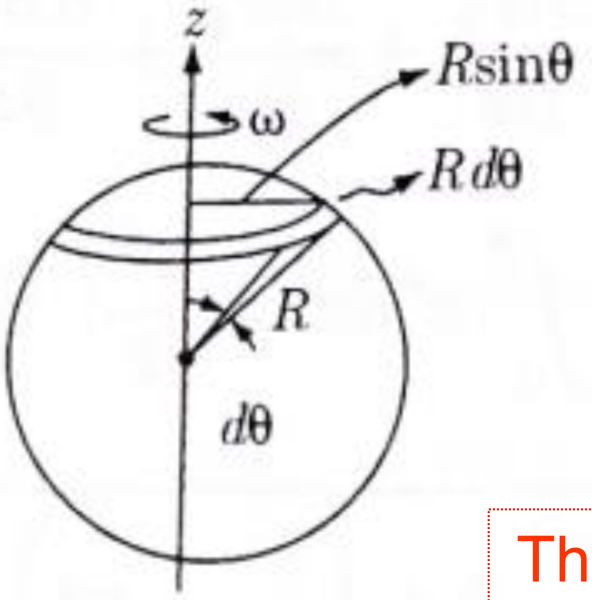
$$\therefore R = \left(\frac{\mu_0 m_0}{2\pi B_0}\right)^{1/3}$$

Can we model the spinning of electrons by this spinning charged sphere?

Spinning Charged Sphere

$$\left|\mathbf{m}\right| = \sigma\omega\pi R^4 \int_0^\pi \sin^3\theta d\theta = \frac{4\pi R^2\sigma}{3}\omega R^2$$

$$= \frac{Q}{3}\omega R^2 = \frac{Q}{3}\frac{2\pi}{T}R^2[\text{MKS}] = \frac{2\pi Q}{3Tc}R^2[\text{cgs}]$$



Rotation speed at the equator: $\frac{2\pi R}{T} = \frac{3mc}{QR} \sim 200c!$

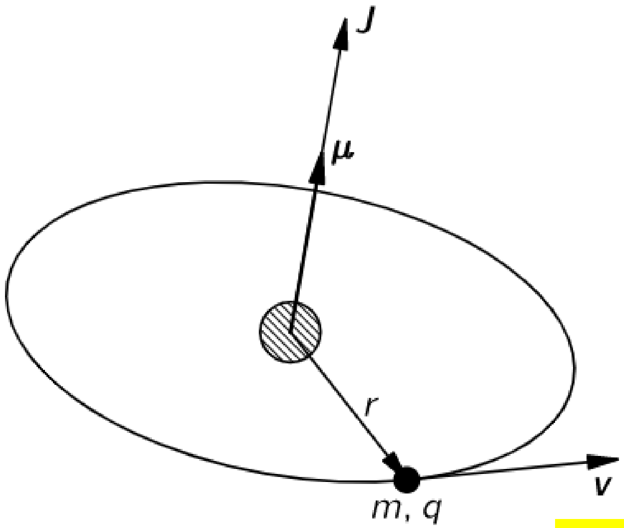
The spinning electron (particle?) is not "physically spinning" as the charged sphere does…… It's an *intrinsic property like charge*!

How exactly does an electron cause the magnetic moment?

Classical Gyromagnetic ratio

Classically, a magnetic moment of an electron in an atom is composed of

1. A moving electric charge of the electron forms a current → orbital motion of an electron around a nucleus generates a magnetic moment by Ampère's circuital law.
2. The inherent rotation, or spin, of the electron, has a spin magnetic moment.



1. Orbital angular momentum: $J = mvr$

2. Orbital magnetic moment: $\mu = IA = \dfrac{qT}{2\pi r}\pi r^2 = \dfrac{qvr}{2}$

$$\Rightarrow \mu = \frac{q}{2m}J \Rightarrow \boldsymbol{\mu}_{\mathbf{e}} = \frac{-q_e}{2m}\mathbf{J}_{\mathbf{e}} \ \text{(electron orbit)}$$

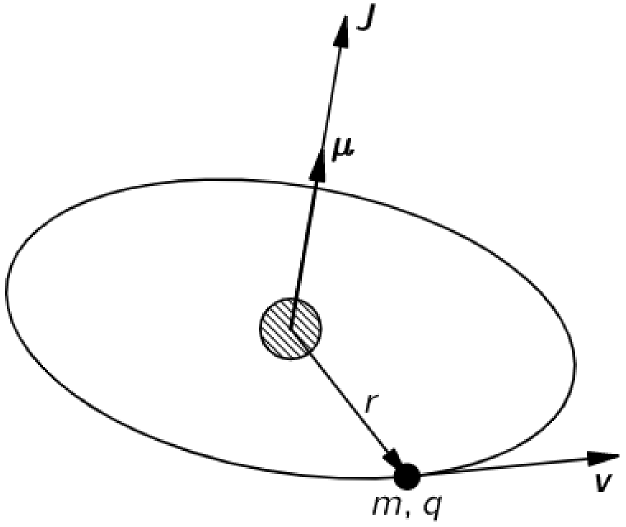
The ratio is independent of the velocity or the radius!

The electron also has a spin rotation about its own axis.

Classical Gyromagnetic ratio

Quantum mechanics:

$$\boldsymbol{\mu}_\mathbf{e} = -g\left(\frac{q_e}{2m}\right)\mathbf{J}_\mathbf{e}$$



Experiments indicate g~2. Dirac’s relativistic electron theory got the 2 right, and Feynman, Schwinger, and Tomonaga later calculated tiny further corrections. Determining the electron’s magnetic dipole moment remains the finest achievement of quantum electrodynamics and exhibits perhaps the most stunningly precise agreement between theory and experiment in all of physics. Incidentally, the quantity ($e\hbar/2m$) is called the Bohr magneton.

This indicates that “***All magnetic phenomena are NOT due to electric charges in motion***.”

Misleading Concept in Chap. 6

6.1.1 ■ Diamagnets, Paramagnets, Ferromagnets

pp. 266 in Griffiths, 4th edition

If you ask the average person what "magnetism" is, you will probably be told about refrigerator decorations, compass needles, and the North Pole—none of which has any obvious connection with moving charges or current-carrying wires. Yet all magnetic phenomena are due to electric charges in motion, and in fact, if you could examine a piece of magnetic material on an atomic scale you *would* find tiny currents: electrons orbiting around nuclei and spinning about their axes. For macroscopic ... all that we may treat them as magnetic dipo... ut because of the random orientation of the... plied, a net alignment of these magnetic d... s magnetically polarized, or **magnetized**.

You should bear in mind that this statement is correct only in classical electrodynamics and is WRONG in general!

Misleading Concept in Chap. 6

—*Feynman Lectures Vol. 2, Sec.34.1* —

"It is not possible to understand the magnetic effects of materials in an honest way from the point of view of classical physics. Such magnetic effects are a completely quantum mechanical phenomenon."

Even Ampere and Maxwell appreciated that something beyond their electromagnetism was needed to understand magnetic materials. Something beyond, of course, turned out to be quantum mechanics and the **intrinsic magnetic moment of the electron**.

See J. C. Maxwell, Electricity and Magnetism, Chap. XXII.

All magnetic phenomena are NOT due to electric charges in motion

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Reply to: All magnetic phenomena are NOT due to electric charges in motion [Am. J. Phys. 90, 7-8 (2022)]

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Could the classical theories be extended to include point particles with intrinsic spin and permanent magnetic moments? Interesting question! Semi-classical theories of spin were introduced in the 1940s (and perhaps earlier) and are under active consideration to the present day*. They tend to be awkward and esoteric, but never mind—they are certainly **worth exploring**. Fahy and O'Sullivan make a valid and important point. The bald assertion that **"all magnetic phenomena are due to electric charges in motion" should always be prefaced by "in classical electrodynamics"** and accompanied by a clear acknowledgment that the *quantum story is quite different, incorporating intrinsic spin (for which the naive classical picture of* ***a tiny rotating sphere is fundamentally defective***).

*See, for instance, J. Barandes, "On magnetic forces and work," Found. Phys. 51, 79–96 (2021).