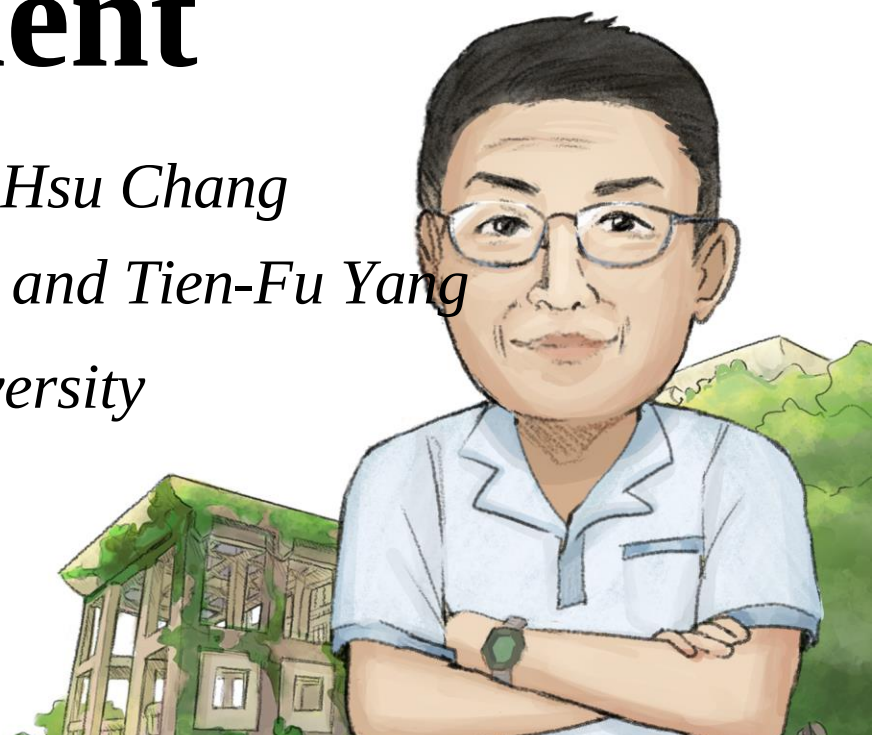




Chap.3 Supplement

*For the EM Course Lectured by Prof. Tsun-Hsu Chang
Teaching Assistants: Hung-Chun Hsu, Yi-Wen Lin, and Tien-Fu Yang
2022 Fall at National Tsing Hua University*





Homework Exercises

Griffiths:

11, 13, 16, 20, 27, 43, 54, 56,

7, 8, 12, 19, 23, 28, 29, 32, 44, 49





Prob 3.2 in Griffiths

Problem 3.2 In one sentence, justify **Earnshaw's Theorem**: *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.* As an example, consider the cubical arrangement of fixed charges in Fig. 3.4. It *looks*, off hand, as though a positive charge at the center would be suspended in midair, since it is repelled away from each corner. Where is the leak in this “electrostatic bottle”? [To harness nuclear fusion as a practical energy source it is necessary to heat a plasma (soup of charged particles) to fantastic temperatures—so hot that contact would vaporize any ordinary pot. Earnshaw's theorem says that electrostatic containment is also out of the question. Fortunately, it *is* possible to confine a hot plasma magnetically.]

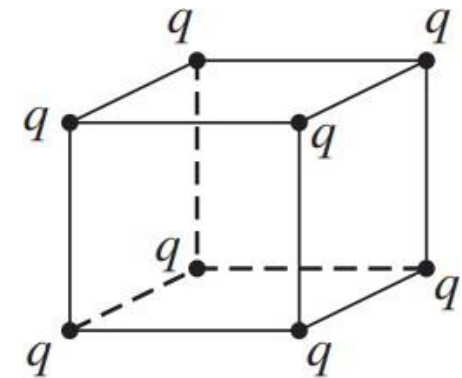
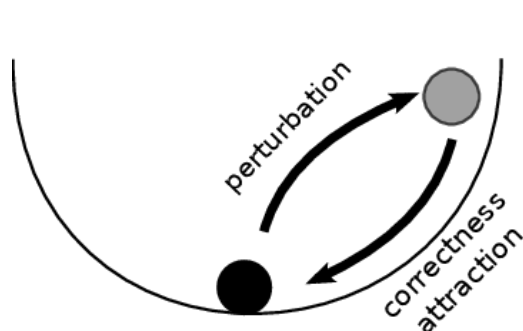


FIGURE 3.4

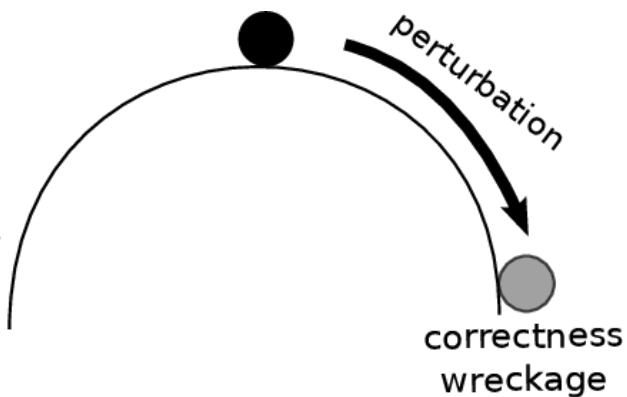




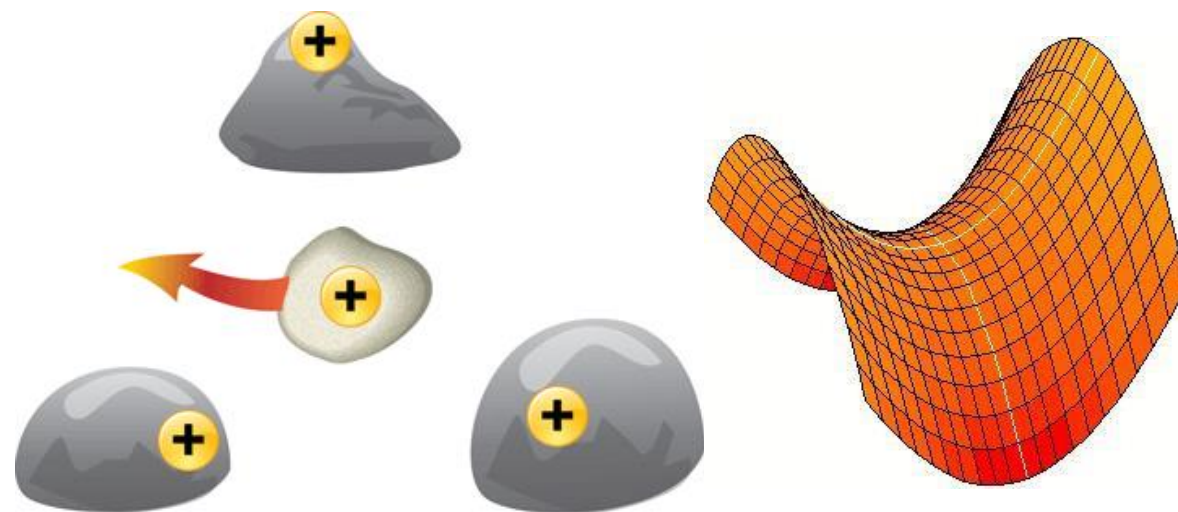
Earnshaw's Theorem



Stable equilibrium



Unstable equilibrium



Picture Credit: B. Danglot et al., *Empir. Softw. Eng.* 23, 2086 (2018).





Earnshaw's Theorem

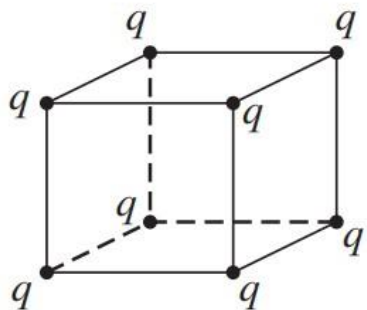


FIGURE 3.4

NO WAY!



Stable equilibrium: small perturbations ("pushes") on the particle in any direction should not break the equilibrium.



Particle "falls back" to its previous position



Force field lines around the particle's equilibrium position should all point inward, toward that position.



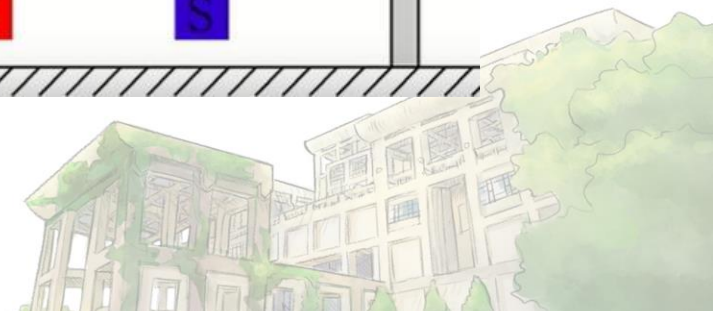
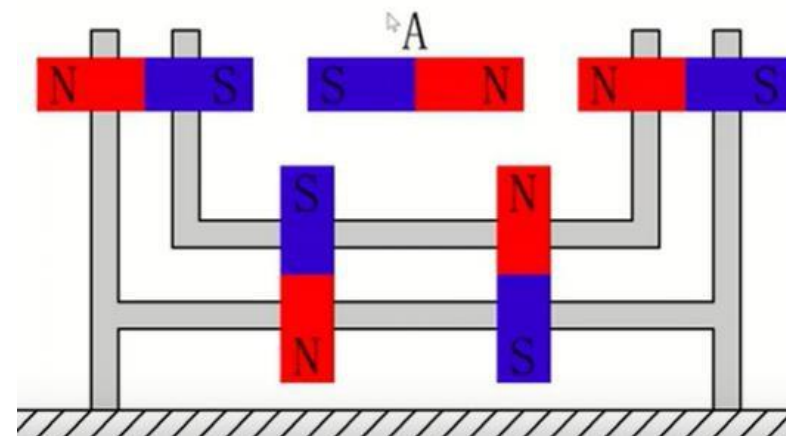
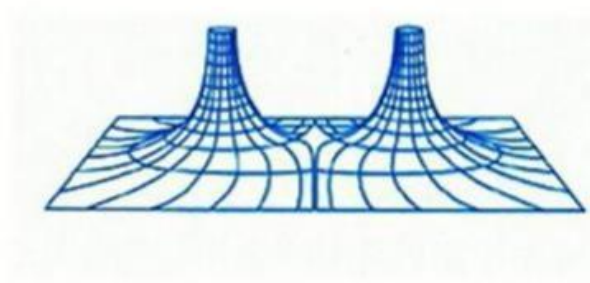
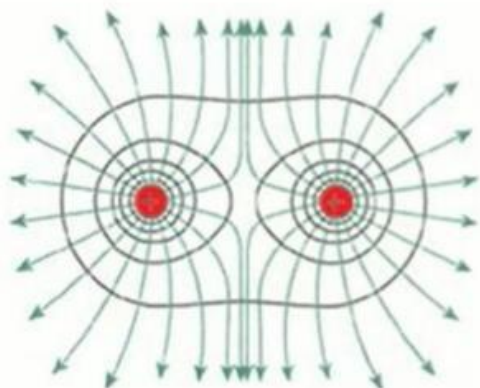
Divergence of the field at that point must be negative.

Gauss's law says that the divergence of any possible electric force field is zero in free space.



Earnshaw's Theorem

Earnshaw's theorem applies to classical inverse-square law forces (electric and gravitational) and also to the magnetic forces of permanent “hard magnets” (the magnets do not vary in strength with external fields). **Earnshaw's theorem forbids magnetic levitation in many common situations. (Hmm...)**



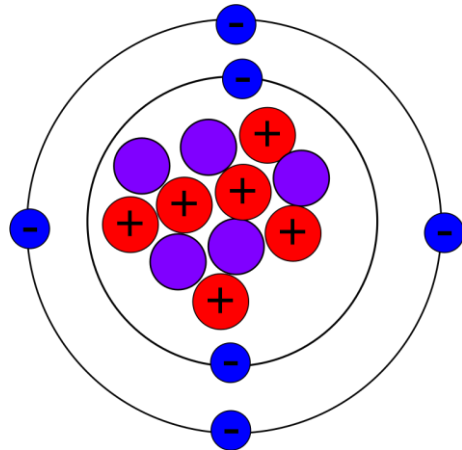


Effect on physics

Configurations of classical charged particles orbiting one another are unstable due to losses of energy by electromagnetic radiation, which, for quite some time, **led to the puzzling question of why matter stays together.**

Static configurations $\rightarrow$ unstable (Earnshaw's theorem)

Electrodynamic configurations $\rightarrow$ radiate energy and decay (**WHY?**)



Quantum Mechanical Explanation

Existence of **stationary (non-radiative) states** where the electron has a **nonzero momentum** (not actually static).

$\rightarrow$ Copenhagen interpretation (probability!)

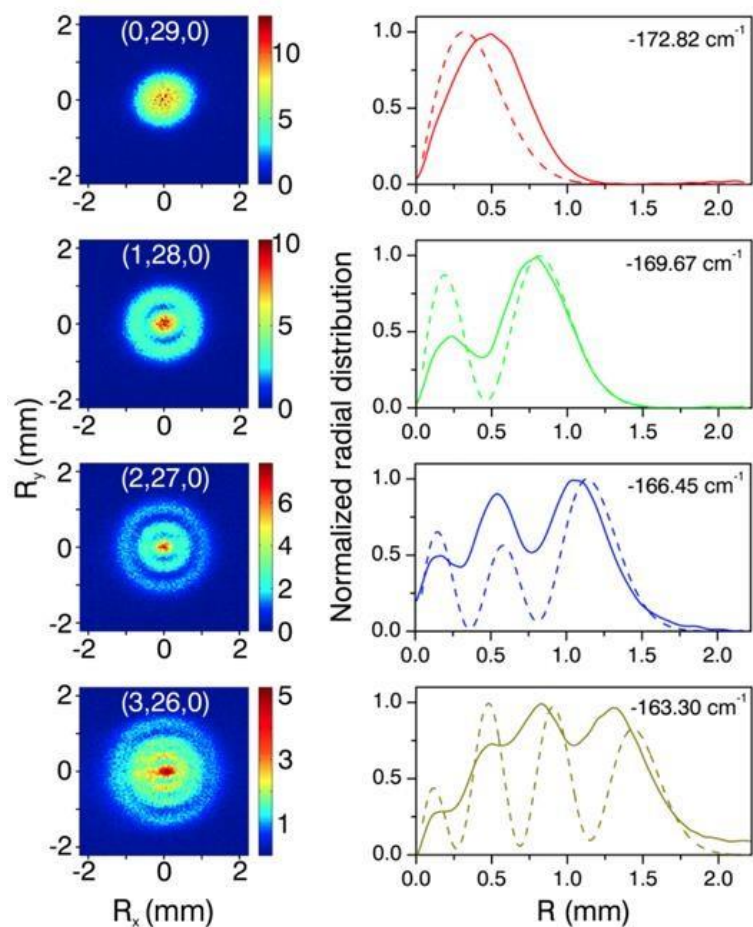
$\rightarrow$ Pauli exclusion principle + discrete electron orbitals

$\rightarrow$ Making bulk matter rigid



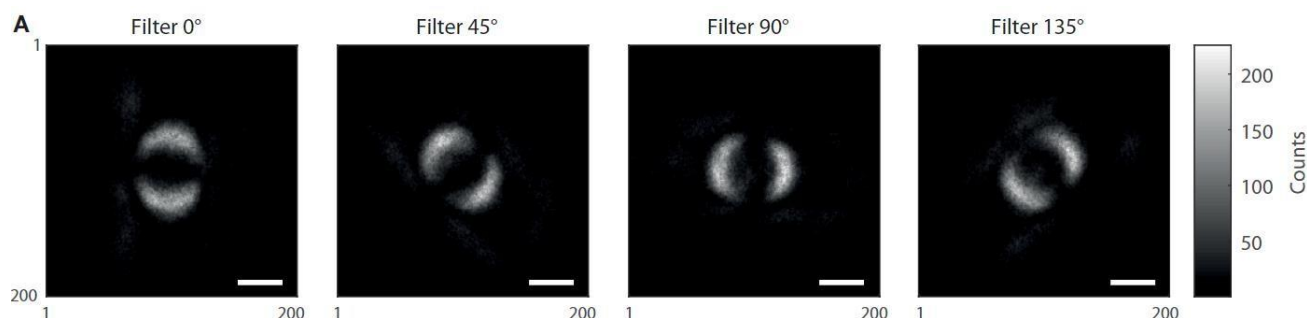
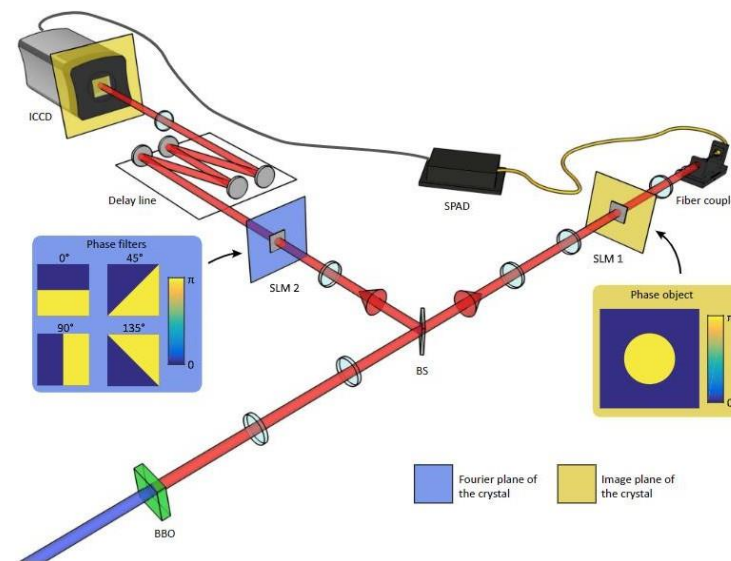


Effect on physics



First image of Hydrogen Atom

Ref: A. S. Stodolna et al., *Phys. Rev. Lett.* **110**, (2013).



First image of quantum entanglement

Ref: P.-A. Moreau et al., *Sci. Adv.* **5**, (2019).



Quick Argument on EM Radiation

Electrodynamic configurations $\rightarrow$ radiate energy and decay (WHY?)

If electromagnetic energy is **not gained from or lost** to other forms of energy within some region
 $\rightarrow$ Locally conserved within the region
 $\rightarrow$ Continuity Equation

$$\nabla \cdot \mathbf{S} = -\frac{\partial}{\partial t} u$$

Energy Density $\nearrow$

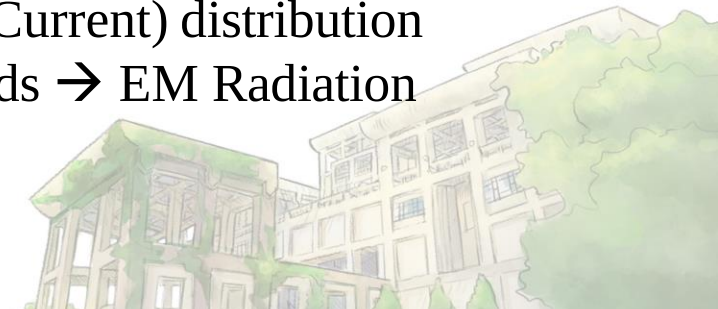
$\downarrow$

Energy flux / Power flow: $\mathbf{S} = \mathbf{E} \times \mathbf{B}$

Time-varying Electromagnetic Fields $\rightarrow$ Radiation

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

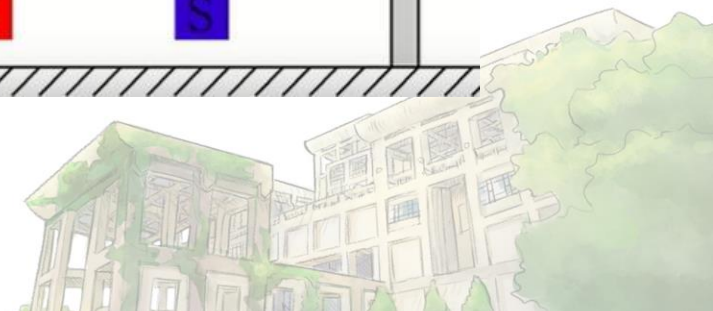
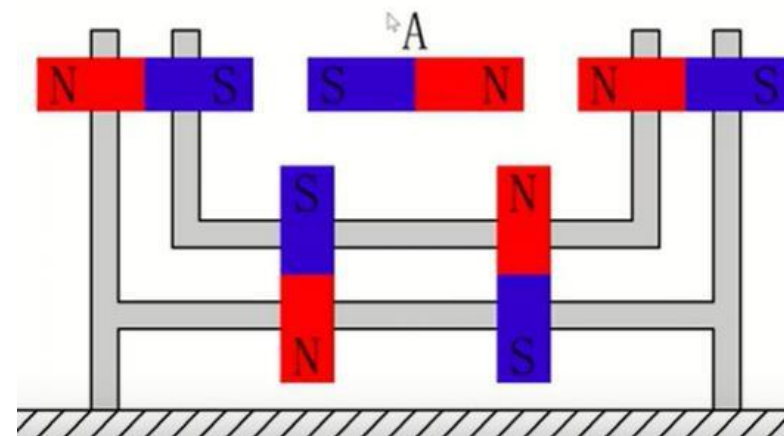
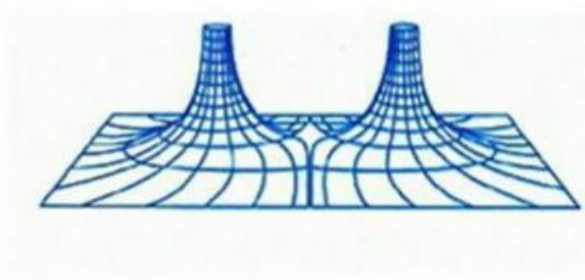
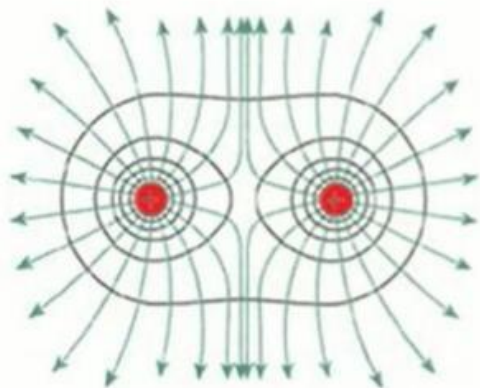
Time-varying Charge (or Current) distribution
 $\rightarrow$ Time-varying EM Fields $\rightarrow$ EM Radiation





Earnshaw's Theorem

Earnshaw's theorem applies to classical inverse-square law forces (electric and gravitational) and also to the magnetic forces of permanent “hard magnets” (the magnets do not vary in strength with external fields). **Earnshaw's theorem forbids magnetic levitation in many common situations. (Hmm...)**





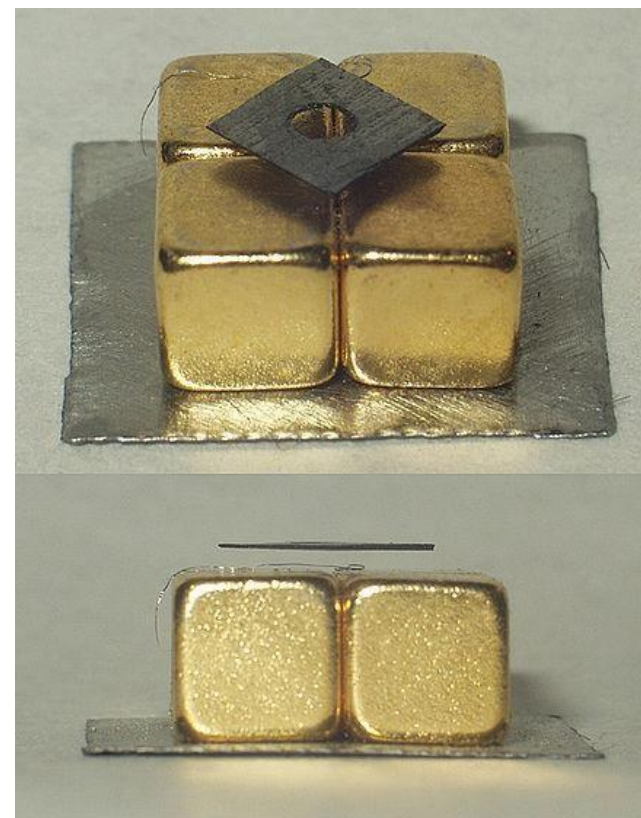
Magnetic Levitation

Earnshaw's theorem has no exceptions for **non-moving** **permanent** ferromagnets.

Magnetic levitation with a feedback loop.



Pseudo-levitation with a mechanical support



Diamagnetic levitation of pyrolytic carbon



Magnetic Levitation

Earnshaw's theorem has no exceptions for **non-moving** **permanent** ferromagnets.



Spin-stabilized
magnetic levitation





Magnetic Levitation

Levitation of a human being requires a magnetic field of ~45Tesla!

Water: predominantly diamagnetic → Levitation of live animals



Andre Geim
(1958~)

“Frankly, I value both my Ig Nobel prize and Nobel prize at the same level and for me, Ig Nobel prize was the manifestation that I can take jokes. [A little bit of self-deprecation always helps.](#)”





Electrodynamic Ion Traps





Quantum Computing

