Chap.3 Supplement

For the EM Course Lectured by Prof. Tsun-Hsu Chang

Teaching Assistants: Hung-Chun Hsu, Yi-Wen Lin, and Tien-Fu Yang

2022 Fall at National Tsing Hua University

Homework Exercises

Griffiths:
11, 13, 16, 20, 27, 43, 54, 56,
7, 8, 12, 19, 23, 28, 29, 32, 44, 49

Prob 3.2 in Griffiths

Problem 3.2 In one sentence, justify **Earnshaw's Theorem:** *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.* As an example, consider the cubical arrangement of fixed charges in Fig. 3.4. It *looks,* off hand, as though a positive charge at the center would be suspended in midair, since it is repelled away from each corner. Where is the leak in this "electrostatic bottle"? [To harness nuclear fusion as a practical energy source it is necessary to heat a plasma (soup of charged particles) to fantastic temperatures—so hot that contact would vaporize any ordinary pot. Earnshaw's theorem says that electrostatic containment is also out of the question. Fortunately, it *is* possible to confine a hot plasma magnetically.]

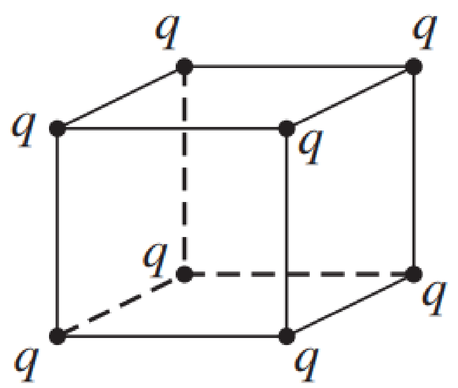
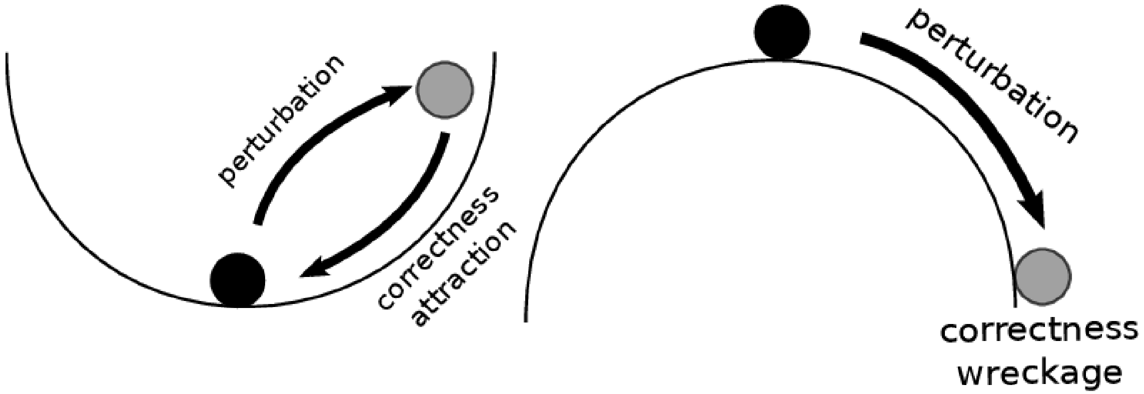


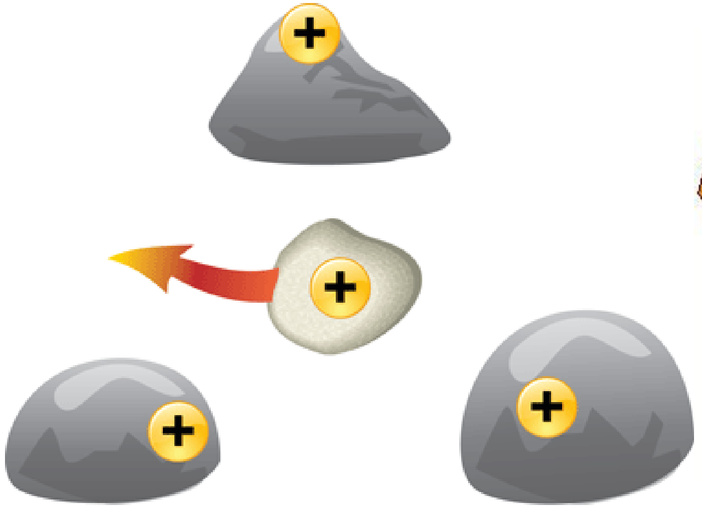
FIGURE 3.4

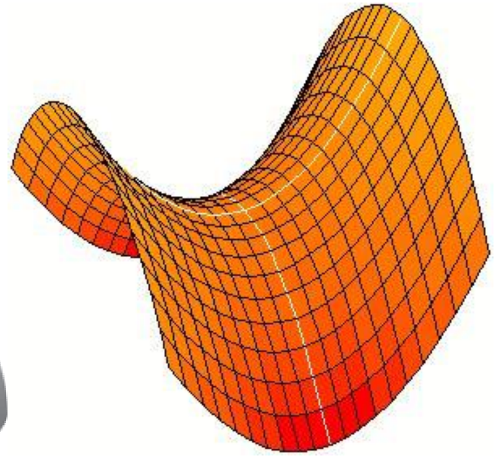
Earnshaw's Theorem



Stable equilibrium | Unstable equilibrium

Picture Credit: B. Danglot et al., *Empir. Softw. Eng.* **23**, 2086 (2018).





Earnshaw’s Theorem

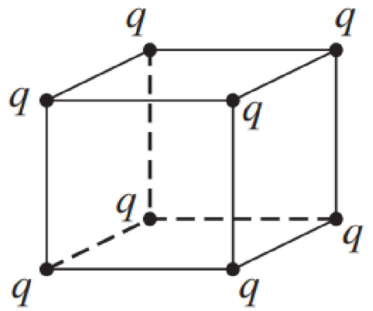
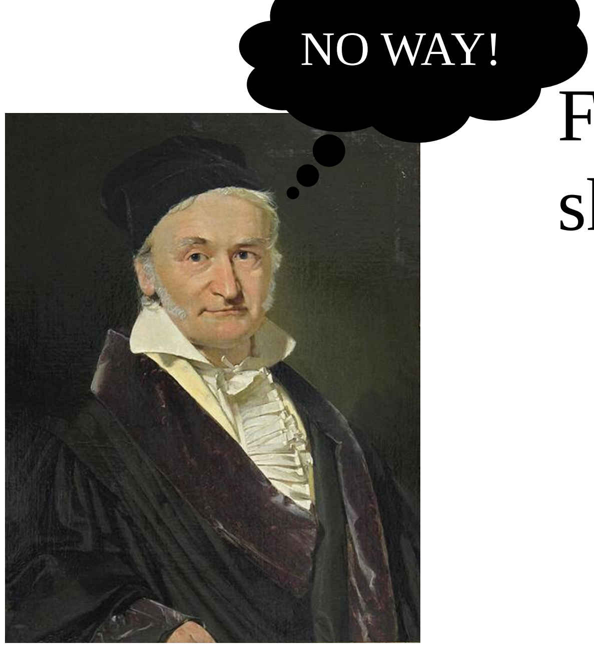


FIGURE 3.4



Stable equilibrium: small perturbations ("pushes") on the particle in any direction should not break the equilibrium.

⬇

Particle "falls back" to its previous position

⬇

Force field lines around the particle's equilibrium position should all point inward, toward that position.

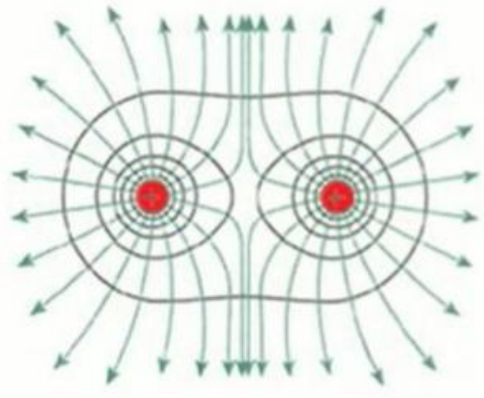
⬇

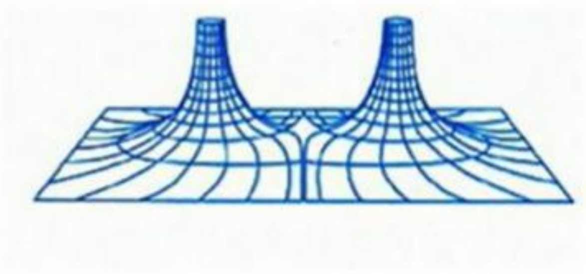
Divergence of the field at that point must be negative.

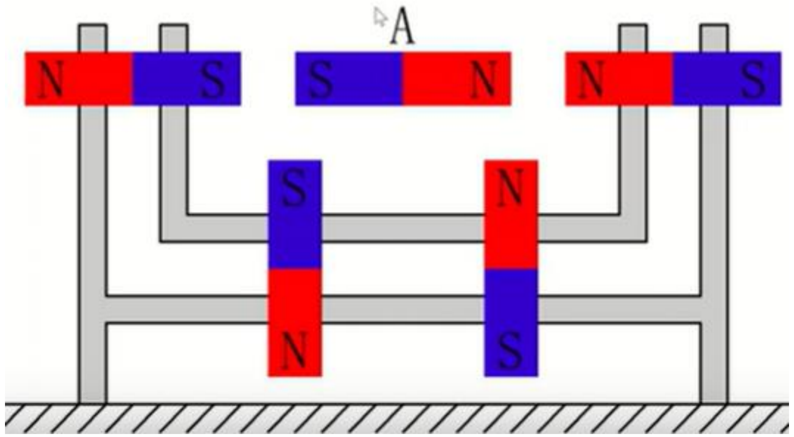
Gauss's law says that the divergence of any possible electric force field is zero in free space.

Earnshaw’s Theorem

Earnshaw's theorem applies to classical inverse-square law forces (electric and gravitational) and also to the magnetic forces of permanent “hard magnets” (the magnets do not vary in strength with external fields). Earnshaw's theorem forbids magnetic levitation in many common situations. (Hmm…)





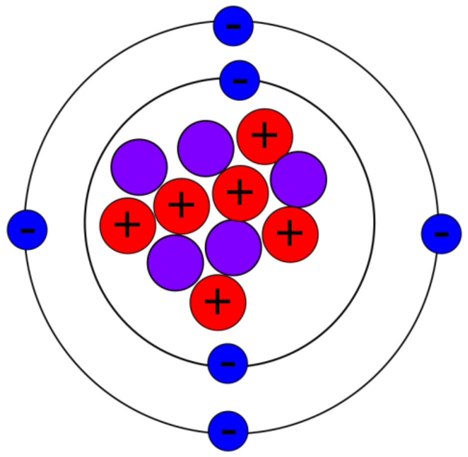


Effect on physics

Configurations of classical charged particles orbiting one another are unstable due to losses of energy by electromagnetic radiation, which, for quite some time, led to the puzzling question of why matter stays together.

Static configurations → unstable (Earnshaw's theorem)

Electrodynamic configurations → radiate energy and decay (WHY?)



Quantum Mechanical Explanation

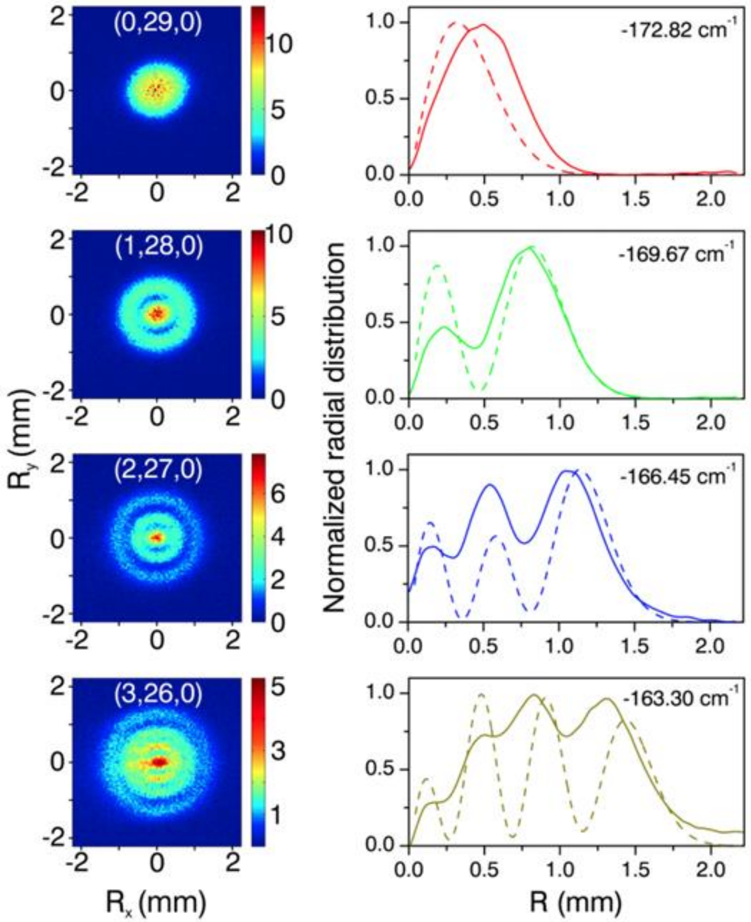
Existence of stationary (non-radiative) states where the electron has a nonzero momentum (not actually static).

→ Copenhagen interpretation (probability!)

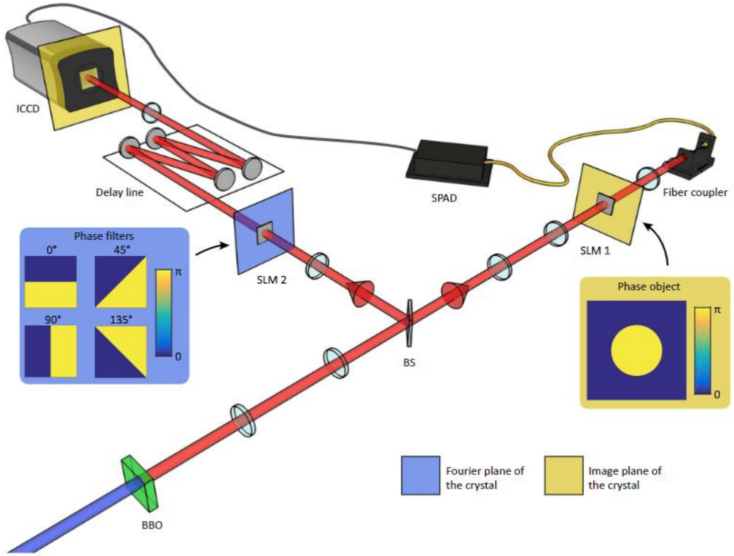
→ Pauli exclusion principle + discrete electron orbitals

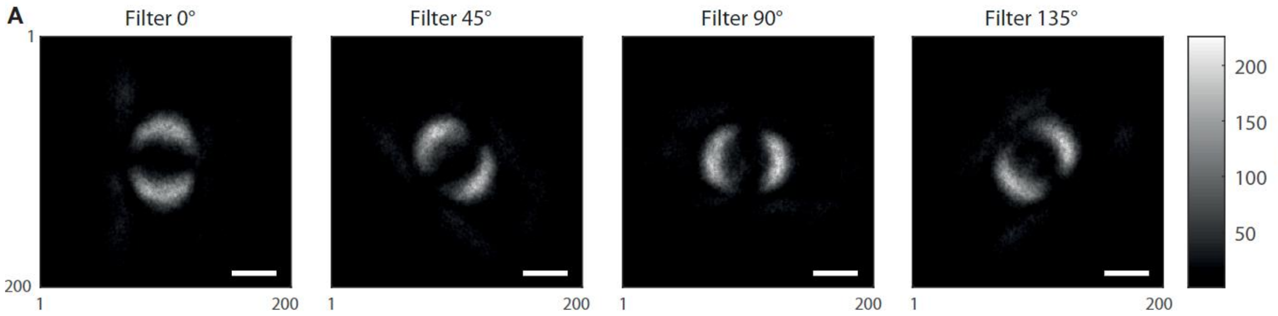
→ Making bulk matter rigid

Effect on physics



First image of Hydrogen Atom
Ref: A. S. Stodolna et al., *Phys. Rev. Lett.* **110**, (2013).





First image of quantum entanglement
Ref: P.-A. Moreau et al., *Sci. Adv.* **5**, (2019).

Quick Argument on EM Radiation

Electrodynamic configurations → radiate energy and decay (WHY?)

If electromagnetic energy is not gained from or lost to other forms of energy within some region
→ Locally conserved within the region
→ Continuity Equation

Energy Density

$$\nabla \cdot \mathbf{S} = -\frac{\partial}{\partial t} u$$

Energy flux / Power flow: $\mathbf{S} = \mathbf{E} \times \mathbf{B}$

Time-varying Electromagnetic Fields → Radiation

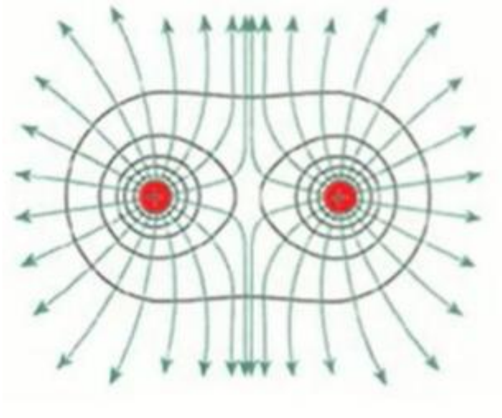
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

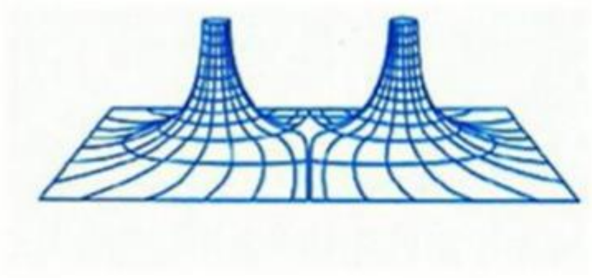
$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

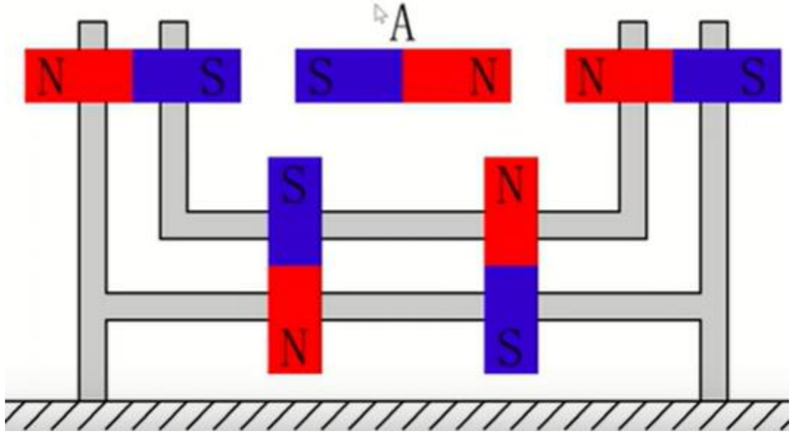
Time-varying Charge (or Current) distribution
→ Time-varying EM Fields → EM Radiation

Earnshaw’s Theorem

Earnshaw's theorem applies to classical inverse-square law forces (electric and gravitational) and also to the magnetic forces of permanent “hard magnets” (the magnets do not vary in strength with external fields). Earnshaw's theorem forbids magnetic levitation in many common situations. (Hmm…)





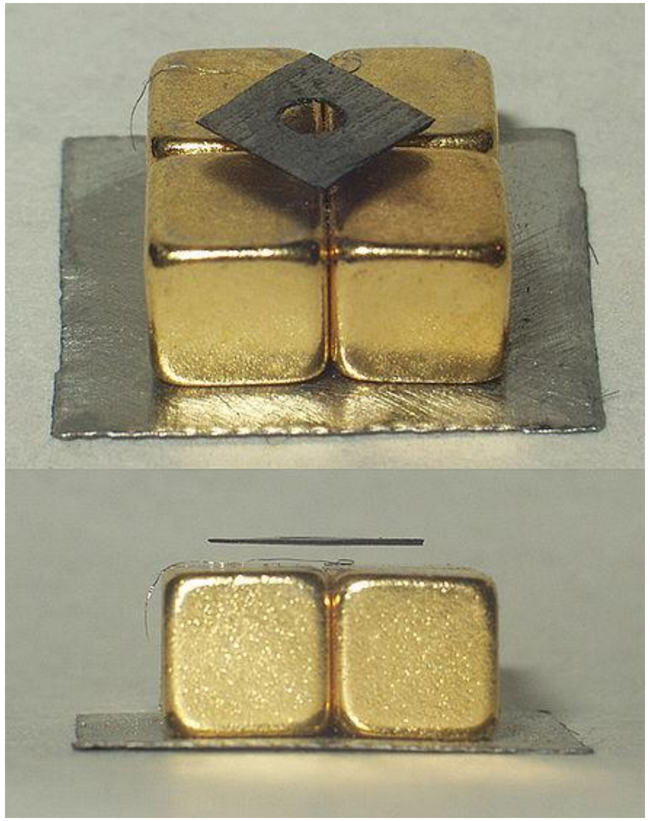


Magnetic Levitation

Earnshaw’s theorem has no exceptions for non-moving permanent ferromagnets.

Magnetic levitation with a feedback loop.

Pseudo-levitation with a mechanical support



Diamagnetic levitation of pyrolytic carbon

Source: https://www.youtube.com/watch?v=Tuat0uwdz_A

Ref: M. D. Simon et al., *Am. J. Phys.* **65**, 286 (1997).

Magnetic Levitation

Earnshaw's theorem has no exceptions for non-moving permanent ferromagnets.

Spin-stabilized magnetic levitation

Source: https://www.youtube.com/watch?v=KlJsVqc0ywM

Magnetic Levitation

Levitation of a human being requires a magnetic field of ~45Tesla!

Water: predominantly diamagnetic → Levitation of live animals

Andre Geim
(1958~)

"Frankly, I value both my Ig Nobel prize and Nobel prize at the same level and for me, Ig Nobel prize was the manifestation that I can take jokes. A little bit of self-deprecation always helps."

Source: https://www.youtube.com/watch?v=tT9e_QrrJzM

Electrodynamic Ion Traps

Source: https://www.youtube.com/watch?v=aV1wL5jsfRU

Quantum Computing